Linear Algebra with Applications

Tenth Edition

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Dedication

For their encouragement, patience, and the joy they bring to my life, thank you to my wonderful husband, Jan Lindheim, and my brilliant daughters, Lydia, Sarah, and Alexandra.

For their non-stop enthusiastic support, for teaching me to love learning and to never stop asking why, thank you to my parents.

And for bringing me alongside, and giving me the opportunity to join in the ongoing work of this book, thank you to my co-author Steven Leon.

Lisette de Pillis

To the memory of Judith Russ Leon, my lover and companion for more than 46 years.

Steven J. Leon

Contents

Preface

We are pleased to see the text reach its tenth edition. The continued support and enthusiasm of its many users have been most gratifying. Linear algebra is more exciting now than at almost any time in the past. Its applications continue to spread to more and more fields. Largely due to the computer revolution of the last 75 years, linear algebra has risen to a role of prominence in the mathematical curriculum rivaling that of calculus. Modern software has also made it possible to dramatically improve the way the course is taught.

The first edition of this book was published in 1980. Each of the following editions has seen significant modifications including the addition of comprehensive sets of MATLAB computer exercises, a dramatic increase in the number of applications, and many revisions in the various sections of the book. We have been fortunate to have had outstanding reviewers, and their suggestions have led to many important improvements in the book.

What's New in the Tenth Edition?

You may have noticed something new on the cover of the book. Another author! Yes, after nearly 40 years as a "solo act," Steve Leon has a partner. New co-author Lisette de Pillis is a professor at Harvey Mudd College and brings her passion for teaching and solving real-world problems to this revision.

The focus of this revision was transforming it from a primarily print-based learning tool to a digital learning tool. The eText is therefore filled with content and tools that will help bring the entire course to life for students in new ways and help you improve instruction. Specifically,

- **Interactive figures and utilities.** We have added a number of opportunities for students to interact with content in a dynamic manner in order to build and enhance understanding. Interactive figures allow students to explore concepts geometrically in ways that are not possible without technology. Examples here include:
  - In Chapter 3, Visualizing the span of vectors—Figures 3.2.3, 3.2.4, 3.2.6(a), 3.2.6(b)
  - In Chapter 4, Visualizing linear transformations
    - Simple linear transformations—Figures 4.1.1 through 4.1.4
    - Dilations, reflections, rotations—Figure 4.2.3
    - Yaw, pitch, and roll of an airplane—Figure 4.2.5

  - In Chapter 6, Visualization tools for 2×2 matrices
    - Eigenvectors—Figure 6.1.1
    - Singular vectors—Figure 6.5.1
- **Hints.** For selected exercises, we've included hints for students to consider if they get stuck.
- **Notes, Labels, and Highlights.** Notes allow instructors to add their personal teaching style to important topics, call out need-to-know information, or clarify difficult concepts. Students can make their eText their own by creating highlights with meaningful labels and notes, helping them focus on what they need to study. The customizable Notebook allows students to filter, arrange, and group their notes in a way that makes sense to them.
- **Dashboard**. Instructors can create reading assignments and see the time spent in the eText so that they can plan more effective instruction.
- **Portability**. Portable access lets students read their eText whenever they have a moment in their day, on Android and iOS mobile phones and tablets. Even without an Internet connection, offline reading ensures students never miss a chance to learn.
- **Ease-of-Use.** Straightforward setup makes it easy for instructors to get their class up and reading quickly on the first day of class. In addition, Learning Management System (LMS) integration provides institutions, instructors, and students with single sign-on access to the eText via many popular LMSs.

Overview of Text

This book is suitable for either a lower or upper division Linear Algebra course. The student should have some familiarity with the basics of differential and integral calculus. This prerequisite can be met by either one semester or two quarters of elementary calculus.

If the text is used for a lower-level course, the instructor should probably spend more time on the early chapters and omit many of the sections in the later chapters. For more advanced courses, a quick review of the topics in the first two chapters and then a more complete coverage of the later chapters would be appropriate. The explanations in the text are given in sufficient detail so that beginning students should have little trouble reading and understanding the material. To further aid the student, a large number of examples have been worked out completely. Additionally, computer exercises at the end of each chapter give students the opportunity to perform numerical experiments and try to generalize the results. Applications are presented throughout the book. These applications can be used to motivate new material or to illustrate the relevance of material that has already been covered.

The text contains all the topics recommended by the National Science Foundation (NSF) sponsored Linear Algebra Curriculum Study Group (LACSG) and much more. Although there is more material than can be covered in a single course, it is our belief that it is easier for an instructor to leave out or skip material than it is to supplement

a book with outside material. Even if many topics are omitted, the book should still provide students with a feeling for the overall scope of the subject matter. Furthermore, students may use the book later as a reference and consequently may end up learning omitted topics on their own.

Suggested Course Outlines

We include here a number of outlines for one-semester courses at either the lower or upper-division levels, and with either a matrix-oriented emphasis or a slightly more theoretical emphasis.

1. One-Semester Lower Division Course

A. Basic Lower Level Course

Chapter 1	Sections 1–6	7 lectures
Chapter 2	Sections 1–2	2 lectures
Chapter 3	Sections 1–6	9 lectures
Chapter 4	Sections 1–3	4 lectures
Chapter 5	Sections 1–6	9 lectures
Chapter 6	Sections 1–3	4 lectures
		Total 35 lectures

B. LACSG Matrix-Oriented Course

The core course recommended by the LACSG involves only the Euclidean vector spaces. Consequently, for this course you should omit Section 1 of Chapter 3 (on general vector spaces) and all references and exercises involving function spaces in Chapters 3 to 6. All the topics in the LACSG core syllabus are included in the text. It is not necessary to introduce any supplementary materials. The LACSG recommended 28 lectures to cover the core material. This is possible if the class is taught in lecture format with an additional recitation section meeting once a week. If the course is taught without recitations, it is our contention that the following schedule of 35 lectures is perhaps more reasonable.

Chapter 1	Sections 1–6	7 lectures
Chapter 2	Sections 1–2	2 lectures
Chapter 3	Sections 2–6	7 lectures
Chapter 4	Sections 1–3	2 lectures
Chapter 5	Sections 1–6	9 lectures
Chapter 6	Sections 1, 3–5	8 lectures
		Total 35 lectures

2. One-Semester Upper-Level Courses

The coverage in an upper-division course is dependent on the background of the students. Following are two possible courses.

Option A: Minimal background in linear algebra

Chapter 1	Sections 1–6	6 lectures
Chapter 2	Sections 1–2	2 lectures
Chapter 3	Sections 1–6	7 lectures
Chapter 5	Sections 1–6	9 lectures
Chapter 6	Sections 1–7, 8*	10 lectures
Chapter 7	Section 4	1 lecture
		Total 35 lectures

* If time allows.

Option B: Some background in linear algebra

Review of Topics in Chapters 1–3		5 lectures
Chapter 4	Sections 1–3	2 lectures
Chapter 5	Sections 1–6	10 lectures
Chapter 6	Sections 1–7, 8*	11 lectures
Chapter 7	Sections 1–3*, 4–7	7 lectures
Chapter 8	Sections 1–2*	2 lectures
		Total 37 lectures

* If time allows.

3. **Two-Semester Sequence**
   Although two semesters of linear algebra have been recommended by the LACSG, it is still not practical at many universities and colleges. At present, there is no universal agreement on a core syllabus for a second course. In a two-semester sequence, it is possible to cover all 43 sections of the book. You might also consider adding a lecture or two in order to demonstrate how to use MATLAB.

Computer Exercises

The text contains a section of computing exercises at the end of each chapter. These exercises are based on the software package MATLAB. The MATLAB Appendix in the book explains the basics of using the software. MATLAB has the advantage that it is a powerful tool for matrix computations, yet it is easy to learn. After reading the Appendix, students should be able to do the computing exercises without having to refer to any other software books or manuals. To help students get started, we recommend a one 50-minute classroom demonstration of the software. The assignments can be done either as ordinary homework assignments or as part of a formally scheduled computer laboratory course.

Although the course can be taught without any reference to a computer, we believe that computer exercises can greatly enhance student learning and provide a new dimension to linear algebra education. One of the recommendations of the LASCG is

that technology should be used in a first course in linear algebra. That recommendation has been widely accepted, and it is now common to see mathematical software packages used in linear algebra courses.

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