

# DIFFERENTIAL EQUATIONS

---

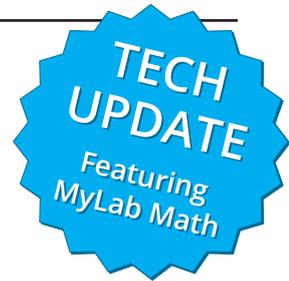
## Computing and Modeling



# DIFFERENTIAL EQUATIONS

---

Computing and Modeling



Fifth Edition

**C. Henry Edwards**

**David E. Penney**

*The University of Georgia*

**David Calvis**

*Baldwin Wallace University*



Director, Portfolio Management: Deirdre Lynch  
Executive Editor: Jeff Weidenaar  
Editorial Assistant: Jon Krebs  
Content Producer: Tara Corpuz  
Managing Producer: Scott Disanno  
Producer: Jean Choe  
Manager, Courseware QA: Mary Durnwald  
Manager, Content Development: Kristina Evans  
Product Marketing Manager: Yvonne Vannatta  
Field Marketing Manager: Evan St. Cyr  
Marketing Assistant: Jon Bryant  
Senior Author Support/Technology Specialist: Joe Vetere  
Manager, Rights and Permissions: Gina Cheselka  
Manufacturing Buyer: Carol Melville, LSC Communications  
Text Design, Production Coordination, Composition: Dennis Kletzing, Kletzing Typesetting Corp.  
Illustrations: George Nichols  
Cover Design: Studio Montage  
Cover Image: Onne van der Wal/Corbis

Copyright © 2019, 2015, 2008 by Pearson Education, Inc. All Rights Reserved. Printed in the United States of America. This publication is protected by copyright, and permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise. For information regarding permissions, request forms and the appropriate contacts within the Pearson Education Global Rights & Permissions department, please visit [www.pearsoned.com/permissions/](http://www.pearsoned.com/permissions/).

PEARSON, ALWAYS LEARNING, and MYLAB are exclusive trademarks owned by Pearson Education, Inc. or its affiliates in the U.S. and/or other countries.

Unless otherwise indicated herein, any third-party trademarks that may appear in this work are the property of their respective owners and any references to third-party trademarks, logos or other trade dress are for demonstrative or descriptive purposes only. Such references are not intended to imply any sponsorship, endorsement, authorization, or promotion of Pearson's products by the owners of such marks, or any relationship between the owner and Pearson Education, Inc. or its affiliates, authors, licensees or distributors.

**The Library of Congress has cataloged the fifth edition as follows:**

Edwards, C. H. (Charles Henry)

Differential equations and boundary value problems : computing and modeling / C. Henry

Edwards, David E. Penney, The University of Georgia, David Calvis, Baldwin Wallace College. --

Fifth edition.

pages cm

ISBN 978-0-321-79698-1 (hardcover)

1. Differential equations. 2. Boundary value problems. I. Penney, David E. II. Calvis,

David. III. Title.

QA371.E28 2015

515 . .35--dc23

2013040067

1 17



ISBN 13: 978-0-13-485047-4

ISBN 10: 0-13-485047-5

# CONTENTS

---

Application Modules   vii  
Preface   viii

## CHAPTER

# 1

### First-Order Differential Equations   1

- 1.1   Differential Equations and Mathematical Models   1
- 1.2   Integrals as General and Particular Solutions   10
- 1.3   Slope Fields and Solution Curves   17
- 1.4   Separable Equations and Applications   30
- 1.5   Linear First-Order Equations   45
- 1.6   Substitution Methods and Exact Equations   57

## CHAPTER

# 2

### Mathematical Models and Numerical Methods   75

- 2.1   Population Models   75
- 2.2   Equilibrium Solutions and Stability   86
- 2.3   Acceleration–Velocity Models   93
- 2.4   Numerical Approximation: Euler's Method   104
- 2.5   A Closer Look at the Euler Method   116
- 2.6   The Runge–Kutta Method   126

## CHAPTER

# 3

### Linear Equations of Higher Order   136

- 3.1   Introduction: Second-Order Linear Equations   136
- 3.2   General Solutions of Linear Equations   149
- 3.3   Homogeneous Equations with Constant Coefficients   162
- 3.4   Mechanical Vibrations   172
- 3.5   Nonhomogeneous Equations and Undetermined Coefficients   184
- 3.6   Forced Oscillations and Resonance   197
- 3.7   Electrical Circuits   209
- 3.8   Endpoint Problems and Eigenvalues   216

<b>CHAPTER</b> <b>4</b>	<b>Introduction to Systems of Differential Equations</b> <b>228</b>
	4.1 First-Order Systems and Applications 228
	4.2 The Method of Elimination 239
	4.3 Numerical Methods for Systems 249
<b>CHAPTER</b> <b>5</b>	<b>Linear Systems of Differential Equations</b> <b>264</b>
	5.1 Matrices and Linear Systems 264
	5.2 The Eigenvalue Method for Homogeneous Systems 282
	5.3 A Gallery of Solution Curves of Linear Systems 296
	5.4 Second-Order Systems and Mechanical Applications 322
	5.5 Multiple Eigenvalue Solutions 335
	5.6 Matrix Exponentials and Linear Systems 349
	5.7 Nonhomogeneous Linear Systems 363
<b>CHAPTER</b> <b>6</b>	<b>Nonlinear Systems and Phenomena</b> <b>372</b>
	6.1 Stability and the Phase Plane 372
	6.2 Linear and Almost Linear Systems 383
	6.3 Ecological Models: Predators and Competitors 395
	6.4 Nonlinear Mechanical Systems 408
	6.5 Chaos in Dynamical Systems 426
<b>CHAPTER</b> <b>7</b>	<b>Laplace Transform Methods</b> <b>437</b>
	7.1 Laplace Transforms and Inverse Transforms 437
	7.2 Transformation of Initial Value Problems 447
	7.3 Translation and Partial Fractions 458
	7.4 Derivatives, Integrals, and Products of Transforms 467
	7.5 Periodic and Piecewise Continuous Input Functions 474
	7.6 Impulses and Delta Functions 484

References for Further Study	494
Appendix: Existence and Uniqueness of Solutions	497
Answers to Selected Problems	511
Index	558

# APPLICATION MODULES

---

The modules listed here follow the indicated sections in the text. Most provide computing projects that illustrate the content of the corresponding text sections. These projects typically provide brief segments of appropriate computer syntax at the point of student need; over time, the student develops the ability to use technology to address a wide range of problems in differential equations.

*Maple*, *Mathematica*, MATLAB, and/or Python versions of these investigations are included in the website that accompanies this text as well as in MyLab Math. Within the sections of this textbook, students are provided with short URLs that link directly to the relevant online resources. Go to [goo.gl/UBgyWt](http://goo.gl/UBgyWt) for a page containing links to all of these online materials.

- |  |   |
|--|---|
| <b>1.3</b> Computer-Generated Slope Fields and Solution Curves | <b>5.1</b> Automatic Solution of Linear Systems                     |
| <b>1.4</b> The Logistic Equation                               | <b>5.2</b> Automatic Calculation of Eigenvalues and Eigenvectors    |
| <b>1.5</b> Indoor Temperature Oscillations                     | <b>5.3</b> Dynamic Phase Plane Graphics                             |
| <b>1.6</b> Computer Algebra Solutions                          | <b>5.4</b> Earthquake-Induced Vibrations of Multistory Buildings    |
| <b>2.1</b> Logistic Modeling of Population Data                | <b>5.5</b> Defective Eigenvalues and Generalized Eigenvectors       |
| <b>2.3</b> Rocket Propulsion                                   | <b>5.6</b> Automated Matrix Exponential Solutions                   |
| <b>2.4</b> Implementing Euler's Method                         | <b>5.7</b> Automated Variation of Parameters                        |
| <b>2.5</b> Improved Euler Implementation                       | <b>6.1</b> Phase Plane Portraits and First-Order Equations          |
| <b>2.6</b> Runge–Kutta Implementation                          | <b>6.2</b> Phase Plane Portraits of Almost Linear Systems           |
| <b>3.1</b> Plotting Second-Order Solution Families             | <b>6.3</b> Your Own Wildlife Conservation Preserve                  |
| <b>3.2</b> Plotting Third-Order Solution Families              | <b>6.4</b> The Rayleigh, van der Pol, and FitzHugh-Nagumo Equations |
| <b>3.3</b> Approximate Solution of Linear Equations            | <b>7.1</b> Computer Algebra Transforms and Inverse Transforms       |
| <b>3.5</b> Automated Variation of Parameters                   | <b>7.2</b> Transforms of Initial Value Problems                     |
| <b>3.6</b> Forced Vibrations                                   | <b>7.3</b> Damping and Resonance Investigations                     |
| <b>4.1</b> Gravitation and Kepler's Laws of Planetary Motion   | <b>7.5</b> Engineering Functions                                    |
| <b>4.2</b> Computer Algebra Solution of Systems                |   |
| <b>4.3</b> Comets and Spacecraft                               |   |

# PREFACE

---

This is a textbook for the standard introductory differential equations course taken by science and engineering students. Its updated content reflects the wide availability of technical computing environments like *Maple*, *Mathematica*, and MATLAB that now are used extensively by practicing engineers and scientists. The traditional manual and symbolic methods are augmented with coverage of qualitative and computer-based methods that employ numerical computation and graphical visualization to develop greater conceptual understanding. A bonus of this more comprehensive approach is accessibility to a wider range of more realistic applications of differential equations.

## Principal Features of This Revision

---

### New to the Tech Update of the 5th Edition

This text is now supported by MyLab Math, which provides online homework, the eText, videos, and more. See pages xii and xiii of the Preface for more information.

We have also inserted short URLs within sections of the text that take students to an Expanded Applications page containing support for that application, including code in *Maple*, *Mathematica*, MATLAB, and/or Python. Go to [goo.gl/UBgyWt](http://goo.gl/UBgyWt) for a page containing links to all of the Expanded Applications.

Note that this Tech Update is completely compatible with the regular 5th edition of the text.

This 5th edition is a comprehensive and wide-ranging revision.

In addition to fine-tuning the exposition (both text and graphics) in numerous sections throughout the book, new applications have been inserted (including biological), and we have exploited throughout the new interactive computer technology that is now available to students on devices ranging from desktop and laptop computers to smart phones and graphing calculators. It also utilizes computer algebra systems such as *Mathematica*, *Maple*, and MATLAB as well as online web sites such as Wolfram|Alpha.

However, with a single exception of a new section inserted in Chapter 5 (noted below), the class tested table of contents of the book remains unchanged. Therefore, instructors' notes and syllabi will not require revision to continue teaching with this new edition.

A conspicuous feature of this edition is the insertion of about 75 new computer-generated figures, many of them illustrating how interactive computer applications with slider bars or touchpad controls can be used to change initial values or parameters in a differential equation, allowing the user to immediately see in real time the resulting changes in the structure of its solutions.

Some illustrations of the various types of revision and updating exhibited in this edition:

**New Interactive Technology and Graphics** New figures inserted throughout illustrate the facility offered by modern computing technology platforms for the user to interactively vary initial conditions and other parameters in real time. Thus, using a mouse or touchpad, the initial point for an initial value problem can be dragged to a new location, and the corresponding solution curve is automatically redrawn and dragged along with its initial point. For instance, see the application modules for Sections 1.3 (page 28) and 3.1 (page 148). Using slider bars in an interactive graphic, the coefficients or other parameters in a linear system can be varied, and the corresponding changes in its direction field and phase plane portrait are automatically shown; see the Section 5.3 application module (page 319).

**New Exposition** In a number of sections, new text and graphics have been inserted to enhance student understanding of the subject matter. For instance, see the new introductory treatments of separable equations in Section 1.4 (page 30), of linear equations in Section 1.5 (page 45), of isolated critical points in Sections 6.1 (page 372) and 6.2 (page 383) and the new example in Section 9.6 (on page 618) showing a vibrating

string with a momentary “flat spot.” Examples and accompanying graphics have been updated in Sections 2.4–2.6 and 4.2–4.3 to illustrate new graphing calculators.

**New Content** The single entirely new section for this edition is Section 5.3, which is devoted to the construction of a “gallery” of phase plane portraits illustrating all the possible geometric behaviors of solutions of the 2-dimensional linear system  $\mathbf{x} = \mathbf{A}\mathbf{x}$ . In motivation and preparation for the detailed study of eigenvalue-eigenvector methods in subsequent sections of Chapter 5 (which then follow in the same order as in the previous edition), Section 5.3 shows how the particular arrangements of eigenvalues and eigenvectors of the coefficient matrix  $\mathbf{A}$  correspond to identifiable patterns—“fingerprints,” so to speak—in the phase plane portrait of the system  $\mathbf{x} = \mathbf{A}\mathbf{x}$ . The resulting gallery is shown in the two pages of phase plane portraits that comprise Figure 5.3.16 (pages 315–316) at the end of the section. The new 5.3 application module (on dynamic phase plane portraits, page 319) shows how students can use interactive computer systems to “bring to life” this gallery, by allowing initial conditions, eigenvalues, and even eigenvectors to vary in real time. This dynamic approach is then illustrated with several new graphics inserted in the remainder of Chapter 5. Finally, for a new biological application, see the application module for Section 6.4, which now includes a substantial investigation (page 423) of the nonlinear FitzHugh-Nagumo equations in neuroscience, which were introduced to model the behavior of neurons in the nervous system.

## Computing Features

---

The following features highlight the computing technology that distinguishes much of our exposition.

- Over 500 *computer-generated figures* show students vivid pictures of direction fields, solution curves, and phase plane portraits that bring symbolic solutions of differential equations to life.
- About 30 *application modules* follow key sections throughout the text. Most of these applications outline “technology neutral” investigations illustrating the use of technical computing systems and seek to actively engage students in the application of new technology.
- A fresh *numerical emphasis* that is afforded by the early introduction of numerical solution techniques in Chapter 2 (on mathematical models and numerical methods). Here and in Chapter 4, where numerical techniques for systems are treated, a concrete and tangible flavor is achieved by the inclusion of numerical algorithms presented in parallel fashion for systems ranging from graphing calculators to MATLAB.

## Modeling Features

---

Mathematical modeling is a goal and constant motivation for the study of differential equations. To sample the range of applications in this text, take a look at the following questions:

- What explains the commonly observed time lag between indoor and outdoor daily temperature oscillations? (Section 1.5)
- What makes the difference between doomsday and extinction in alligator populations? (Section 2.1)
- How do a unicycle and a twoaxle car react differently to road bumps? (Sections 3.7 and 5.4)

- How can you predict the time of next perihelion passage of a newly observed comet? (Section 4.3)
- Why might an earthquake demolish one building and leave standing the one next door? (Section 5.4)
- What determines whether two species will live harmoniously together, or whether competition will result in the extinction of one of them and the survival of the other? (Section 6.3)
- Why and when does non-linearity lead to chaos in biological and mechanical systems? (Section 6.5)
- If a mass on a spring is periodically struck with a hammer, how does the behavior of the mass depend on the frequency of the hammer blows? (Section 7.6)

## Organization and Content

---

We have reshaped the usual approach and sequence of topics to accommodate new technology and new perspectives. For instance:

- After a precis of first-order equations in Chapter 1 (though with the coverage of certain traditional symbolic methods streamlined a bit), Chapter 2 offers an early introduction to mathematical modeling, stability and qualitative properties of differential equations, and numerical methods—a combination of topics that frequently are dispersed later in an introductory course. Chapter 3 includes the standard methods of solution of linear differential equations of higher order, particularly those with constant coefficients, and provides an especially wide range of applications involving simple mechanical systems and electrical circuits; the chapter ends with an elementary treatment of endpoint problems and eigenvalues.
- Chapters 4 and 5 provide a flexible treatment of linear systems. Motivated by current trends in science and engineering education and practice, Chapter 4 offers an early, intuitive introduction to first-order systems, models, and numerical approximation techniques. Chapter 5 begins with a self-contained treatment of the linear algebra that is needed, and then presents the eigenvalue approach to linear systems. It includes a wide range of applications (ranging from railway cars to earthquakes) of all the various cases of the eigenvalue method. Section 5.5 includes a fairly extensive treatment of matrix exponentials, which are exploited in Section 5.6 on nonhomogeneous linear systems.
- Chapter 6 on nonlinear systems and phenomena ranges from phase plane analysis to ecological and mechanical systems to a concluding section on chaos and bifurcation in dynamical systems. Section 6.5 presents an elementary introduction to such contemporary topics as period-doubling in biological and mechanical systems, the pitchfork diagram, and the Lorenz strange attractor (all illustrated with vivid computer graphics).
- Laplace transform methods (Chapter 7) follow the material on linear and nonlinear systems, but can be covered at any earlier point (after Chapter 3) the instructor desires.

This book includes enough material appropriately arranged for different courses varying in length from one quarter to two semesters. The longer version *Differential Equations and Boundary Value Problems: Computing and Modeling* 0-13-483739-8 contains additional chapters on power series methods, Fourier series, separation of variables and partial differential equations.

## Student and Instructor Resources

---

The answer section now includes the answers to most odd-numbered problems plus a good many even-numbered ones. The **Instructor's Solutions Manual** (0-321-79701-9), available at [www.pearson.com](http://www.pearson.com) and within MyLab Math, provides worked-out solutions for most of the problems in the book, and the **Student Solutions Manual** (available in print—ISBN 0-321-79700-0—or within MyLab Math) contains solutions for most of the odd-numbered problems. Both manuals have been reworked extensively for this edition with improved explanations and more details inserted in the solutions.

The effectiveness of the 45 application modules located throughout the text is greatly enhanced by the material at the new Expanded Applications website. Nearly all of the application modules in the text are marked with  and a unique short URL—a web address that leads directly to an Expanded Applications page containing a wealth of resources supporting that module. Typical Expanded Applications materials include an enhanced and expanded PDF version of the text with further discussion or additional applications, together with computer files in a variety of platforms, including *Mathematica*, *Maple*, *MATLAB*, and in some cases Python and/or TI calculator. These files provide all code appearing in the text as well as equivalent versions in other platforms, allowing students to immediately use the material in the Application Module on the computing platform of their choice. In addition to the URLs dispersed throughout the text, the Expanded Applications can be accessed via this homepage: [goo.gl/UBgyWt](http://goo.gl/UBgyWt).

## Acknowledgments

---

In preparing this revision, we profited greatly from the advice and assistance of the following very capable and perceptive reviewers:

Anthony Aidoo, *Eastern Connecticut State University*

Brent Solie, *Knox College*

Elizabeth Bradley, *University of Louisville*

Gregory Davis, *University of Wisconsin-Green Bay*

Zoran Grujic, *University of Virginia*

Richard Jardine, *Keene State College*

Yang Kuang, *Arizona State University*

Dening Li, *West Virginia University*

Francisco Sayas-Gonzalez, *University of Delaware*

Luther White, *University of Oklahoma*

Hong-Ming Yin, *Washington State University*

Morteza Shafii-Mousavi, *Indiana University-South Bend*

It is a pleasure to (once again) credit Dennis Kletzing and his extraordinary T<sub>E</sub>Xpertise for the attractive presentation of the text and the art in this book. We are grateful to our editor, Jeff Weidenaar, for his support and inspiration of this revision; to Jean Choe and Kristina Evans for coordination of the technology tools and content for the text; to Tara Corpuz for her supervision of the production of this book; and to Joe Vetere for his assistance with technical aspects of the development of its extensive supplementary resources. Finally, we dedicate this edition to our colleague David E. Penney who passed away on June 3, 2014.

Henry Edwards

[h.edwards@mindspring.com](mailto:h.edwards@mindspring.com)

David Calvis

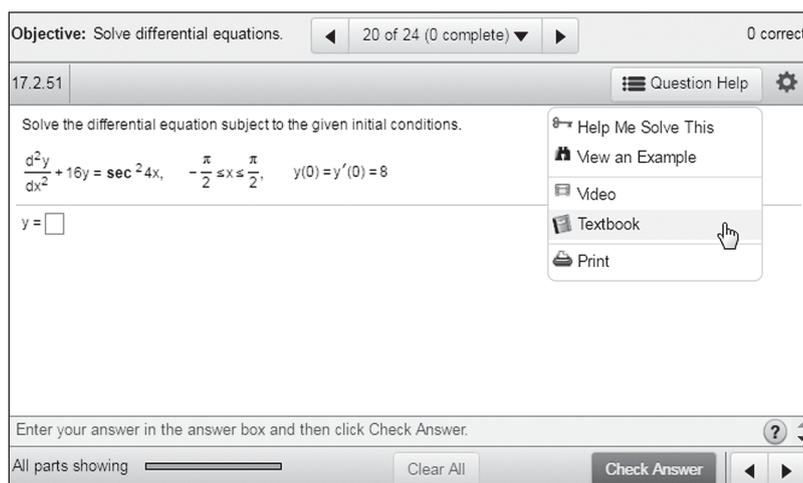
[dcalvis@bw.edu](mailto:dcalvis@bw.edu)

## MyLab Math Online Course (access code required)

MyLab™ Math is the teaching and learning platform that empowers you to reach every student. By combining trusted author content with digital tools and a flexible platform, MyLab Math personalizes the learning experience and improves results for each student.

### Learning and Teaching Tools.

- Exercises with immediate feedback**—Hundreds of assignable exercises are based on the textbook exercises, and regenerate algorithmically to give students unlimited opportunity for practice and mastery. MyLab Math provides helpful feedback when students enter incorrect answers and includes optional learning aids including Help Me Solve This, View an Example, videos, and an eText.



- Help for gaps in prerequisite skills**—One of the biggest challenges in differential equations courses is making sure students are adequately prepared with the prerequisite skills needed to successfully complete their course work. MyLab Math supports students with just-in-time remediation of key calculus and precalculus skills.
- Learning Catalytics™** is a student response tool that uses students' smartphones, tablets, or laptops to engage them in interactive tasks and thinking. Learning Catalytics fosters student engagement and peer-to-peer learning with real-time analytics.

differential equations (23151365)
0:08

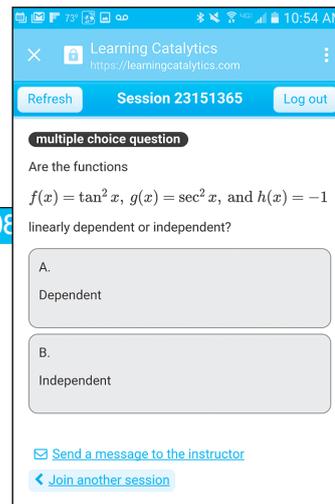
Are the functions

$$f(x) = \tan^2 x, g(x) = \sec^2 x, \text{ and } h(x) = -1$$

linearly dependent or independent?

A. Dependent

B. Independent



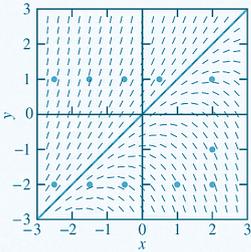
- **Instructional videos**, featuring the authors of the textbook, are available as learning aids within exercises and for self-study within the Multimedia Library. The Guide to Video-Based Assignments makes it easy to assign videos for homework by showing which MyLab Math exercises correspond to each video.

**Problem Statement**

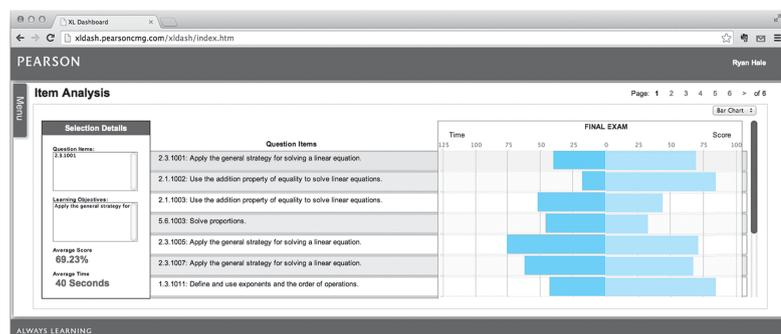
- Given the slope field of the differential equation

$$\frac{dy}{dx} = y - x + 1,$$

sketch likely solution curves through the points marked.



- **Lecture slides** are available in both Beamer (LaTeX) and PDF for all sections of the text.
- **The complete eText** is available to students through their MyLab Math courses for the lifetime of the edition, giving students unlimited access to the eText within any course using that edition of the textbook.
- **Accessibility**—Pearson works continuously to ensure our products are as accessible as possible to all students. We are working toward achieving WCAG 2.0 Level AA and Section 508 standards, as expressed in the Pearson Guidelines for Accessible Educational Web Media, [www.pearson.com/mylab/math/accessibility](http://www.pearson.com/mylab/math/accessibility).
- **A comprehensive gradebook** with enhanced reporting functionality allows you to efficiently manage your course.
  - ◇ **The Reporting Dashboard** provides insight to view, analyze, and report learning outcomes. Student performance data is presented at the class, section, and program levels in an accessible, visual manner so you'll have the information you need to keep your students on track.
  - ◇ **Item Analysis** tracks class-wide understanding of particular exercises so you can refine your class lectures or adjust the course/department syllabus.



MyLab Math comes from an experienced partner with educational expertise and an eye on the future. Whether you are just getting started with MyLab Math, or have a question along the way, we're here to help you learn about our technologies and how to incorporate them into your course. To learn more about how MyLab Math helps students succeed, visit [www.pearson.com/mylab/math](http://www.pearson.com/mylab/math) or contact your Pearson representative.

