Preface

The goal of *Calculus for Biology and Medicine* has remained constant from its inception:

*To show students how calculus is used to analyze phenomena in nature without compromising the rigor of the presentation of calculus principles.*

The result of this goal is a calculus text that has plentiful life and health sciences applications and that provides students with the knowledge and skills necessary to analyze and interpret mathematical models of a diverse array of phenomena in the living world. Since this text is written for college freshmen, the examples were chosen so that no formal training in biology is needed.

The rigor of the text prepares students well for more advanced courses in mathematics and statistics. Our hope is that students will find calculus concepts easier to understand and more interesting if they are related to their major and career aspirations. While the table of contents resembles that of a traditional calculus text, the content does not: Abstract calculus concepts are introduced in a biological context, and students learn how to transfer and apply these concepts to biological situations.

**New to This Edition**

- **Modeling** – The 4th Edition places much more emphasis on modeling biological situations. Students are instructed in the processes of modeling real-world situations and given many opportunities to practice these techniques in problems.
- **Applications** – The applications in the text have been greatly expanded in number. New applications include population genetics, pharmacology, and the evolution of microbial cooperation. Many of these applications are adapted from published studies and other current sources.
- **Technology** – The 4th Edition now includes clear student instructions on using spreadsheets to numerically solve equations, visualize data, and model biological processes. This material is clearly labeled so that instructors who prefer not to use it can easily omit it, or assign technology sections as optional readings to their students.
- **Approach** – The level of rigor of the text has been maintained, but we have made adjustments to how some topics are introduced to make the presentation accessible to students, better bridging the gap between what students already know and what they are attempting to learn. We have also streamlined or made optional material that is not useful for life sciences students (formal discussion of limits, continuity in multivariate functions, etc.). This material is maintained so that instructors may continue to teach it, and students who may transfer out of life sciences calculus courses into physical sciences and engineering calculus will still find the material that is covered in physical calculus; but the main current of the book is through topics that are directly needed for life sciences.
- **Writing** – Every attempt has been made in the new edition to use language that will enable students to better use the text as an independent learning resource. In some sections this required lengthening explanations that were overly terse in order to make them more accessible; in others, topics are introduced informally using examples directly taken from life sciences to motivate the more formal mathematical material that was the strength of the previous editions.
- **Prerequisites** – We added a Precalculus Skills Diagnostic Test at the beginning of the text to help students gauge whether review of precalculus topics is needed. Answers to the quiz are provided in the back of the book along with tips on what to review in Chapter 1 if refreshers are needed.
- **Design** – The book has been redesigned in full color to help students better use it and to help motivate students as they put in the hard work to learn the mathematics.
• **Figures** – Many figures were revised to take advantage of the new full-color design. Most notably, the 3-dimensional figures were re-rendered using the latest software. See the figure at the left for an example.

• **Biology Notes** – New “Bio Info” notes provide optional background information to support the narrative and exercises.

• **“Help Text” within Examples** – We added text (in blue type) within examples to explain the mathematical principle(s) applied in the steps of the solution. This text helps students understand the solution and emphasizes that each step in a mathematical argument is carefully justified.

• **Topic Coverage** – Based on feedback from reviewers, some new topics have been added to the text. The most significant among these are the following:
  - Many new mathematical models, including models for microbial cooperation, evolution, and epidemiology, and multicompartment models in pharmacology.
  - Expanded applications for optimization methods.
  - Tools for fitting models to real data.
  - Expanded discussion of methods for visualizing multivariate functions.

**MyLab™ Math Online Homework** – Last, but not least, the text now has online homework within MyLab Math. The MyLab Math course contains hundreds of algorithmically generated exercises that provide students with instant feedback, optional learning aids for many exercises, and the complete eBook. See below for additional features of MyLab Math for this text.

### Features of the Text

A distinguishing feature of this text is the biological examples and exercises, which are notably *real* (many from published studies or other current sources), *relevant*, and *varied*. The References section at the back of this text contains an exhaustive list of the sources we used.

**Examples and Explanations**  Each topic is inspired by biological examples. These motivating introductions are followed by a thorough discussion outside the life science context to enable students to become familiar with both the meaning and the mechanics of the mathematical topic. Finally, biological examples are presented to teach students how to apply the material in a life science context. Examples in the text are completely worked out, and the steps in the solutions are explained in blue text to the right of each step.

**Exercises**  Calculus cannot be learned by watching someone do it. Because of this, *Calculus for Biology and Medicine* provides students with skill-based exercises as well as word problems. Word problems are an integral part of teaching calculus in a life science context. The word problems contained in the text are up-to-date and are adapted from either standard biology texts or original research. The exercises and word problems are at the end of each section and are organized by subsection to help students refer to specific subsections of content while completing homework. This also aids instructors in assigning homework problems.

**Technology**  *Calculus for Biology and Medicine* assumes the availability of graphing calculators. This allows students to develop a much better visual understanding of the concepts in calculus. Beyond this, no special software is required.

### Reflections and Outlook

Like many schools now, both UCLA and University of Minnesota offer life science students their own calculus track. Other universities are increasingly adopting separate calculus tracks to deal with the different needs of life sciences majors and physical science/engineering majors. There are many ways to design curricula for these courses, and faculty venturing into the recommendations offered by reports on new
needs for life sciences education (such as Bio2010: Transforming Undergraduate Education for Future Research Biologists from the National Research Council and the National Academies, or Scientific Foundations for Future Physicians from the Association of the American Medical Colleges and the Howard Hughes Medical Institute) may be overwhelmed by the amount of quantitative training that is now expected for life science students and by how it goes far beyond what students can be prepared for with a single year of calculus. In this fourth edition of the textbook we have focused on retaining the strengths of the third edition, including giving students access to the rigorous foundations of mathematical ideas that will enable them to take further classes in math that are increasingly necessary for quantitative minded biologists. However, we have rewritten much of the material with an eye to eliminating barriers to study (e.g., by avoiding using expressions with multiple unknown constants in them).

We are also very mindful of the future needs of students to handle the large data streams created by new innovations in omics, personal medicine, and remote sensing. Much of the math underlying these new areas is outside what can be covered in this book, but Chapters 9 and 12 lay foundations for students who will go on to study bioinformatics. Additionally, we have brought data (and data fitting) increasingly into the book, especially in support of the new mathematical modeling topics we have introduced. Study of algorithms is supported by explicit directions on using spreadsheets to implement the algorithms. Any spreadsheet software can be used, but we have found Google Sheets to be especially effective in the classroom, since it allows spreadsheets to be simultaneously shared and edited across dozens of computers.

**Chapter Summary**

**Chapter 1** This chapter reviews precalculus tools, including functions and methods for graphing data. Many students will have studied this material in their precalculus classes, so summaries are kept brief. Section 1.1 includes a diagnostic test that students can take (either by itself, or in conjunction with MyLab Math) to review their knowledge of these topics. The basic tools from algebra and trigonometry are summarized in Section 1.2. Section 1.3 then describes the functions that students need to be familiar with for this book, including exponential and logarithmic functions. Section 1.4 focuses on graphing, including log-log and semi-log plots and translating verbal descriptions of biological phenomena into graphs.

**Chapter 2** This chapter covers recurrence equations (or discrete time models) and sequences. Importantly, we use this chapter to introduce $\sum$ notation for summing series. This notation is used throughout the text. We also use this chapter to introduce mathematical modeling, including the assumptions that are built into models, parsing verbal descriptions, and comparing models against data. Our examples here are drawn from population growth and physiological modeling of how drugs pass through a patient’s body.

**Chapter 3** Limits and continuity are key concepts for understanding the conceptual parts of calculus. Visual intuition is emphasized before the theory is discussed. We show how the bisection method can be used as a practical tool for solving equations. The formal definition of limits is given at the end of the chapter in an optional section.

**Chapter 4** We start with an intuitive and visual description of the derivative before giving a formal definition. Then, before we go into the mechanics of differentiation, we describe interpretations of the derivative in different contexts (including chemical reactions), building students’ intuition further. Differentiation rules are discussed and broken into readily digestible chunks to give students time to acquaint themselves with them. Error propagation and differential equations are the main applications.
Chapter 5  This chapter presents biological and more traditional applications of differentiation. We maintain the 3rd Edition's approach that derives results on functional extrema rigorously from the Mean Value Theorem. But we also explain to skeptical students why calculus-based tools for analyzing functions and drawing their graphs are still relevant when computers allow functions to be so readily plotted. We have also enlarged the number of applications for optimization, including models from physiology and population genetics. We also added a new section on differential equation-based models (again focusing on population growth and the passage of drugs through the body), so that students encounter these vital applications before they meet integration. Finally, we introduce antiderivatives, in anticipation of studying integration in Chapter 6. Analysis of recurrence equations is covered in an optional section.

Chapter 6  Integration is motivated geometrically. We also describe the definition of the integral via Riemann sums in a way that has been greatly simplified from the 3rd Edition. In particular, students can study this material without needing to know \( \sum \)-notation and without the full formalism of partitions. In our experience, this makes this difficult topic much easier for students without an overall compromise on the level of rigor in their understanding. The fundamental theorem of calculus and its consequences are discussed in depth. We discuss applications for integration, but have reduced the amount of required material in this chapter to focus only on applications that are directly relevant to life sciences, such as calculating the mean of a function and its cumulative change.

Chapter 7  This chapter contains integration techniques, focusing on the techniques that are immediately necessary for solving differential equations, including integration by parts, by substitution, and a tailored introduction to the method of partial fractions. Material on Taylor polynomials and on using tables of integrals (a technique now largely made redundant by computer algebra packages) is covered in optional sections at the end of the chapter.

Chapter 8  This chapter provides an introduction to differential equations, covering separable equations and linear first order equations. The treatment is not complete, but it will equip students with both analytical and graphical skills to analyze differential equations. The chapter showcases and interprets mathematical models from many areas of biology, including microbial cooperation, ecology, and epidemiology. Addition of integrating factors allows us to discuss two-compartment models, which are widely used for studying the movement of drugs through the body.

Chapter 9  Linear models, and the matrix methods needed to solve them, are central to modern methods in bioinformatics. The material in this chapter introduces students to the basic concepts needed to study multivariate functions in Chapters 10 and 11. However, although the treatment of eigenvalues and eigenvectors emphasizes their importance to models of change (both recurrence equations and systems of differential equations), matrix math is introduced in a way designed to provide students with a firm platform for further study in linear algebra for bioinformatics applications.

Chapter 10  This is an introduction to multidimensional calculus. Since students often struggle with the transition from univariate functions to multivariate functions, we have expanded the introductory material to build up student intuition more gradually, with more examples (including practical examples like heat index) and larger discussion of how functions can be visualized. The main mathematical topics are partial derivatives and linearization of vector-valued functions. We cover at length finding extrema of functions (including under constraints). Although this topic is not needed for Chapter 11, optimization has many uses (and we highlight its application to least squares estimation of fitting parameters), and we find this section worthy of class time. The final sections provide optional material on systems of recurrence equations and on partial differential equation models.

Chapter 11  Both graphical and analytical tools are developed to enable students to analyze systems of differential equations. The material is divided into linear and
nonlinear systems. Understanding the stability of linear systems in terms of vector fields, eigenvectors, and eigenvalues helps students to master the more difficult analysis of nonlinear systems. Theory is explained before applications are given. Extensive examples (with accompanying problems) showcase applications of these tools in ecology, epidemiology, and physiology.

Chapter 12 This chapter introduces some fundamental probabilistic and statistical tools, taking students from counting (i.e., combinatorial) approaches to probability, through important distributions that arise when modeling stochastic processes. Throughout students are introduced to fundamental ideas for working with data: estimating probability distributions from histograms, fitting linear models, and calculating and interpreting summary statistics.

How to Use This Book
By design this book contains more material than can be covered in one year. The intent is to allow for schools and instructors to have more flexibility in the choice of material covered. Topics labeled with an asterisk (*) in the Table of Contents may be omitted at the instructor’s discretion. They include sections going more rigorously into the definitions of limits and continuity as well as many of the modeling and applications sections.

The book’s content can be arranged to support any length of course, from one quarter to three semesters. Chapter 1 is precalculus material. Students should have been exposed to this material before starting their first course in calculus. This said, we find it highly useful for students to self-study this material before starting the class, which they can do more easily using the new Precalculus Skills Diagnostic Test. Additionally, we often cover in class the material in Section 1.4 (in particular on how to graph data using logarithmic and semi-logarithmic axes). Sections 2.1 and 2.2 give students a minimal introduction to sequences, series, and \( \sum \)-notation. However, we strongly recommend Section 2.3 as an introduction to deriving, solving, and interpreting mathematical models, before students meet models again in the calculus context.

Chapters 3 and 4 must be covered in that order before any of the other sections are covered. In addition to Chapters 3–4, the following sections can be chosen:

**One semester—integration emphasis** 5.1–5.6, 5.10, 6.1–6.3 (without 6.3.4 and 6.3.5)
**One semester—differential equation emphasis** 5.1–5.6, 5.9–5.10, 6.1, 6.2, 8.2, 8.3 (without solving any of the differential equations)
**One semester—probability emphasis** Chapter 3 (except 3.6), Chapter 4 (without 4.11), 5.1–5.4, 5.10, 6.1, 6.2, 7.1, 72.1, 12.1–12.5 (without 12.5.5), 12.6 (if time permits)
**Two quarters** 5.1–5.6, 5.8, 5.10, 6.1–6.2, 6.3.1 and 6.3.2, Chapter 7, Chapter 8
**Two semesters or three quarters** 5.1–5.6, 5.10, 6.1–6.2, 6.3.1 and 6.3.2, Chapters 7, 8, and 9 (without 9.2.4 or 9.4), 10.1, 10.3, 10.4, 11.1–11.4
**Four quarters or three semesters** All sections that are not labeled optional (with *); optional sections should be chosen as time permits

MyLab Math Online Course (access code required)
Used by over 3 million students a year, MyLab Math is the world’s leading online program for teaching and learning mathematics. MyLab Math delivers assessment, tutorials, and multimedia resources that provide engaging and personalized experiences for each student, so learning can happen in any environment. For the first time, instructors teaching with Calculus for Biology and Medicine can assign text-specific online homework and other resources to students outside of the classroom.
To learn more about how MyLab Math combines proven learning applications with powerful assessment, visit [pearson.com/mylab/math](http://pearson.com/mylab/math) or contact your Pearson representative.

**Preparedness**

One of the biggest challenges in calculus courses is making sure students are adequately prepared with the prerequisite skills needed to successfully complete their course work. MyLab Math supports students with precalculus content and just-in-time remediation. Instructors can create quizzes to assess necessary prerequisite skills, then automatically assign personalized remediation for any gaps in skills that are identified.

**Building Understanding**

MyLab Math’s online homework offers students immediate feedback and tutorial assistance that helps them build understanding of key concepts.

- **Exercises with immediate feedback**—the assignable exercises for this text regenerate algorithmically to give students unlimited opportunity for practice and mastery. MyLab Math provides helpful feedback when students enter incorrect answers and includes optional learning aids including Help Me Solve This, View an Example, and an eText.

- **Setup and Solve Exercises** ask students to first describe how they will set up and approach the problem. This reinforces students’ conceptual understanding of the process they are applying and promotes long-term retention of the skill.
Additional Conceptual Questions focus on deeper, theoretical understanding of the key concepts in calculus. These questions were written by faculty at Cornell University under a National Science Foundation grant and are also assignable through Learning Catalytics.

Interactive Figures have been added to support teaching and learning. The figures are designed to be used in lecture as well as by students independently. They are editable using the freely available GeoGebra software.

Learning Catalytics™ is a student response tool that uses students’ smartphones, tablets, or laptops to engage them in more interactive tasks and thinking during lecture. Learning Catalytics fosters student engagement and peer-to-peer learning with real-time analytics. Learning Catalytics is available to all MyLab Math users.

Complete eText is available to students through their MyLab Math courses for the lifetime of the edition, giving students unlimited access to the eText within any course using that edition of the textbook.
- **Mathematica** manual and projects, **Maple** manual and projects, and **TI Graphing Calculator** manual utilize the most current versions of Maple and Mathematica, as well as the TI-84 Plus and TI-89. Each provides detailed guidance for integrating the software package or graphing calculator throughout the course, including syntax and commands.

- **Accessibility** and achievement go hand in hand. MyLab Math is compatible with the JAWS screen reader, and enables multiple-choice and free-response problem types to be read and interacted with via keyboard controls and math notation input. MyLab Math also works with screen enlargers, including ZoomText, MAGic, and SuperNova. More information is available at [www.pearson.com/mylab/math/accessibility](http://www.pearson.com/mylab/math/accessibility).

**Instructor Support**

- **Comprehensive gradebook** with enhanced reporting functionality allows you to efficiently manage your course. The gradebook meets all FERPA requirements.
- **Reporting Dashboard** provides insight to view, analyze, and report learning outcomes. Student performance data is presented at the class, section, and program levels in an accessible, visual manner so you’ll have the information you need to keep your students on track.
- **Item Analysis** tracks class-wide understanding of particular exercises so you can refine your class lectures or adjust the course/department syllabus. Just-in-time teaching has never been easier!

**Supplements**

**Instructor’s Solutions Manual (download only)** Provides fully worked-out solutions to every textbook exercise, including the Chapter Review problems. Available for download online within MyLab Math.


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