DIFFERENTIAL EQUATIONS AND BOUNDARY VALUE PROBLEMS

Computing and Modeling

TECH UPDATE
Featuring MyLab Math

Fifth Edition

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APPLICATION MODULES

The modules listed here follow the indicated sections in the text. Most provide computing projects that illustrate the content of the corresponding text sections. These projects typically provide brief segments of appropriate computer syntax at the point of student need; over time, the student develops the ability to use technology to address a wide range of problems in differential equations.

Maple, *Mathematica*, MATLAB, and/or Python versions of these investigations are included in the website that accompanies this text as well as in MyLab Math. Within the sections of this textbook, students are provided with short URLs that link directly to the relevant online resources. Go to `goo.gl/UBgyWt` for a page containing links to all of these online materials.

PREFACE

This is a textbook for the standard introductory differential equations course taken by science and engineering students. Its updated content reflects the wide availability of technical computing environments like *Maple*, *Mathematica*, and MATLAB that now are used extensively by practicing engineers and scientists. The traditional manual and symbolic methods are augmented with coverage also of qualitative and computer-based methods that employ numerical computation and graphical visualization to develop greater conceptual understanding. A bonus of this more comprehensive approach is accessibility to a wider range of more realistic applications of differential equations.

Principal Features of This Revision

This 5th edition is a comprehensive and wide-ranging revision.

In addition to fine-tuning the exposition (both text and graphics) in numerous sections throughout the book, new applications have been inserted (including biological), and we have exploited throughout the new interactive computer technology that is now available to students on devices ranging from desktop and laptop computers to smart phones and graphing calculators. It also utilizes computer algebra systems such as *Mathematica*, *Maple*, and MATLAB as well as online web sites such as Wolfram|Alpha.

However, with a single exception of a new section inserted in Chapter 5 (noted below), the class tested table of contents of the book remains unchanged. Therefore, instructors' notes and syllabi will not require revision to continue teaching with this new edition.

A conspicuous feature of this edition is the insertion of about 80 new computer-generated figures, many of them illustrating how interactive computer applications with slider bars or touchpad controls can be used to change initial values or parameters in a differential equation, allowing the user to immediately see in real time the resulting changes in the structure of its solutions.

Some illustrations of the various types of revision and updating exhibited in this edition:

New Interactive Technology and Graphics New figures inserted throughout illustrate the facility offered by modern computing technology platforms for the user to interactively vary initial conditions and other parameters in real time. Thus, using a mouse or touchpad, the initial point for an initial value problem can be dragged to a new location, and the corresponding solution curve is automatically redrawn and dragged along with its initial point. For instance, see the application modules for Sections 1.3 (page 28) and 3.1 (page 148). Using slider bars in an interactive graphic, the coefficients or other parameters in a linear system can be varied, and the corresponding changes in its direction field and phase plane portrait are automatically shown; for instance see the application module for Section 5.3 (page 319). The number of terms used from an infinite series solution of a differential equation can be varied, and the resulting graphical change in the corresponding approximate solution is shown immediately; see the Section 8.2 application module (page 516).

New Exposition In a number of sections, new text and graphics have been inserted to enhance student understanding of the subject matter. For instance, see the

New to the Tech Update of the 5th Edition

This text is now supported by MyLab Math, which provides online homework, the eText, videos, and more. See pages xii and xiii of the Preface for more information.

We have also inserted short URLs within sections of the text that take students to an Expanded Applications page containing support for that application, including code in *Maple*, *Mathematica*, MATLAB, and/or Python. Go to goo.gl/UBgyWt for a page containing links to all of the Expanded Applications.

Note that this Tech Update is completely compatible with the regular 5th edition of the text.

treatments of separable equations in Section 1.4 (page 30), of linear equations in Section 1.5 (page 45), of isolated critical points in Sections 6.1 (page 372) and 6.2 (page 383) and the new example in Section 9.6 (on page 618) showing a vibrating string with a momentary "flat spot." Examples and accompanying graphics have been updated in Sections 2.4–2.6 and 4.2–4.3 to illustrate new graphing calculators.

New Content The single entirely new section for this edition is Section 5.3, which is devoted to the construction of a "gallery" of phase plane portraits illustrating all the possible geometric behaviors of solutions of the 2-dimensional linear system $\mathbf{x}' = \mathbf{A}\mathbf{x}$. In motivation and preparation for the detailed study of eigenvalue-eigenvector methods in subsequent sections of Chapter 5 (which then follow in the same order as in the previous edition), Section 5.3 shows how the particular arrangements of eigenvalues and eigenvectors of the coefficient matrix **A** correspond to identifiable patterns—"fingerprints," so to speak—in the phase plane portrait of the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$. The resulting gallery is shown in the two pages of phase plane portraits that comprise Figure 5.3.16 (pages 315–316) at the end of the section. The new 5.3 application module (on dynamic phase plane portraits, page 319) shows how students can use interactive computer systems to "bring to life" this gallery, by allowing initial conditions, eigenvalues, and even eigenvectors to vary in real time. This dynamic approach is then illustrated with several new graphics inserted in the remainder of Chapter 5. Finally, for a new biological application, see the application module for Section 6.4, which now includes a substantial investigation (page 423) of the nonlinear FitzHugh-Nagumo equations in neuroscience, which were introduced to model the behavior of neurons in the nervous system.

Computing Features

The following features highlight the computing technology that distinguishes much of our exposition.

- Over 750 *computer-generated figures* show students vivid pictures of direction fields, solution curves, and phase plane portraits that bring symbolic solutions of differential equations to life.
- About 45 *application modules* follow key sections throughout the text. Most of these applications outline "technology neutral" investigations illustrating the use of technical computing systems and seek to actively engage students in the application of new technology.
- A fresh *numerical emphasis* that is afforded by the early introduction of numerical solution techniques in Chapter 2 (on mathematical models and numerical methods). Here and in Chapter 4, where numerical techniques for systems are treated, a concrete and tangible flavor is achieved by the inclusion of numerical algorithms presented in parallel fashion for systems ranging from graphing calculators to MATLAB.

Modeling Features

Mathematical modeling is a goal and constant motivation for the study of differential equations. To sample the range of applications in this text, take a look at the following questions:

- What explains the commonly observed time lag between indoor and outdoor daily temperature oscillations? (Section 1.5)
- What makes the difference between doomsday and extinction in alligator populations? (Section 2.1)

- How do a unicycle and a twoaxle car react differently to road bumps? (Sections 3.7 and 5.4)
- How can you predict the time of next perihelion passage of a newly observed comet? (Section 4.3)
- Why might an earthquake demolish one building and leave standing the one next door? (Section 5.4)
- What determines whether two species will live harmoniously together, or whether competition will result in the extinction of one of them and the survival of the other? (Section 6.3)
- Why and when does non-linearity lead to chaos in biological and mechanical systems? (Section 6.5)
- If a mass on a spring is periodically struck with a hammer, how does the behavior of the mass depend on the frequency of the hammer blows? (Section 7.6)
- Why are flagpoles hollow instead of solid? (Section 8.6)
- What explains the difference in the sounds of a guitar, a xylophone, and drum? (Sections 9.6, 10.2, and 10.4)

Organization and Content

We have reshaped the usual approach and sequence of topics to accommodate new technology and new perspectives. For instance:

- After a precis of first-order equations in Chapter 1 (though with the coverage of certain traditional symbolic methods streamlined a bit), Chapter 2 offers an early introduction to mathematical modeling, stability and qualitative properties of differential equations, and numerical methods—a combination of topics that frequently are dispersed later in an introductory course. Chapter 3 includes the standard methods of solution of linear differential equations of higher order, particularly those with constant coefficients, and provides an especially wide range of applications involving simple mechanical systems and electrical circuits; the chapter ends with an elementary treatment of endpoint problems and eigenvalues.
- Chapters 4 and 5 provide a flexible treatment of linear systems. Motivated by current trends in science and engineering education and practice, Chapter 4 offers an early, intuitive introduction to first-order systems, models, and numerical approximation techniques. Chapter 5 begins with a self-contained treatment of the linear algebra that is needed, and then presents the eigenvalue approach to linear systems. It includes a wide range of applications (ranging from railway cars to earthquakes) of all the various cases of the eigenvalue method. Section 5.5 includes a fairly extensive treatment of matrix exponentials, which are exploited in Section 5.6 on nonhomogeneous linear systems.
- Chapter 6 on nonlinear systems and phenomena ranges from phase plane analysis to ecological and mechanical systems to a concluding section on chaos and bifurcation in dynamical systems. Section 6.5 presents an elementary introduction to such contemporary topics as period-doubling in biological and mechanical systems, the pitchfork diagram, and the Lorenz strange attractor (all illustrated with vivid computer graphics).
- Laplace transform methods (Chapter 7) and power series methods (Chapter 8) follow the material on linear and nonlinear systems, but can be covered at any earlier point (after Chapter 3) the instructor desires.

- Chapters 9 and 10 treat the applications of Fourier series, separation of variables, and Sturm-Liouville theory to partial differential equations and boundary value problems. After the introduction of Fourier series, the three classical equations—the wave and heat equations and Laplace's equation—are discussed in the last three sections of Chapter 9. The eigenvalue methods of Chapter 10 are developed sufficiently to include some rather significant and realistic applications.

This book includes enough material appropriately arranged for different courses varying in length from one quarter to two semesters. The briefer version *Differential Equations: Computing and Modeling* (0-13-485047-5) ends with Chapter 7 on Laplace transform methods (and thus omits the material on power series methods, Fourier series, separation of variables and partial differential equations).

Student and Instructor Resources

The answer section has been expanded considerably to increase its value as a learn-ing aid. It now includes the answers to most odd-numbered problems plus a good many even-numbered ones. The **Instructor's Solutions Manual** (0-321-79701-9), available at www.pearson.com and within MyLab Math, provides worked-out solutions for most of the problems in the book, and the **Student Solutions Manual** (available in print—ISBN 0-321-79700-0—or within MyLab Math) contains solutions for most of the odd-numbered problems. These manuals have been reworked extensively for this edition with improved explanations and more details inserted in the solutions.

The effectiveness of the 45 application modules located throughout the text is greatly enhanced by the material at the new Expanded Applications website. Nearly all of the application modules in the text are marked with and a unique short URL—a web address that leads directly to an Expanded Applications page containing a wealth of resources supporting that module. Typical Expanded Applications materials include an enhanced and expanded PDF version of the text with further discussion or additional applications, together with computer files in a variety of platforms, including *Mathematica*, *Maple*, MATLAB, and in some cases Python and/or TI calculator. These files provide all code appearing in the text as well as equivalent versions in other platforms, allowing students to immediately use the material in the Application Module on the computing platform of their choice. In addition to the URLs dispersed throughout the text, the Expanded Applications can be accessed via this homepage: goo.gl/UBgyWt.

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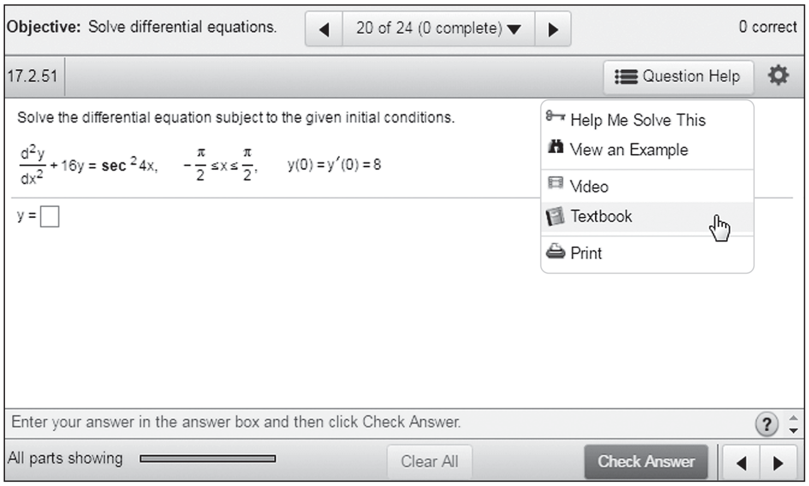
MyLab Math Online Course (access code required)

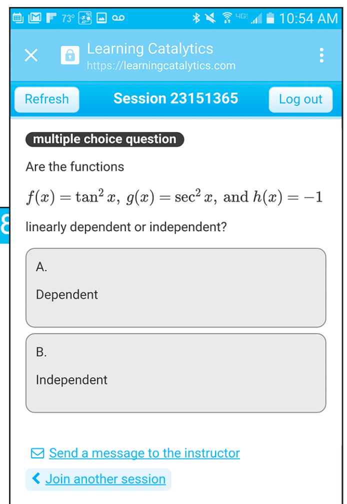
MyLab™ Math is the teaching and learning platform that empowers you to reach *every* student. By combining trusted author content with digital tools and a flexible platform, MyLab Math personalizes the learning experience and improves results for each student.

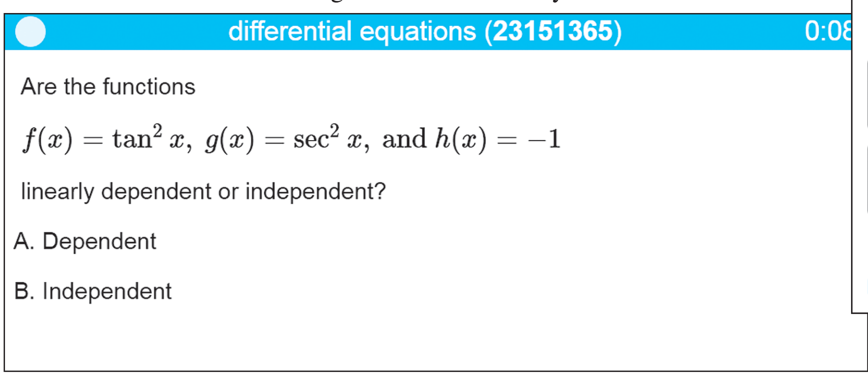
Learning and Teaching Tools.

- **Exercises with immediate feedback**—Hundreds of assignable exercises are based on the textbook exercises, and regenerate algorithmically to give students unlimited opportunity for practice and mastery. MyLab Math provides helpful feedback when students enter incorrect answers and includes optional learning aids including Help Me Solve This, View an Example, videos, and an eText.

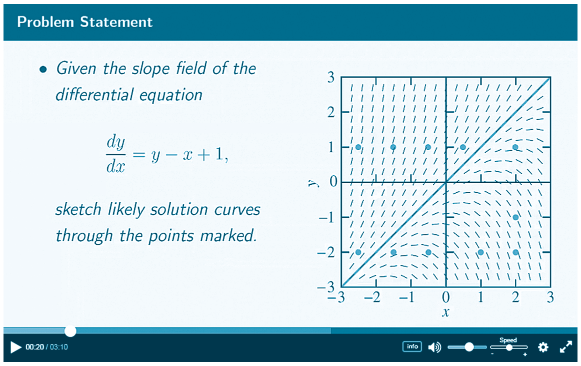


- **Help for gaps in prerequisite skills**—One of the biggest challenges in differential equations courses is making sure students are adequately prepared with the prerequisite skills needed to successfully complete their course work. MyLab Math supports students with just-in-time remediation of key calculus and precalculus skills.
- **Learning Catalytics™** is a student response tool that uses students' smartphones, tablets, or laptops to engage them in interactive tasks and thinking. Learning Catalytics fosters student engagement and peer-to-peer learning with real-time analytics.

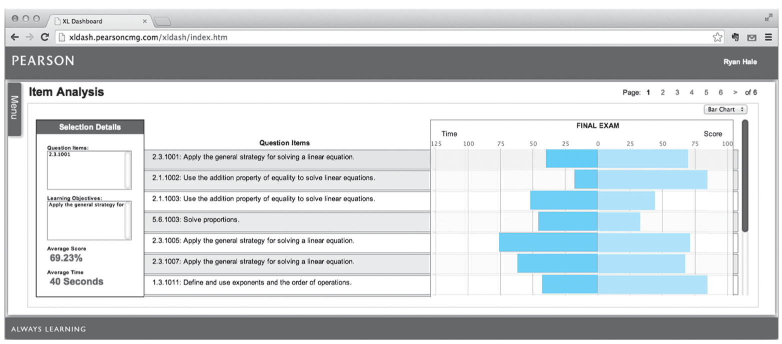




- **Instructional videos**, featuring the authors of the textbook, are available as learning aids within exercises and for self-study within the Multimedia Library. The Guide to Video-Based Assignments makes it easy to assign videos for homework by showing which MyLab Math exercises correspond to each video.



- **Lecture slides** are available in both Beamer (LaTeX) and PDF for all sections of the text.
- **The complete eText** is available to students through their MyLab Math courses for the lifetime of the edition, giving students unlimited access to the eText within any course using that edition of the textbook.
- **Accessibility**—Pearson works continuously to ensure our products are as accessible as possible to all students. We are working toward achieving WCAG 2.0 Level AA and Section 508 standards, as expressed in the Pearson Guidelines for Accessible Educational Web Media, `www.pearson.com/mylab/math/accessibility`.
- **A comprehensive gradebook** with enhanced reporting functionality allows you to efficiently manage your course.
    - ◊ **The Reporting Dashboard** provides insight to view, analyze, and report learning outcomes. Student performance data is presented at the class, section, and program levels in an accessible, visual manner so you'll have the information you need to keep your students on track.
    - ◊ **Item Analysis** tracks class-wide understanding of particular exercises so you can refine your class lectures or adjust the course/department syllabus.



MyLab Math comes from an experienced partner with educational expertise and an eye on the future. Whether you are just getting started with MyLab Math, or have a question along the way, we're here to help you learn about our technologies and how to incorporate them into your course. To learn more about how MyLab Math helps students succeed, visit `www.pearson.com/mylab/math` or contact your Pearson representative.