TENTH EDITION

Elementary and Middle School Mathematics

Teaching Developmentally

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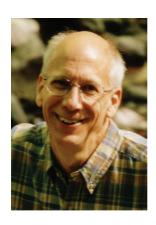
About the Authors

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The late John A. Van de Walle was a professor emeritus at Virginia Commonwealth University. He was a leader in mathematics education who regularly gave professional development workshops for K–8 teachers in the United States and Canada focused on mathematics instruction that engaged students in mathematical reasoning and problem solving. He visited and taught in many classrooms and worked with teachers to implement student-centered mathematics lessons. He co-authored the *Scott Foresman-Addison Wesley Mathematics K–6* series and contributed to the original Pearson School mathematics program *enVisionMATH*. Additionally, John was very active in the National Council of Teachers of Mathematics (NCTM), writing book chapters and journal articles, serving on the board of directors, chairing the educational materials committee, and speaking at national and regional meetings.

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Brief Contents

PART I Teaching Mathematics: Foundations and Perspectives

PART II Development of Mathematical Concepts and Procedures

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PART I   Teaching Mathematics: Foundations and Perspectives

The fundamental core of effective teaching of mathematics combines an understanding of how students learn, how to promote that learning by teaching through problem solving, and how to plan for and assess that learning daily. That is the focus of these first six chapters, providing discussion, examples, and activities that develop the core ideas of learning, teaching, planning, and assessment for each and every student.

PART II   Teaching Student-Centered Mathematics

Each of these chapters *applies* the core ideas of Part I to the content taught in K–8 mathematics. Clear discussions are provided for how to teach the topic, what a learning progression for that topic might be, and what worthwhile tasks look like. Hundreds of problem-based, engaging tasks and activities are provided to show how the concepts can be developed with students. These chapters are designed to help you develop pedagogical strategies now, and serve as a resource and reference for your teaching now and in the future.

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Preface

All students can learn mathematics with understanding. It is through the teacher's actions that every student can have this experience. We believe that teachers must create a classroom environment in which students are given opportunities to solve problems and work together, using their ideas and strategies, to solve them. Effective mathematics instruction involves posing tasks that engage students in the mathematics they are expected to learn. Then, by allowing students to interact with and productively struggle with *their own mathematical ideas* and *their own strategies*, they will learn to see the connections among mathematical topics and the real world. Students value mathematics and feel empowered to use it.

Creating a classroom in which students design solution pathways, engage in productive struggle, and connect one mathematical idea to another, is complex. Questions arise, such as, "How do I get students to wrestle with problems if they just want me to show them how to do it? What kinds of tasks lend themselves to this type of engagement? Where can I learn the mathematics content I need to be able to teach in this way?" With these and other questions firmly in mind, we have several objectives in the tenth edition of this textbook:

1. Illustrate what it means to teach mathematics using a problem-based approach.
2. Serve as a go-to reference for all of the mathematics content suggested for grades preK–8 as recommended in the Common Core State Standards (NGA Center & CCSSO, 2010) and in standards used in other states, and for the research-based strategies that illustrate how students best learn this content.
3. Present a practical resource of robust, problem-based activities and tasks that can engage students in the use of significant mathematical concepts and skills.
4. Focus attention on student thinking, including the ways students might reason about numbers, and possible challenges and misconceptions they might have.

We are hopeful that you will find that this book is a valuable resource for teaching and learning mathematics!

New to this Edition

The following are highlights of the most significant changes in the tenth edition.

Common Challenges and Misconceptions

Every chapter in Part II offers at least one table that summarizes common challenges students encounter in learning that topic (Chapter 15, Fraction Operations has three). The table includes the challenge, provides an example of what that might look like in either a sample of student work or a statement, and then offers some brief ideas of what you might do to help. Knowing common student challenges and misconceptions is a critical part of planning and can greatly influence how a lesson is structured and what problems you use. The research from many sources has been merged into these practical references.

Routines

More and more classrooms are using innovative lesson designs and short discussion routines to help students develop number sense, flexibility, and the mathematical practices. In Chapter 4, we have added se new sections on: 3-Act Tasks, Number Talks, and Worked Examples. For

example, worked examples are mentioned in some of the tables identifying student challenges, because there is research to suggest that analyzing worked examples is effective in helping students learn.

Mathematical Modeling

Since the ninth edition, there has been significant national dialogue about the importance of mathematical modeling and what this might look like across the grades. The *Guidelines for Assessment & Instruction in Mathematical Modeling Education* (GAIMME) Report (COMAP & SIAM, 2016) provides excellent guidance. Therefore, the section in Chapter 13 on mathematical modeling was completely rewritten to reflect the GAIMME report, as well as to showcase a number of excellent books and articles that have emerged recently.

Infusion of Technology

You may notice that Chapter 7 (Technology) from the previous edition is gone. Readers and reviewers have commented that this chapter is not needed in part because using technology is much more commonly understood and used, and in part because it makes far more sense to talk about technology *as it relates to the mathematics.* We have heard you and we have integrated technology discussions, tools, and ideas throughout the book.

MyLab Education

Digital learning and assessment resources have been expanded significantly via MyLab Education. The following resources have been designed to help you develop the pedagogical knowledge *and* content knowledge needed to be a successful teacher of mathematics:

- **Video examples:** Embedded throughout all chapters, these examples allow you to see key concepts in action through authentic classroom video, as well as clips of children solving math problems. Additional videos feature your authors and other experts introducing and briefly explaining strategies for teaching important topics.
- **Self-checks:** Designed for self-study, these multiple-choice items are tied to each chapter learning outcome, and help you assess how well you have mastered the concepts covered in the reading. These exercises are self-grading and provide a rationale for the correct answer.
- **Application exercises:** Video and scenario-based exercises appear throughout the chapters and provide an opportunity for you to apply what you have learned to real classroom situations. There are also ten exercises on *observing and responding to student thinking* that include video clips of children talking through and solving problems on a whiteboard app; accompanying questions ask you to analyze and child's reasoning, identify any misconceptions, and explain any actions or prompts you might use as the teacher to guide the student's learning. Expert feedback is provided after submitting your response.
- **Math practice:** Located at the end of most content chapters, these sets of questions provide an opportunity to practice or refresh your own mathematics skills through solving exercises associated with the content from that chapter. These questions are also self-grading.
- **Blackline masters, activity pages, and expanded lessons:** These documents are linked throughout each chapter and make it easy for instructors and students to download and print classroom-ready handouts that can be used in a methods class or school settings.

Major Changes to Specific Chapters

Every chapter in the tenth edition has been revised to reflect the most current research, standards, and exemplars. This is evident in the approximately *300 new references* in the tenth

edition! This represents our ongoing commitment to synthesize and present the most current evidence of effective mathematics teaching. Here we share changes to what we consider the most significant (and that have not already been mentioned above).

Teaching Mathematics in the 21st Century (Chapter 1)

The new Association of Mathematics Teacher Educators (AMTE) Standards for Preparing Teachers of Mathematics (AMTE, 2017) are described in Chapter 1. We added a section on how to create a whole school agreement with a cohesive mathematics message.

Exploring What It Means to Know and Do Mathematics (Chapter 2)

Chapter 2 was revised in several significant ways, including revisions to the exemplar tasks (one in each content domain) to each have a common format, and to each have a stronger focus on multiple strategies. The discussions on theory were condensed, and making connections between theory and teaching were revised to be more succinct and explicit.

Teaching through Problem Solving (Chapter 3)

The NCTM Teaching Practices (2014) have been integrated into Chapter 3. A completely revamped section, now titled Developing Procedural Fluency, focuses on the importance of connecting conceptual and procedural knowledge, and includes a new list of ways to adapt drill-related tasks to emphasize understanding and connections (Boaler, 2016). Talk moves in the Discourse section have been revised to include eight talk moves (Chapin, O'Conner, & Anderson, 2013).

Teaching through Problem Solving (Chapter 4)

Beyond the new routines section (described above), the families section was heavily revised and the lesson plan steps condensed and formatted for easier readability.

Teaching Mathematics Equitably to All Students (Chapter 6)

We expanded our emphasis on using an asset-based approach, focusing on students' strengths rather than deficits. We emphasize a focus on using students' prior knowledge and experiences to drive instructional decisions. There is also a revamping of the section on gifted and talented students including attention to an excellence gap (students who may be overlooked).

Basic Facts (Chapter 9)

Recent research (e.g., Baroody et al., 2016) has uncovered a new and effective addition reasoning strategy—Use 10, which has been added to this chapter, along with new visuals and insights on teaching subtraction facts effectively.

Developing Strategies for Multiplication and Division (Chapter 12)

In new updates in this chapter, there are expanded examinations of the written records of computing multiplication and division problems including lattice multiplication, open arrays, and partial quotients. There is also a new section of the use of the break apart or decomposition strategy for division. A conversation about the selection of numbers for computational estimation problems is also shared.

Algebraic Thinking, Equations, and Functions (Chapter 13)

In addition to the new section on mathematical modeling, there are several new ideas and strategies for supporting algebraic thinking, including adapting the hundreds chart to explore patterns and options for creating tables with more structure to help students notice relationships.

Developing Fraction Concepts (Chapters 14)

Fraction concepts has an expanded focus on the fundamental ideas of sharing and iterating. This chapter also has been reorganized, has more contexts for comparing fractions, and more attention to student challenges in understanding fractions.

Ratios, Proportions, and Proportional Reasoning (Chapter 17)

The sections on additive and multiplicative reasoning have been significantly revised, including a new discussion on social justice mathematics. Additionally, significantly more literature connections are provided in this chapter and new activities.

Developing Concepts of Data Analysis (Chapter 20)

This chapter had numerous enhancements and changes! In addition to four new figures and completely updated technology options, the discussion of variability is woven throughout the chapter (including more attention to measures that are resistant to outlier), and sections on boxplots, histograms, and bivariate data were expanded and revised (see new subsection on bivariate categorical data).

An Introduction to *Teaching Developmentally*

If you look at the table of contents, you will see that the chapters are separated into two distinct sections. The first section consists of six chapters and covers important ideas that cross the boundaries of specific areas of content. The second section, consisting of 16 chapters, offers teaching suggestions and activities for every major mathematics topic in the preK–8 curriculum. Chapters in Part I offer perspectives on the challenging task of helping students learn mathematics. Having a feel for the discipline of mathematics— that is, to know what it means to "do mathematics"—is critical to learning how to teach mathematics well. In addition, understanding constructivist and sociocultural perspectives on learning mathematics and how they are applied to teaching through problem solving provides a foundation and rationale for how to teach and assess preK–8 students. You will be teaching diverse students including students who are English learners, are gifted, or have disabilities. In this text, you will learn how to apply instructional strategies in ways that support and challenge *all* learners. Formative assessment strategies and strategies for diverse learners are addressed in specific chapters in Part I (Chapters 5, and 6, respectively), and throughout Part II chapters.

Each chapter of Part II focuses on one of the major content areas in preK–8 mathematics curriculum. It begins with identifying the big ideas for that content, and provides guidance on how students best learn that content through many problem-based activities to engage them in understanding mathematics, as well as considering what challenges they may encounter and how you might help them.

Hundreds of tasks and activities are embedded in the text. Take out pencil and paper, or use technology, and try the problems, thinking about how you might solve them *and* how students at the intended grades might solve them. This is one way to actively engage in *your learning* about *students learning* mathematics. In so doing, this book will increase your own understanding of mathematics, the students you teach, and how to teach them effectively.

Some Special Features of This Text

By flipping through the book, you will notice many section headings, a large number of figures, and various special features. All are designed to make the book more useful as a long-term resource. Here are a few things to look for.

CHAPTER

14

Developing Fraction Concepts

LEARNER OUTCOMES

After reading this chapter and engaging in the embedded activities and reflections, you should be able to:

14.1 Describe and give examples for fractions constructs and fraction models.

14.2 Explain foundational concepts of fractional parts, including iteration and partitioning.

14.3 Illustrate the concept of equivalence across fraction models.

14.4 Describe strategies for comparing fractions and ways to teach this topic conceptually.

Fractions are one of the most important topics students need to understand to be successful in algebra and beyond, yet it is an area in which U.S. students, as well as students in many countries, struggle (OECD, 2014). National Assessment of Educational Progress (NAEP) results have consistently shown that students have a weak understanding of fraction concepts (Sowder & Wearne, 2006; Wearne & Kouba, 2000). This lack of understanding is then translated into difficulties with fraction computation, decimal and percent concepts, and algebra (Bailey, Hoard, Nugent, & Geary, 2012; Booth & Newton, 2012; Brown & Quinn, 2007; National Mathematics Advisory Panel, 2008; Siegler, Fazio, Bailey, & Zhou, 2013). Therefore, it is critical that you teach fractions well, present fractions as interesting and important, and commit to helping students understand the meaning of fractions.

BIG IDEAS

- Fractions can and should be represented across different interpretations (e.g., part-whole and division) and different models: area (e.g., $\frac{1}{3}$ of a garden), length (e.g., $\frac{3}{4}$ of an inch), and set (e.g., $\frac{1}{2}$ of the marbles).
- Fractions are equal shares of a whole or a unit. Therefore, equal sharing activities (e.g., 2 sandwiches shared with 4 friends) build on whole-number knowledge to introduce fractional quantities.
- Partitioning and iterating are ways students to understand the meaning of fractions. Partitioning can be thought of as splitting the whole equally (e.g., splitting a whole into fourths), and iterating can be thought of as making a copy of each piece and counting them (e.g., one-fourth, two-fourths, etc.).
- Equivalent fractions are ways of describing the same amount by using different-sized fractional parts.
- Fractions can be compared by reasoning about the relative size of the fractions. Estimation and reasoning are important in teaching understanding of fractions.

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◀ **Learning Outcomes**

To help readers know what they should expect to learn, each chapter begins with learning outcomes. Self-checks are numbered to cover and thus align with each learning outcome.

◀ **Big Ideas**

Much of the research and literature espousing a student-centered approach suggests that teachers plan their instruction around big ideas rather than isolated skills or concepts. At the beginning of each chapter in Part II, you will find a list of the big mathematical ideas associated with the chapter. Teachers find these lists helpful to quickly envision the mathematics they are to teach.

Activity 21.9   CCSS-M: 7.G.B.4; 7.SP.C.6; 7.SP.C.7b

Chance of Hitting the Target?

Project a target such as the one illustrated here with concentric circles having radii of 2 inches, 6 inches, 8 inches, and 10 inches, each region shaded a different color. Ask students to determine the fraction and percent of each colored region in the circle.

Ask students to discuss what the probability for landing on the center (assuming all throws land on the circle and are thrown randomly). Ask students to discuss why data may or may not match the percent of the area that is covered (e.g., people with good aim will be able to hit the smaller areas more often). Then, have students propose what point values they would assign to each region. Students may assign values in various ways. For example, they may think the skinny outer circle is harder to land on and give it more points than other sections, even though the area of that region may be more. Allow them time to share their reasoning and to critique others' ways of assigning points.

◄ **Activities**

The numerous activities found in every chapter of Part II have always been rated by readers as one of the most valuable parts of the book. Some activity ideas are described directly in the text and in the illustrations. Others are presented in the numbered Activity boxes. Every activity is a problem-based task (as described in Chapter 3) and is designed to engage students in doing mathematics.

Adaptations for Students with Special Needs and English Learners ▶

Chapter 6 provides detailed background and strategies for how to support students with special needs and English learners (ELs). But, many adaptations are specific to a activity or task. Therefore, Part II chapters offer adaptations and instructions within activities (look for the icon) that can meet the needs of students with special needs and ELs.

Activity 9.2   CCSS-M: 1.OA.A.1; 1.OA.C.6; 2.OA.B.2

How Many Feet in the Bed?

Read *How Many Feet in the Bed?* by Diane Johnston Hamm. On the second time through the book, ask students how many more feet are in the bed when a new person gets in. Ask students to record the equation (e.g., 6 + 2) and tell how many. Two less can be considered as family members get out of the bed. Find opportunities to make the connection between counting on and adding using a number line. For ELs, be sure that they know what the phrases "two more" and "two less" mean (and clarify the meaning of *foot,* which is also used for measuring). Acting out with students in the classroom can be a great illustration for both ELs and students with disabilities.

ENGLISH LEARNERS

STUDENTS with SPECIAL NEEDS

FORMATIVE ASSESSMENT Notes. To assess understanding of division algorithms, call on different students to explain individual steps using the appropriate terminology that connects to the concept of division. Use an Observation Checklist to record students' responses, indicating how well they understand the algorithm. For students who are having difficulty, you may want to conduct a short diagnostic interview to explore their level of understanding in more detail. Begin by having the student complete $115 \div 9$ and ask them to talk about what they are thinking as they carry out specific steps in the process. If there is difficulty explaining, have the student use base-ten materials to directly model the problem and attempt to link the actions to the procedure. Then ask them to discuss verbally the connections between what was done with the models and what was written symbolically. ■

◄ **Formative Assessment Notes**

Assessment is an integral process within instruction. Similarly, it makes sense to think about what to be listening for (assessing) as you read about different areas of content development. Throughout the content chapters, there are formative assessment notes with brief descriptions of ways to assess the topic in that section. Reading these assessment notes as you read the text can help you understand how best to assist students who struggle.

Technology Notes ▶

Infusing technological tools is important in learning mathematics. We have infused technology notes throughout Part II. A technology icon is used to identify places within the text or activity where a technology idea or resource is discussed. Descriptions include open-source (free) software, applets, and other Web-based resources, as well as ideas for calculator use.

TECHNOLOGY Note. An amazing computer tool for drawing two-dimensional views of block buildings is the Isometric Drawing Tool, available at the NCTM Illuminations website. Using mouse clicks students can draw either whole cubes, faces, or just lines. The drawings, however, are actually "buildings" and can be viewed as three-dimensional objects that when rotated can be seen from any vantage point. Prepared investigations lead students through the features of the tool. ■

◄ Standards for Mathematical Practice Margin Notes

Connections to the eight Standards of Mathematical Practice from the *Common Core State Standards* are highlighted in the margins. The location of the note indicates an example of the identified practice in the nearby text.

CCSS **Standards for Mathematical Practice**

MP1. Make sense of problems and persevere in solving them.

◄ End of Chapter Resources

The end of each chapter there are *Resources*, which include "Literature Connections" (found in all Part II chapters) and "Recommended Readings."

Literature Connections. Here you will find examples of great children's literature for launching into the mathematics concepts in the chapter just read. For each title suggested, there is a brief description of how the mathematics concepts in the chapter can be connected to the story. These literature-based mathematics activities will help you engage students in interesting contexts for doing mathematics.

Recommended Readings. In this section, you will find an annotated list of articles and books to augment the information found in the chapter. These recommendations include NCTM articles and books, and other professional resources designed for the classroom teacher.

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RESOURCES FOR CHAPTER 14

LITERATURE CONNECTIONS

The Doorbell Rang
Hutchins (1986)
Often used to investigate whole-number operations of multiplication and division, this book is also an excellent early introduction to fractions. The story is a simple tale of two children preparing to share a plate of 12 cookies. Just as they have figured out how to share the cookies, the doorbell rings and more children arrive. You can change the number of children to create a sharing situation that requires fractions (e.g., 8 children).

The Man Who Counted: A Collection of Mathematical Adventures
Tahan (1993)
This book contains a story, "Beasts of Burden," about a wise mathematician, Beremiz, and the narrator, who are traveling together on one camel. They are asked by three brothers to solve an argument: *Their father has left them 35 camels to divide among them: half to one brother; one-third to another; and one-ninth to the third brother.* The story is an excellent context for fractional parts of sets (and adding fractions). Changing the number of camels to 36 or 34, does not solve the challenge because the sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{9}$ will never be one whole, no matter how many camels are involved. See Bresser (1995) for three days of activities with this book.

Apple Fractions
Pallotta (2002)
This book offers interesting facts about apples while introducing fractions as fair shares (of apples, a healthier option than books that focus on chocolate and cookies!). In addition, the words for fractions are used and connected to fraction symbols, making it a good connection for fractions in grades 1–3.

RECOMMENDED READINGS

Articles
Clarke, D. M., Roche, A., & Mitchell, A. (2008). Ten practical tips for making fractions come alive and make sense. *Mathematics Teaching in the Middle School, 13*(7), 373–380.

Ten excellent tips for teaching fractions are discussed and favorite activities are shared. An excellent overview of teaching fractions.

Lewis, R. M., Gibbons, L. K., Kazemi, E., & Lind T. (2015). Unwrapping students ideas about fractions. *Teaching Children Mathematics, 22*(3), 158–168.

This excellent read provides a how-to for implementing sharing tasks, including sequencing of tasks, questions to pose, and formatives assessment tool to monitor student understanding.

Freeman, D. W., & Jorgensen, T. A. (2015). Moving beyond brownies and pizzas. *Teaching Children Mathematics, 21*(7), 412–420.

This article describes student thinking as they compare fractions. In the more4U pages, they offer excellent sets of tasks with a range of contexts, each set focusing on a different reasoning strategy for comparing fractions.

Books
Lamon, S. (2012). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies.* New York, NY: Taylor & Francis Group.

As the title implies, this book has a wealth of information to help with better understanding fractions and teaching fractions well. Many rich tasks and student work are provided throughout.

McNamara, J., & Shaughnessy, M. M. (2010). *Beyond pizzas and pies: 10 essential strategies for supporting fraction sense (grades 3–5).* Sausalito, CA: Math Solutions Publications.

This book has it all—classroom vignettes, discussion of research on teaching fractions, and many activities, including student work.

Websites
Rational Number Project (http://www.cehd.umn.edu/ci/rationalnumberproject/rnp1-09.html).

This project offers excellent lessons and other materials for teaching fraction concepts effectively.

Supplements for Instructors

Qualified college adopters can contact their Pearson sales representatives for information on ordering any of the supplements described below. These instructor supplements are all posted and available for download (click on Educators) from the Pearson Instructor Resource Center at www.pearsonhighered.com/irc. The IRC houses the following:

- **Instructor's resource manual:**   The Instructor's Resource Manual for the tenth edition includes a wealth of resources designed to help instructors teach the course, including chapter notes, activity suggestions, and suggested assessment and test questions.
- **Electronic test bank:**   An electronic test bank (TB) contains hundreds of challenging questions as multiple-choice or short-answer questions. Instructors can choose from these questions and create their own customized exams.
- **PowerPoint™ presentation:**   Ideal for instructors to use for lecture presentations or student handouts, the PowerPoint presentation provides ready-to-use graphics and text images tied to the individual chapters and content development of the text.

Acknowledgments

Many talented people have contributed to the success of this book and we are deeply grateful to all those who have assisted over the years. Without the success of the first edition, there would certainly not have been a second, much less ten editions. The following people worked closely with John on the first edition, and he was sincerely indebted to Warren Crown, John Dossey, Bob Gilbert, and Steven Willoughby, who gave time and great care in offering detailed comments on the original manuscript.

In preparing this tenth edition, we have received thoughtful input from the following mathematics teacher educators who offered comments on the ninth edition. Each reviewer challenged us to think through important issues. Many specific suggestions have found their way into this book, and their feedback helped us focus on important ideas. Thank you to Jessica Cohen, Western Washington University; Shea Mosely Culpepper, University of Houston; Shirley Dissler, High Point University; Cynthia Gautreau, California State University in Fullerton; Kevin LoPresto, Radford University; Ryan Nivens, East Tennessee State University; Adrienne Redmond-Sanogo, Oklahoma State University; and Douglas Roebuck, Ball State University. We are indebted to you for your dedicated and professional insight.

Additionally, we are very grateful for the ideas and reviews as we developed the tenth edition. Graham Fletcher, mathematics specialist, Atlanta, Georgia, provided strong support in the development of high leverage routines, including, of course, the new section on 3-Act Tasks. Susan Peters, mathematics teacher educator, University of Louisville, provided critical feedback and helpful ideas for developing concepts of data analysis. Their input resulted in significant improvements to those chapters. We continue to seek suggestions from teachers who use this book so please email us at teachingdevelopmentally@gmail.com with any advice, ideas, or insights you would like to share.

We are extremely grateful to our Pearson team of editors! Each of them has worked hard to turn our ideas (and yours) into a reality. And that is why we have been able to continue to evolve this book in a way to make it accessible online and via hard copy. Drew Bennett, our editor, has helped us define the direction of this edition and make important decisions that would make the book a better product for pre-service and in-service teachers. Our development editor, Kim Norbuta, has been supportive and positive, keeping us on target, even with the tightest of deadlines. Our content producer Yagnesh Jani was always available with the missing resources and answers we needed. Finally, we are very grateful to Jason Hammond and his editing team at SPi-Global, who carefully and conscientiously assisted in preparing this edition for publication. It has been a pleasure to interact with each of them and they have given us peace of mind to have knowledgeable, strong support.