Preface

This book has grown out of an undergraduate course developed and taught by us in MIT’s Department of Electrical Engineering and Computer Science. Our course is typically taken by third- and fourth-year undergraduate students from many engineering branches, as well as undergraduate and graduate students from applied science. There are two formal prerequisites for the course, and for this book: an introductory subject in time- and frequency-domain analysis of signals and systems, and an introductory subject in probability. These two subjects are typically taken by most engineering students early in their degree programs. The signals and systems subject almost invariably builds on an earlier course in differential equations, ideally with some basic linear algebra folded into it.

In many engineering departments, students with a strong interest in applied mathematics have then traditionally gone on to a more specialized undergraduate subject in control, signal processing, or communication. In addition to being specialized, such subjects often focus on deterministic signals and systems. Our aim instead was to build broadly on the prerequisite material, folding together signals, systems, and probability in ways that could make our course relevant and interesting to a wider range of students. The course could then serve both as a terminal undergraduate subject and as a sufficiently rigorous basis for more advanced undergraduate subjects or introductory graduate subjects in many engineering and applied science departments.

The course that gave rise to this book teaches students about signals and signal descriptions that are typically new to them, for example, random signals and their characterization through correlation functions and power spectral densities. It introduces them to new kinds of systems and system properties, such as state-space models, reachability and observability, optimum filters, and group delay. And it highlights model-based approaches to inference, particularly in the context of state estimation, signal estimation, and signal detection.
Although some parts of our course are well covered by existing textbooks, we did not find one that fit our needs across the range of topics. This led to lecture notes, which was the easier part, and then eventually this book. In the process, we continually experimented with and refined the content and order of presentation. Along the way we also at times included other material or excluded some that is now back in the book. Among the conclusions of these experiments was that we did not have time in a one-semester class to fold in even basic notions of information theory, despite its central importance to communication systems and, more generally, to inference.

As suggested in the Prologue to this book, signals, systems and probability have been and will continue to be usefully combined in studying fields such as signal processing, control, communication, financial engineering, biomedicine, and many others that involve dynamically varying processes operating in continuous or discrete time, and affected by disturbances, noise, or uncertainty. This premise forms the basis for the overall organization and content of our course and this text.

The book can be thought of as comprising four parts, outlined below. A more detailed overview of the individual chapters is captured in the table of contents. Chapters 1 and 2 present a brief review of the assumed prerequisites in signals and linear time-invariant (LTI) systems, though some portions of the material may be less familiar. A key intent in these chapters is to establish uniform notation and concepts on which to build in the chapters that follow. Chapter 3 discusses the application of some of this prerequisite material in the setting of digital communication by pulse amplitude modulation.

Chapters 4–6 are devoted to state-space models, concentrating on the single-input single-output LTI case. The development is largely built around the eigenmodes of such systems, under the simplifying assumption of distinct natural frequencies. This part of the book introduces the idea of model-based inference in the context of state observers for LTI systems, and examines associated feedback control strategies.

Chapters 7–9 provide a brief review of the assumed probability prerequisites, including estimation and hypothesis testing for static random variables. As with Chapters 1 and 2, we felt it important to set out our notation and perspectives on the concepts while making contact with what students might have encountered in their earlier probability subject. Again, some parts of this material, particularly on hypothesis testing, may be previously unfamiliar to some students.

In Chapters 10–13, we characterize wide-sense stationary random signals, and the outputs that result from LTI filtering of such signals. The associated properties and interpretations of correlation functions and power spectral densities are then used to study canonical signal estimation and signal detection problems. The focus in Chapter 12 is on linear minimum mean square error signal estimation, i.e., Wiener filtering. In Chapter 13, the emphasis is on signal detection for which optimum solutions involve matched filtering.

As is often said, the purpose of a course is to uncover rather than to cover a subject. In this spirit, each chapter includes a final section with some
suggestions for further reading. Our intent in these brief sections is not to be exhaustive but rather to suggest the wealth of learning opened up by the material in this text. We have pointed exclusively to books rather than to papers in the research literature, and have in each case listed only a fraction of the books that could have been listed.

Each chapter contains a rich set of problems, which have been divided into Basic, Advanced, and Extension. Basic problems are likely to be easy for most students, while the Advanced problems may be more demanding. The Extension problems often involve material somewhat beyond what is developed in the chapter. Certain problems require simulation or computation using some appropriate computational package. Given the variety and ubiquity of such packages, we have intentionally not attempted to structure the computational exercises around any specific platform.

There is more material in this book than can be taught comfortably in a one-semester course. This allows the instructor or self-learner to choose different routes through the text, and over the years we have experimented with various paths. For a course that is more oriented towards communication or signal processing, Chapters 4, 5 and 6 (state-space models) can be omitted, or addressed only briefly. For a course with more of a control orientation, Chapter 3 (pulse amplitude modulation), Chapter 9 (hypothesis testing) and Chapter 13 (signal detection) can perhaps be considered optional.

A third version of the course, and the one that we currently teach, is outlined in a little more detail below. This version involves two weekly lectures over a semester of approximately thirteen weeks. The lectures are interleaved with an equal number of small-group recitation sections, devoted to more interactive discussion of specific problems that illustrate the lectures and help address the weekly homework. In addition, we staff optional small-group tutorials. Finally an optional evening “common room” that we run several times each week allows students in the class to congregate and interact with each other and with a member of the teaching staff while they work on their homework.

In our teaching in general, we like to emphasize that the homework is intended to provide an occasion for learning and engaging with the concepts and mechanics, rather than being an exam. We recommend that the end-of-chapter problems in this book be approached in the same spirit. In particular, we encourage students to work constructively together, sharing insights and approaches. Our grading of the problems is primarily for feedback to the students and to provide some accountability and motivation. The course does typically have a midterm quiz and a final exam, and many of the end-of-chapter problems in this text were first created as quiz or exam problems. There are also many possibilities for term projects that can grow out of the material in the class, if desired.

An introductory lecture in the same spirit as the Prologue to this text is followed by a brief review of the signals and systems material in Chapter 1. The focus in class is on what might be less familiar from the prerequisite subject, and students are tasked with reviewing the rest on their own, guided by appropriate homework problems. We then move directly to the state-space
material in Chapters 4, 5 and 6. Even if students have had some prior exposure to state-space models, there is much that is likely to be new to them here, though they generally relate easily to the material. We have not held students responsible for the more detailed proofs, such as those on eigenvalue placement for LTI observers or state feedback, but do expect them to develop an understanding of the relevant results and how to apply them to small examples. An important lesson from the state-space observer framework is the role of a system model in going from measured signals to inferences about the system.

Our course then turns to probabilistic models and random signals. The probability review in Chapter 7 is mostly woven into lectures covering minimum mean square error (MMSE) and linear MMSE (LMMSE) estimation, which are dealt with in Chapter 8. In order to move more quickly to random signals rather than linger on review of material from the prerequisite probability course, we defer the study of hypothesis testing in Chapter 9 to the end of the course, using it as a lead-in to the signal detection material in Chapter 13. Part of the rationale is also that Chapters 9 and 13 are devoted to making inferences about discrete random quantities, namely the hypotheses, whereas Chapters 8 and 12 on (L)MMSE estimation deal with inferences about continuous random variables. We therefore move directly from Chapter 8 to Chapter 10, studying random signals, i.e., stochastic processes, focusing on the time-domain analysis of wide-sense stationary (WSS) processes, and LTI filtering of such processes.

The topic of power spectral density in Chapter 11 connects back to the development of transforms and energy spectral density in Chapter 1, and also provides the opportunity to refer to relevant sections of Chapter 2 on all-pass filters and spectral factorization. These topics are again important in Chapter 12, on LMMSE (or Wiener) filtering for WSS processes. In most offerings of the course, we omit the full causal Wiener filter development, instead only treating the case of prediction of future values of a process from past values of the same process.

The last part of the course refers strongly back to Chapter 3, using the context of digital communication via pulse amplitude modulation to motivate the hypothesis testing problem. The return to Chapter 3 can also involve reference to the material in Chapter 2 on channel distortions and group delay. The hypothesis testing paradigm is then treated as in Chapter 9. This serves as the foundation for the study of signal detection in the last chapter, Chapter 13.

The breadth of this book, and the different backgrounds we brought to the project, meant that we had much to learn from each other. We also learn each term from the very engaged students, teaching assistants and faculty colleagues who are involved in the course, as well as from the literature on the subjects treated here. This book will have amply met its objectives if it sparks and supports a similar voyage of discovery in its readers, as they construct their own individual re-synthesis of the themes of signals, systems and inference.

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