

**PART 1**

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*Transport Processes:  
Momentum, Heat,  
and Mass*

**FOR PUBLIC  
RELEASE**

# *Introduction to Engineering Principles and Units*

## **1.1 CLASSIFICATION OF TRANSPORT PROCESSES AND SEPARATION PROCESSES (UNIT OPERATIONS)**

### **1.1A Introduction**

In the chemical and other physical processing industries and the food and biological processing industries, many similarities exist in the manner in which the entering feed materials are modified or processed into final materials of chemical and biological products. We can take these seemingly different chemical, physical, or biological processes and break them down into a series of separate and distinct steps that were originally called *unit operations*. However, the term “unit operations” has largely been superseded by the more modern and descriptive term “separation processes.” These *separation processes* are common to all types of diverse process industries.

For example, the separation process *distillation* is used to purify or separate alcohol in the beverage industry and hydrocarbons in the petroleum industry. Drying of grain and other foods is similar to drying of lumber, filtered precipitates, and wool. The separation process *absorption* occurs in absorption of oxygen from air in a fermentation process or in a sewage treatment plant and in absorption of hydrogen gas in a process for liquid hydrogenation of oil. Evaporation of salt solutions in the chemical industry is similar to evaporation of sugar solutions in the food industry. Settling and sedimentation of suspended solids in the sewage industry and the mining industry are similar. Flow of liquid hydrocarbons in the petroleum refinery and flow of milk in a dairy plant are carried out in a similar fashion.

Many of these separation processes have certain fundamental and basic principles or mechanisms in common. For example, the mechanism of diffusion or mass transfer occurs in drying, membrane separation, absorption, distillation, and crystallization. Heat transfer occurs in drying, distillation, evaporation, and so on. The following classification of a more fundamental nature is often made, according to transfer or transport processes.

## 1.1B Fundamental Transport Processes

1. *Momentum transfer.* This is concerned with the transfer of momentum which occurs in moving media, such as in the separation processes of fluid flow, sedimentation, mixing, and filtration.

2. *Heat transfer.* In this fundamental process, we are concerned with the transfer of heat from one place to another; it occurs in the separation processes of drying, evaporation, distillation, and others.

3. *Mass transfer.* Here mass is being transferred from one phase to another distinct phase; the basic mechanism is the same whether the phases are gas, solid, or liquid. This includes distillation, absorption, liquid–liquid extraction, membrane separation, adsorption, crystallization, and leaching.

## 1.1C Classification of Separation Processes

The separation processes deal mainly with the transfer and change of energy and the transfer and change of materials, primarily by physical means but also by physical–chemical means. The important separation processes, which can be combined in various sequences in a process and which are covered in this text, are described next.

1. *Evaporation.* This refers to the evaporation of a volatile solvent such as water from a nonvolatile solute such as salt or any other material in solution.

2. *Drying.* In this operation volatile liquids, usually water, are removed from solid materials.

3. *Distillation.* This is an operation whereby components of a liquid mixture are separated by boiling because of their differences in vapor pressure.

4. *Absorption.* In this process a component is removed from a gas stream by treatment with a liquid.

5. *Membrane separation.* This process involves the separation of a solute from a fluid by diffusion of this solute from a liquid or gas through a semipermeable membrane barrier to another fluid.

6. *Liquid–liquid extraction.* In this case a solute in a liquid solution is removed by contacting with another liquid solvent that is relatively immiscible with the solution.

7. *Adsorption.* In this process a component of a gas or liquid stream is removed and adsorbed by a solid adsorbent.

8. *Ion exchange.* Certain ions in solution are removed from a liquid by an ion-exchange solid.

9. *Liquid–solid leaching.* This involves treating a finely divided solid with a liquid that dissolves out and removes a solute contained in the solid.

10. *Crystallization.* This concerns the removal of a solute such as a salt from a solution by precipitating the solute from the solution.

11. *Mechanical–physical separations.* These involve separation of solids, liquids, or gases by mechanical means, such as filtration, settling, centrifugation, and size reduction.

### 1.1D Arrangement in Parts 1 and 2

This text is arranged in two parts:

*Part 1: Transport Processes: Momentum, Heat, and Mass.* These fundamental principles are covered extensively in Chapters 1 through 7 in order to provide the basis for study of separation processes in Part 2 of this text.

*Part 2: Separation Process Principles (Includes Unit Operations).* The various separation processes and their applications to process areas are studied in Part 2 of this text.

There are a number of elementary engineering principles, mathematical techniques, and laws of physics and chemistry that are basic to a study of the principles of momentum, heat, and mass transfer and the separation processes. These are reviewed for the reader in this first chapter. Some readers, especially chemical engineers, agricultural engineers, civil engineers, and chemists, may be familiar with many of these principles and techniques and may wish to omit all or parts of this chapter.

Homework problems at the end of each chapter are arranged in different sections, each corresponding to the number of a given section in the chapter.

## 1.2 SI SYSTEM OF BASIC UNITS USED IN THIS TEXT AND OTHER SYSTEMS

There are three main systems of basic units employed at present in engineering and science. The first and most important of these is the *SI* (Système International d'Unités) *system*, which has as its three basic units the meter (m), the kilogram (kg), and the second (s). The others are the English foot (ft)–pound (lb)–second (s), or *English system* and the centimeter (cm)–gram (g)–second (s), or *cgs system*.

At present the SI system has been adopted officially for use exclusively in engineering and science, but the older English and cgs systems will still be used for some time. Much of the physical and chemical data and empirical equations are given in these latter two systems. Hence, the engineer not only should be proficient in the SI system but must also be able to use the other two systems to a limited extent.

### 1.2A SI System of Units

The basic quantities used in the SI system are as follows: the unit of length is the meter (m); the unit of time is the second (s); the unit of mass is the kilogram (kg); the unit of temperature is the kelvin (K); and the unit of an element is the kilogram mole (kg mol). The other standard units are derived from these basic quantities.

The basic unit of force is the newton (N), defined as

$$1 \text{ newton (N)} = 1 \text{ kg} \cdot \text{m/s}^2$$

The basic unit of work, energy, or heat is the newton-meter, or joule (J).

$$1 \text{ joule (J)} = 1 \text{ newton} \cdot \text{m (N} \cdot \text{m)} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$$

Power is measured in joules/s or watts (W).

$$1 \text{ joule/s (J/s)} = 1 \text{ watt (W)}$$

The unit of pressure is the newton/m<sup>2</sup> or pascal (Pa).

$$1 \text{ newton/m}^2 \text{ (N/m}^2\text{)} = 1 \text{ pascal (Pa)}$$

[Pressure in atmospheres (atm) is not a standard SI unit but is being used during the transition period.] The standard acceleration of gravity is defined as

$$1 g = 9.80665 \text{ m/s}^2$$

A few of the standard prefixes for multiples of the basic units are as follows: giga (G) = 10<sup>9</sup>, mega (M) = 10<sup>6</sup>, kilo (k) = 10<sup>3</sup>, centi (c) = 10<sup>-2</sup>, milli (m) = 10<sup>-3</sup>, micro ( $\mu$ ) = 10<sup>-6</sup>, and nano (n) = 10<sup>-9</sup>. The prefix c is not a preferred prefix.

Temperatures are defined in kelvin (K) as the preferred unit in the SI system. However, in practice, wide use is made of the degree Celsius ( $^{\circ}\text{C}$ ) scale, which is defined by

$$t^{\circ}\text{C} = T(\text{K}) - 273.15$$

Note that  $1^{\circ}\text{C} = 1 \text{ K}$  and that in the case of temperature difference,

$$\Delta t^{\circ}\text{C} = \Delta T \text{ K}$$

The standard preferred unit of time is the second (s), but time can be in nondecimal units of minutes (min), hours (h), or days (d).

## 1.2B CGS System of Units

The cgs system is related to the SI system as follows:

$$1 \text{ g mass (g)} = 1 \times 10^{-3} \text{ kg mass (kg)}$$

$$1 \text{ cm} = 1 \times 10^{-2} \text{ m}$$

$$1 \text{ dyne (dyn)} = 1 \text{ g} \cdot \text{cm/s}^2 = 1 \times 10^{-5} \text{ newton (N)}$$

$$1 \text{ erg} = 1 \text{ dyn} \cdot \text{cm} = 1 \times 10^{-7} \text{ joule (J)}$$

The standard acceleration of gravity is

$$g = 980.665 \text{ cm/s}^2$$

## 1.2C English fps System of Units

The English system is related to the SI system as follows:

$$1 \text{ lb mass (lb}_m\text{)} = 0.45359 \text{ kg}$$

$$1 \text{ ft} = 0.30480 \text{ m}$$

$$1 \text{ lb force (lb}_f\text{)} = 4.4482 \text{ newton (N)}$$

$$1 \text{ ft} \cdot \text{lb}_f = 1.35582 \text{ newton} \cdot \text{m (N} \cdot \text{m)} = 1.35582 \text{ joules (J)}$$

$$1 \text{ psia} = 6.89476 \times 10^3 \text{ newton/m}^2 \text{ (N/m}^2\text{)}$$

$$1.8^\circ\text{F} = 1 \text{ K} = 1^\circ\text{C (centigrade or Celsius)}$$

$$g = 32.174 \text{ ft/s}^2$$

The proportionality factor for Newton's law is

$$g_c = 32.174 \text{ ft} \cdot \text{lb}_m / \text{lb}_f \cdot \text{s}^2$$

The factor  $g_c$  in SI units and cgs units is 1.0 and is omitted.

In Appendix A.1, convenient conversion factors for all three systems are tabulated. Further discussions and use of these relationships are given in various sections of the text.

This text uses the SI system as the primary set of units in the equations, sample problems, and homework problems. However, the important equations derived in the text are given in a dual set of units, SI and English, when these equations differ. Some example problems and homework problems are also given using English units. In some cases, intermediate steps and/or answers in example problems are also stated in English units.

## 1.2D Dimensionally Homogeneous Equations and Consistent Units

A dimensionally homogeneous equation is one in which all the terms have the same units. These units can be the base units or derived ones (for example,  $\text{kg/s}^2 \cdot \text{m}$  or Pa). Such an equation can be used with any system of units provided that the same base or derived units are used throughout the equation. No conversion factors are needed when consistent units are used.

The reader should be careful in using any equation and always check it for dimensional homogeneity. To do this, a system of units (SI, English, etc.) is first selected. Then units are substituted for each term in the equation and like units in each term canceled out.

## 1.3 METHODS OF EXPRESSING TEMPERATURES AND COMPOSITIONS

### 1.3A Temperature

There are two temperature scales in common use in the chemical and biological industries. These are degrees Fahrenheit (abbreviated °F) and Celsius (°C). It is often necessary to convert from one scale to the other. Both use the freezing point and boiling point of water at 1 atmosphere pressure as base points. Often temperatures are expressed as absolute degrees K (SI standard) or degrees Rankine (°R) instead of °C or °F. Table 1.3-1 shows the equivalences of the four temperature scales.

TABLE 1.3-1. *Temperature Scales and Equivalents*

	<i>Centigrade</i>	<i>Fahrenheit</i>	<i>Kelvin</i>	<i>Rankine</i>	<i>Celsius</i>
Boiling water	100°C	212°F	373.15 K	671.67°R	100°C
Melting ice	0°C	32°F	273.15 K	491.67°R	0°C
Absolute zero	-273.15°C	-459.67°F	0 K	0°R	-273.15°C

The difference between the boiling point of water and melting point of ice at 1 atm is 100°C or 180°F. Thus, a 1.8°F change is equal to a 1°C change. Usually, the value of -273.15°C is rounded to -273.2°C and -459.67°F to -460°F. The following equations can be used to convert from one scale to another:

$$^{\circ}\text{F} = 32 + 1.8(^{\circ}\text{C}) \quad (1.3-1)$$

$$^{\circ}\text{C} = \frac{1}{1.8}(^{\circ}\text{F} - 32) \quad (1.3-2)$$

$$^{\circ}\text{R} = ^{\circ}\text{F} + 459.67 \quad (1.3-3)$$

$$\text{K} = ^{\circ}\text{C} + 273.15 \quad (1.3-4)$$

### 1.3B Mole Units, and Weight or Mass Units

There are many methods used to express compositions in gases, liquids, and solids. One of the most useful is molar units, since chemical reactions and gas laws are simpler to express in terms of molar units. A mole (mol) of a pure substance is defined as the amount of that substance whose mass is numerically equal to its molecular weight. Hence, 1 kg mol of methane CH<sub>4</sub> contains 16.04 kg. Also, 1.0 lb mol contains 16.04 lb<sub>m</sub>.

The mole fraction of a particular substance is simply the moles of this substance divided by the total number of moles. In like manner, the weight or mass fraction is the mass of the substance divided by the total mass. These two compositions, which hold for gases, liquids, and solids, can be expressed as follows for component *A* in a mixture:

$$x_A \text{ (mole fraction of } A) = \frac{\text{moles of } A}{\text{total moles}} \quad (1.3-5)$$

$$w_A \text{ (mass or wt fraction of } A) = \frac{\text{mass } A}{\text{total mass}} \quad (1.3-6)$$

#### **EXAMPLE 1.3-1. Mole and Mass or Weight Fraction of a Solution**

A container holds 50 g of water (*B*) and 50 g of NaOH (*A*). Calculate the weight fraction and mole fraction of NaOH. Also, calculate the lb<sub>m</sub> of NaOH (*A*) and H<sub>2</sub>O (*B*).

**Solution:** Taking as a basis for calculation 50 + 50 or 100 g of solution, the following data are calculated:

Component	<i>G</i>	<i>Wt Fraction</i>	<i>Mol Wt</i>	<i>G Moles</i>	<i>Mole Fraction</i>
H <sub>2</sub> O ( <i>B</i> )	50.0	$\frac{50}{100} = 0.500$	18.02	$\frac{50.0}{18.02} = 2.78$	$\frac{2.78}{4.03} = 0.690$
NaOH ( <i>A</i> )	50.0	$\frac{50}{100} = 0.500$	40.0	$\frac{50.0}{40.0} = 1.25$	$\frac{1.25}{4.03} = 0.310$
Total	<u>100.0</u>	<u>1.000</u>		<u>4.03</u>	<u>1.000</u>

Hence,  $x_A = 0.310$  and  $x_B = 0.690$  and  $x_A + x_B = 0.310 + 0.690 = 1.00$ . Also,  $w_A + w_B = 0.500 + 0.500 = 1.00$ . To calculate the lb<sub>m</sub> of each component, Appendix A.1 gives the conversion factor of 453.6 g per 1 lb<sub>m</sub>. Using this,

$$\text{lb mass of } A = \frac{50 \text{ g } A}{453.6 \text{ g } A/\text{lb}_m A} = 0.1102 \text{ lb}_m A$$

Note that the g of  $A$  in the numerator cancels the g of  $A$  in the denominator, leaving  $\text{lb}_m$  of  $A$  in the numerator. The reader is cautioned to put all units down in an equation and cancel those appearing in the numerator and denominator. In a similar manner we obtain  $0.1102 \text{ lb}_m B$  ( $0.0500 \text{ kg } B$ ).

The analyses of solids and liquids are usually given as weight or mass fraction or weight percent, and gases as mole fraction or percent. Unless otherwise stated, analyses of solids and liquids will be assumed to be weight (mass) fraction or percent, and of gases to be mole fraction or percent.

### 1.3C Concentration Units for Liquids

In general, when one liquid is mixed with another miscible liquid, the volumes are not additive. Hence, compositions of liquids are usually not expressed as volume percent of a component but as weight or mole percent. Another convenient way to express concentrations of components in a solution is *molarity*, which is defined as g mol of a component per liter of solution. Other methods used are  $\text{kg}/\text{m}^3$ , g/liter,  $\text{g}/\text{cm}^3$ , lb mol/cu ft,  $\text{lb}_m/\text{cu ft}$ , and  $\text{lb}_m/\text{gallon}$ . All these concentrations depend on temperature, so the temperature must be specified.

The most common method of expressing total concentration per unit volume is density,  $\text{kg}/\text{m}^3$ ,  $\text{g}/\text{cm}^3$ , or  $\text{lb}_m/\text{ft}^3$ . For example, the density of water at 277.2 K ( $4^\circ\text{C}$ ) is  $1000 \text{ kg}/\text{m}^3$ , or  $62.43 \text{ lb}_m/\text{ft}^3$ . Sometimes the density of a solution is expressed as *specific gravity*, which is defined as the density of the solution at its given temperature divided by the density of a reference substance at its temperature. If the reference substance is water at 277.2 K, the specific gravity and density of the substance are numerically equal.

## 1.4 GAS LAWS AND VAPOR PRESSURE

### 1.4A Pressure

There are numerous ways of expressing the pressure exerted by a fluid or system. An *absolute pressure* of 1.00 atm is equivalent to 760 mm Hg at  $0^\circ\text{C}$ , 29.921 in. Hg, 0.760 m Hg, 14.696 lb force per square inch (psia), or 33.90 ft of water at  $4^\circ\text{C}$ . *Gage pressure* is the pressure above the absolute pressure. Hence, a pressure of 21.5 lb per square inch gage (*psig*) is  $21.5 + 14.7$  (rounded off), or 36.2 psia. In SI units, 1 psia =  $6.89476 \times 10^3$  pascal (Pa) =  $6.89476 \times 10^3$  newtons/ $\text{m}^2$ . Also, 1 atm =  $1.01325 \times 10^5$  Pa.

In some cases, particularly in evaporation, one may express the pressure as inches of mercury vacuum. This means the pressure as inches of mercury measured “below” the absolute barometric pressure. For example, a reading of 25.4 in. Hg vacuum is  $29.92 - 25.4$ , or 4.52 in. Hg absolute pressure. Pressure conversion units are given in Appendix A.1.

### 1.4B Ideal Gas Law

An *ideal gas* is defined as one that obeys simple laws. Also, in an ideal gas the gas molecules are considered as rigid spheres which themselves occupy no volume and do not exert forces on one another. No real gases obey these laws exactly, but at ordinary temperatures and pres-



tures of not more than several atmospheres, the ideal laws give answers within a few percent or less of the actual answers. Hence, these laws are sufficiently accurate for engineering calculations.

The *ideal gas law* of Boyle states that the volume of a gas is directly proportional to the absolute temperature and inversely proportional to the absolute pressure. This is expressed as

$$pV = nRT \quad (1.4-1)$$

where  $p$  is the absolute pressure in  $\text{N/m}^2$ ,  $V$  the volume of the gas in  $\text{m}^3$ ,  $n$  the kg mol of the gas,  $T$  the absolute temperature in K, and  $R$  the gas law constant of  $8314.3 \text{ kg} \cdot \text{m}^2/\text{kg mol} \cdot \text{s}^2 \cdot \text{K}$ . When the volume is in  $\text{ft}^3$ ,  $n$  in lb moles, and  $T$  in  $^\circ\text{R}$ ,  $R$  has a value of  $0.7302 \text{ ft}^3 \cdot \text{atm}/\text{lb mol} \cdot ^\circ\text{R}$ . For cgs units (see Appendix A.1),  $V = \text{cm}^3$ ,  $T = \text{K}$ ,  $R = 82.057 \text{ cm}^3 \cdot \text{atm}/\text{g mol} \cdot \text{K}$ , and  $n = \text{g mol}$ .

In order that amounts of various gases may be compared, *standard conditions of temperature and pressure* (abbreviated STP or SC) are arbitrarily defined as 101.325 kPa (1.0 atm) abs and 273.15 K ( $0^\circ\text{C}$ ). Under these conditions the volumes are as follows:

$$\begin{aligned} \text{volume of 1.0 kg mol (SC)} &= 22.414 \text{ m}^3 \\ \text{volume of 1.0 g mol (SC)} &= 22.414 \text{ L (liter)} \\ &= 22\,414 \text{ cm}^3 \\ \text{volume of 1.0 lb mol (SC)} &= 359.05 \text{ ft}^3 \end{aligned}$$

**EXAMPLE 1.4-1. Gas-Law Constant**

Calculate the value of the gas-law constant  $R$  when the pressure is in psia, moles in lb mol, volume in  $\text{ft}^3$ , and temperature in  $^\circ\text{R}$ . Repeat for SI units.

**Solution:** At standard conditions,  $p = 14.7 \text{ psia}$ ,  $V = 359 \text{ ft}^3$ , and  $T = 460 + 32 = 492^\circ\text{R}$  ( $273.15 \text{ K}$ ). Substituting into Eq. (1.4-1) for  $n = 1.0 \text{ lb mol}$  and solving for  $R$ ,

$$\begin{aligned} R &= \frac{pV}{nT} = \frac{(14.7 \text{ psia})(359 \text{ ft}^3)}{(1.0 \text{ lb mol})(492^\circ\text{R})} = 10.73 \frac{\text{ft}^3 \cdot \text{psia}}{\text{lb mol} \cdot ^\circ\text{R}} \\ R &= \frac{pV}{nT} = \frac{(1.01325 \times 10^5 \text{ Pa})(22.414 \text{ m}^3)}{(1.0 \text{ kg mol})(273.15 \text{ K})} = 8314 \frac{\text{m}^3 \cdot \text{Pa}}{\text{kg mol} \cdot \text{K}} \end{aligned}$$

A useful relation can be obtained from Eq. (1.4-1) for  $n$  moles of gas at conditions  $p_1, V_1, T_1$ , and also at conditions  $p_2, V_2, T_2$ . Substituting into Eq. (1.4-1),

$$\begin{aligned} p_1V_1 &= nRT_1 \\ p_2V_2 &= nRT_2 \end{aligned}$$

Combining gives

$$\frac{p_1V_1}{p_2V_2} = \frac{T_1}{T_2} \quad (1.4-2)$$

### 1.4C Ideal Gas Mixtures

Dalton's law for mixtures of ideal gases states that the total pressure of a gas mixture is equal to the sum of the individual partial pressures:

$$P = p_A + p_B + p_C + \cdots \quad (1.4-3)$$

where  $P$  is total pressure and  $p_A, p_B, p_C, \dots$  are the partial pressures of the components  $A, B, C, \dots$  in the mixture.

Since the number of moles of a component is proportional to its partial pressure, the mole fraction of a component is

$$x_A = \frac{p_A}{P} = \frac{p_A}{p_A + p_B + p_C + \cdots} \quad (1.4-4)$$

The volume fraction is equal to the mole fraction. Gas mixtures are almost always represented in terms of mole fractions and not weight fractions. For engineering purposes, Dalton's law is sufficiently accurate to use for actual mixtures at total pressures of a few atmospheres or less.

#### **EXAMPLE 1.4-2. Composition of a Gas Mixture**

A gas mixture contains the following components and partial pressures: CO<sub>2</sub>, 75 mm Hg; CO, 50 mm Hg; N<sub>2</sub>, 595 mm Hg; O<sub>2</sub>, 26 mm Hg. Calculate the total pressure and the composition in mole fraction.

**Solution:** Substituting into Eq. (1.4-3),

$$P = p_A + p_B + p_C + p_D = 75 + 50 + 595 + 26 = 746 \text{ mm Hg}$$

The mole fraction of CO<sub>2</sub> is obtained by using Eq. (1.4-4).

$$x_A(\text{CO}_2) = \frac{p_A}{P} = \frac{75}{746} = 0.101$$

In like manner, the mole fractions of CO, N<sub>2</sub>, and O<sub>2</sub> are calculated as 0.067, 0.797, and 0.035, respectively.

### 1.4D Vapor Pressure and Boiling Point of Liquids

When a liquid is placed in a sealed container, molecules of liquid will evaporate into the space above the liquid and fill it completely. After a time, equilibrium is reached. This vapor will exert a pressure just like a gas and we call this pressure the *vapor pressure* of the liquid. The value of the vapor pressure is independent of the amount of liquid in the container as long as some is present.

If an inert gas such as air is also present in the vapor space, it will have very little effect on the vapor pressure. In general, the effect of total pressure on vapor pressure can be considered as negligible for pressures of a few atmospheres or less.

The vapor pressure of a liquid increases markedly with temperature. For example, from Appendix A.2 for water, the vapor pressure at 50°C is 12.333 kPa (92.51 mm Hg). At 100°C the vapor pressure has increased greatly to 101.325 kPa (760 mm Hg).

The *boiling point* of a liquid is defined as the temperature at which the vapor pressure of a liquid equals the total pressure. Hence, if the atmospheric total pressure is 760 mm Hg, wa-

ter will boil at 100°C. On top of a high mountain, where the total pressure is considerably less, water will boil at temperatures below 100°C.

A plot of vapor pressure  $P_A$  of a liquid versus temperature does not yield a straight line but a curve. However, for moderate temperature ranges, a plot of  $\log P_A$  versus  $1/T$  is a reasonably straight line, as follows:

$$\log P_A = m \left( \frac{1}{T} \right) + b \quad (1.4-5)$$

where  $m$  is the slope,  $b$  is a constant for the liquid  $A$ , and  $T$  is the temperature in K.

## 1.5 CONSERVATION OF MASS AND MATERIAL BALANCES

### 1.5A Conservation of Mass

One of the basic laws of physical science is the *law of conservation of mass*. This law, stated simply, says that mass cannot be created or destroyed (excluding, of course, nuclear or atomic reactions). Hence, the total mass (or weight) of all materials entering any process must equal the total mass of all materials leaving plus the mass of any materials accumulating or left in the process:

$$\text{input} = \text{output} + \text{accumulation} \quad (1.5-1)$$

In the majority of cases there will be no accumulation of materials in a process, and then the input will simply equal the output. Stated in other words, “what goes in must come out.” We call this type of process a *steady-state process*:

$$\text{input} = \text{output (steady state)} \quad (1.5-2)$$

### 1.5B Simple Material Balances

In this section we do simple material (weight or mass) balances in various processes at steady state with no chemical reaction occurring. We can use units of kg, lb<sub>m</sub>, lb mol, g, kg mol, and so on, in our balances. The reader is cautioned to be consistent and not to mix several units in a balance. When chemical reactions occur in the balances (as discussed in Section 1.5D), one should use kg mol units, since chemical equations relate moles reacting. In Section 2.6, overall mass balances will be covered in more detail and in Section 3.6, differential mass balances.

To solve a material-balance problem, it is advisable to proceed by a series of definite steps, as listed below:

1. *Sketch a simple diagram of the process.* This can be a simple box diagram showing each stream entering by an arrow pointing in and each stream leaving by an arrow pointing out. Include on each arrow the compositions, amounts, temperatures, and so on, of that stream. All pertinent data should be on this diagram.
2. *Write the chemical equations involved (if any).*
3. *Select a basis for calculation.* In most cases the problem is concerned with a specific amount of one of the streams in the process, which is selected as the basis.

4. *Make a material balance.* The arrows into the process will be input items and the arrows going out output items. The balance can be a total material balance in Eq. (1.5-2) or a balance on each component present (if no chemical reaction occurs).

Typical processes that do not undergo chemical reactions are drying, evaporation, dilution of solutions, distillation, extraction, and so on. These can be solved by setting up material balances containing unknowns and solving these equations for the unknowns.

**EXAMPLE 1.5-1. Concentration of Orange Juice**

In the concentration of orange juice, a fresh extracted and strained juice containing 7.08 wt % solids is fed to a vacuum evaporator. In the evaporator, water is removed and the solids content increased to 58 wt % solids. For 1000 kg/h entering, calculate the amounts of the outlet streams of concentrated juice and water.

**Solution:** Following the four steps outlined, we make a process flow diagram (step 1) in Fig. 1.5-1. Note that the letter  $W$  represents the unknown amount of water and  $C$  the amount of concentrated juice. No chemical reactions are given (step 2). Basis: 1000 kg/h entering juice (step 3).

To make the material balances (step 4), a total material balance will be made using Eq. (1.5-2):

$$1000 = W + C \tag{1.5-3}$$

This gives one equation and two unknowns. Hence, a component balance on solids will be made:

$$1000 \left( \frac{7.08}{100} \right) = W(0) + C \left( \frac{58}{100} \right) \tag{1.5-4}$$

To solve these two equations, we solve Eq. (1.5-4) first for  $C$  since  $W$  drops out. We get  $C = 122.1$  kg/h concentrated juice.

Substituting the value of  $C$  into Eq. (1.5-3),

$$1000 = W + 122.1$$

and we obtain  $W = 877.9$  kg/h water.

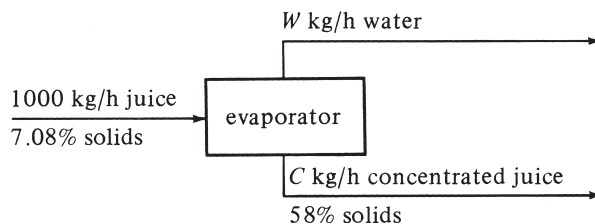


FIGURE 1.5-1. Process flow diagram for Example 1.5-1.

As a check on our calculations, we can write a balance on the water component:

$$1000 \left( \frac{100 - 7.08}{100} \right) = 877.9 + 122.1 \left( \frac{100 - 58}{100} \right) \quad (1.5-5)$$

Solving,

$$929.2 = 877.9 + 51.3 = 929.2$$

In Example 1.5-1 only one unit or separate process was involved. Often, a number of processes in series are involved. Then we have a choice of making a separate balance over each separate process and/or a balance around the complete overall process.

### 1.5C Material Balances and Recycle

Processes that have a recycle or feedback of part of the product into the entering feed are sometimes encountered. For example, in a sewage treatment plant, part of the activated sludge from a sedimentation tank is recycled back to the aeration tank where the liquid is treated. In some food-drying operations, the humidity of the entering air is controlled by recirculating part of the hot, wet air that leaves the dryer. In chemical reactions, the material that did not react in the reactor can be separated from the final product and fed back to the reactor.

#### **EXAMPLE 1.5-2. Crystallization of $KNO_3$ and Recycle**

In a process producing  $KNO_3$  salt, 1000 kg/h of a feed solution containing 20 wt %  $KNO_3$  is fed to an evaporator, which evaporates some water at 422 K to produce a 50 wt %  $KNO_3$  solution. This is then fed to a crystallizer at 311 K, where crystals containing 96 wt %  $KNO_3$  are removed. The saturated solution containing 37.5 wt %  $KNO_3$  is recycled to the evaporator. Calculate the amount of recycle stream  $R$  in kg/h and the product stream of crystals  $P$  in kg/h.

**Solution:** Figure 1.5-2 gives the process flow diagram. As a basis we shall use 1000 kg/h of fresh feed. No chemical reactions are occurring. We can make an overall balance on the entire process for  $KNO_3$  and solve for  $P$  directly:

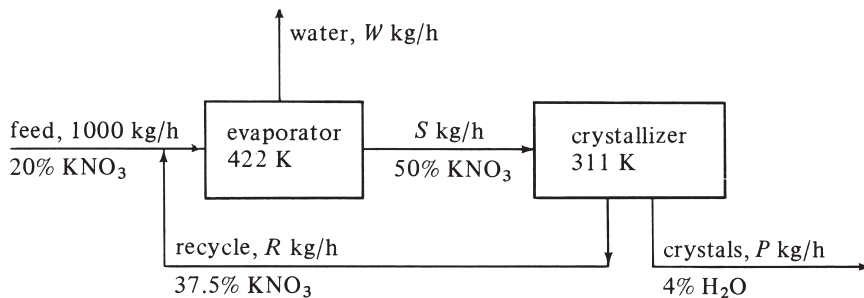


FIGURE 1.5-2. Process flow diagram for Example 1.5-2.

$$1000(0.20) = W(0) + P(0.96) \quad (1.5-6)$$

$$P = 208.3 \text{ kg crystals/h}$$

To calculate the recycle stream, we can make a balance around the evaporator or the crystallizer. Using a balance on the crystallizer, since it now includes only two unknowns,  $S$  and  $R$ , we get for a total balance,

$$S = R + 208.3 \quad (1.5-7)$$

For a  $\text{KNO}_3$  balance on the crystallizer,

$$S(0.50) = R(0.375) + 208.3(0.96) \quad (1.5-8)$$

Substituting  $S$  from Eq. (1.5-7) into Eq. (1.5-8) and solving,  $R = 766.6$  kg recycle/h and  $S = 974.9$  kg/h.

### 1.5D Material Balances and Chemical Reaction

In many cases the materials entering a process undergo chemical reactions in the process, so that the materials leaving are different from those entering. In these cases it is usually convenient to make a molar and not a weight balance on an individual component, such as kg mol  $\text{H}_2$  or kg atom H, kg mol  $\text{CO}_3^-$  ion, kg mol  $\text{CaCO}_3$ , kg atom  $\text{Na}^+$ , kg mol  $\text{N}_2$ , and so on. For example, in the combustion of  $\text{CH}_4$  with air, balances can be made on kg mol of  $\text{H}_2$ , C,  $\text{O}_2$ , or  $\text{N}_2$ .

#### EXAMPLE 1.5-3. Combustion of Fuel Gas

A fuel gas containing 3.1 mol %  $\text{H}_2$ , 27.2% CO, 5.6%  $\text{CO}_2$ , 0.5%  $\text{O}_2$ , and 63.6%  $\text{N}_2$  is burned with 20% excess air (i.e., the air over and above that necessary for complete combustion to  $\text{CO}_2$  and  $\text{H}_2\text{O}$ ). The combustion of CO is only 98% complete. For 100 kg mol of fuel gas, calculate the moles of each component in the exit flue gas.

**Solution:** First, the process flow diagram is drawn (Fig. 1.5-3). On the diagram the components in the flue gas are shown. Let  $A$  be moles of air and  $F$  be moles of flue gas. Next the chemical reactions are given:

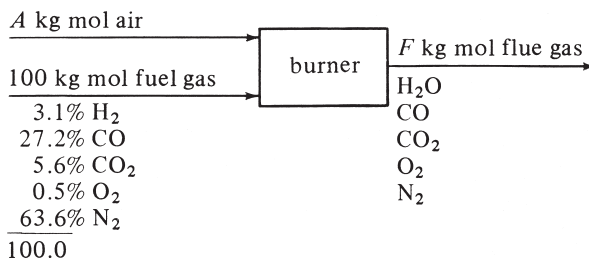


FIGURE 1.5-3. Process flow diagram for Example 1.5-3.

An accounting of the total moles of O<sub>2</sub> in the fuel gas is as follows:

$$\text{mol O}_2 \text{ in fuel gas} = \left(\frac{1}{2}\right)27.2(\text{CO}) + 5.6(\text{CO}_2) + 0.5(\text{O}_2) = 19.7 \text{ mol O}_2$$

For all the H<sub>2</sub> to be completely burned to H<sub>2</sub>O, we need, from Eq. (1.5-10),  $\frac{1}{2}$  mol O<sub>2</sub> for 1 mol H<sub>2</sub>, or  $3.1\left(\frac{1}{2}\right) = 1.55$  total mol O<sub>2</sub>. For completely burning the CO from Eq. (1.5-9), we need  $27.2\left(\frac{1}{2}\right) = 13.6$  mol O<sub>2</sub>. Hence, the amount of O<sub>2</sub> we must add is, theoretically, as follows:

$$\begin{aligned} \text{mol O}_2 \text{ theoretically needed} &= 1.55 + 13.6 - 0.5 \text{ (in fuel gas)} \\ &= 14.65 \text{ mol O}_2 \end{aligned}$$

For a 20% excess, we add  $1.2(14.65)$ , or 17.58 mol O<sub>2</sub>. Since air contains 79 mol % N<sub>2</sub>, the amount of N<sub>2</sub> added is  $(79/21)(17.58)$ , or 66.1 mol N<sub>2</sub>.

To calculate the moles in the final flue gas, all the H<sub>2</sub> gives H<sub>2</sub>O, or 3.1 mol H<sub>2</sub>O. For CO, 2.0% does not react. Hence,  $0.02(27.2)$ , or 0.54 mol CO will be unburned.

A total carbon balance is as follows: inlet moles C =  $27.2 + 5.6 = 32.8$  mol C. In the outlet flue gas, 0.54 mol will be as CO and the remainder of  $32.8 - 0.54$ , or 32.26 mol as CO<sub>2</sub>.

For calculating the outlet mol O<sub>2</sub>, we make an overall O<sub>2</sub> balance:

$$\begin{aligned} \text{O}_2 \text{ in} &= 19.7 \text{ (in fuel gas)} + 17.58 \text{ (in air)} = 37.28 \text{ mol O}_2 \\ \text{O}_2 \text{ out} &= (3.1/2) \text{ (in H}_2\text{O)} + (0.54/2) \text{ (in CO)} + 32.26 \text{ (in CO}_2\text{)} + \text{free O}_2 \end{aligned}$$

Equating inlet O<sub>2</sub> to outlet, the free remaining O<sub>2</sub> = 3.2 mol O<sub>2</sub>. For the N<sub>2</sub> balance, the outlet =  $63.6 \text{ (in fuel gas)} + 66.1 \text{ (in air)}$ , or 129.70 mol N<sub>2</sub>. The outlet flue gas contains 3.10 mol H<sub>2</sub>O, 0.54 mol CO, 32.26 mol CO<sub>2</sub>, 3.20 mol O<sub>2</sub>, and 129.7 mol N<sub>2</sub>.

In chemical reactions with several reactants, the limiting reactant component is defined as that compound which is present in an amount less than the amount necessary for it to react stoichiometrically with the other reactants. Then the percent completion of a reaction is the amount of this limiting reactant actually converted, divided by the amount originally present, times 100.

## 1.6 ENERGY AND HEAT UNITS

### 1.6A Joule, Calorie, and Btu

In a manner similar to that used in making material balances on chemical and biological processes, we can also make energy balances on a process. Often a large portion of the energy entering or leaving a system is in the form of heat. Before such energy or heat balances are made, we must understand the various types of energy and heat units.

In the SI system, energy is given in joules (J) or kilojoules (kJ). Energy is also expressed in btu (British thermal units) or cal (calories). The g calorie (abbreviated cal) is defined as the amount of heat needed to heat 1.0 g water 1.0°C (from 14.5°C to 15.5°C). Also, 1 kcal

(kilocalorie) = 1000 cal. The btu is defined as the amount of heat needed to raise 1.0 lb water 1°F. Hence, from Appendix A.1,

$$1 \text{ btu} = 252.16 \text{ cal} = 1.05506 \text{ kJ} \quad (1.6-1)$$

### 1.6B Heat Capacity

The *heat capacity* of a substance is defined as the amount of heat necessary to increase the temperature by 1 degree. It can be expressed for 1 g, 1 lb, 1 g mol, 1 kg mol, or 1 lb mol of the substance. For example, a heat capacity is expressed in SI units as J/kg mol · K; in other units as cal/g · °C, cal/g mol · °C, kcal/kg mol · °C, btu/lb<sub>m</sub> · °F, or btu/lb mol · °F.

It can be shown that the actual numerical value of a heat capacity is the same in mass units or in molar units. That is,

$$1.0 \text{ cal/g} \cdot ^\circ\text{C} = 1.0 \text{ btu/lb}_m \cdot ^\circ\text{F} \quad (1.6-2)$$

$$1.0 \text{ cal/g mol} \cdot ^\circ\text{C} = 1.0 \text{ btu/lb mol} \cdot ^\circ\text{F} \quad (1.6-3)$$

For example, to prove this, suppose that a substance has a heat capacity of 0.8 btu/lb<sub>m</sub> · °F. The conversion is made using 1.8°F for 1°C or 1 K, 252.16 cal for 1 btu, and 453.6 g for 1 lb<sub>m</sub>, as follows:

$$\begin{aligned} \text{heat capacity} \left( \frac{\text{cal}}{\text{g} \cdot ^\circ\text{C}} \right) &= \left( 0.8 \frac{\text{btu}}{\text{lb}_m \cdot ^\circ\text{F}} \right) \left( 252.16 \frac{\text{cal}}{\text{btu}} \right) \left( \frac{1}{453.6 \text{ g/lb}_m} \right) \left( 1.8 \frac{^\circ\text{F}}{^\circ\text{C}} \right) \\ &= 0.8 \frac{\text{cal}}{\text{g} \cdot ^\circ\text{C}} \end{aligned}$$

The heat capacities of gases (sometimes called *specific heat*) at constant pressure  $c_p$  are functions of temperature and for engineering purposes can be assumed to be independent of pressure up to several atmospheres. In most process engineering calculations, one is usually interested in the amount of heat needed to heat a gas from one temperature  $t_1$  to another at  $t_2$ . Since the  $c_p$  varies with temperature, an integration must be performed or a suitable mean  $c_{pm}$  used. These mean values for gases have been obtained for  $T_1$  of 298 K or 25°C (77°F) and various  $T_2$  values, and are tabulated in Table 1.6-1 at 101.325 kPa pressure or less as  $c_{pm}$  in kJ/kg mol · K at various values of  $T_2$  in K or °C.

#### EXAMPLE 1.6-1. Heating of N<sub>2</sub> Gas

The gas N<sub>2</sub> at 1 atm pressure absolute is being heated in a heat exchanger. Calculate the amount of heat needed in J to heat 3.0 g mol N<sub>2</sub> in the following temperature ranges:

- 298–673 K (25–400°C)
- 298–1123 K (25–850°C)
- 673–1123 K (400–850°C)

**Solution:** For case (a), Table 1.6-1 gives  $c_{pm}$  values at 1 atm pressure or less which can be used up to several atm pressures. For N<sub>2</sub> at 673 K,  $c_{pm} = 29.68$  kJ/kg mol · K or 29.68 J/g mol · K. This is the mean heat capacity for the range 298–673 K:

$$\text{heat required} = M \text{ g mol} \left( c_{pm} \frac{\text{J}}{\text{g mol} \cdot \text{K}} \right) (T_2 - T_1) \text{K} \quad (1.6-4)$$



TABLE 1.6-1. Mean Molar Heat Capacities of Gases Between 298 and TK (25 and T°C) at 101.325 kPa or Less (SI Units:  $c_p = \text{kJ/kg mol} \cdot \text{K}$ )

T(K)	T(°C)	H <sub>2</sub>	N <sub>2</sub>	CO	Air	O <sub>2</sub>	H <sub>2</sub> O	CO <sub>2</sub>	CH <sub>4</sub>	SO <sub>2</sub>
298	25	28.86	29.14	29.16	29.19	29.38	33.59	37.20	35.8	39.9
373	100	28.99	29.19	29.24	29.29	29.66	33.85	38.73	37.6	41.2
473	200	29.13	29.29	29.38	29.40	30.07	34.24	40.62	40.3	42.9
573	300	29.18	29.46	29.60	29.61	30.53	34.39	42.32	43.1	44.5
673	400	29.23	29.68	29.88	29.94	31.01	35.21	43.80	45.9	45.8
773	500	29.29	29.97	30.19	30.25	31.46	35.75	45.12	48.8	47.0
873	600	29.35	30.27	30.52	30.56	31.89	36.33	46.28	51.4	47.9
973	700	29.44	30.56	30.84	30.87	32.26	36.91	47.32	54.0	48.8
1073	800	29.56	30.85	31.16	31.18	32.62	37.53	48.27	56.4	49.6
1173	900	29.63	31.16	31.49	31.48	32.97	38.14	49.15	58.8	50.3
1273	1000	29.84	31.43	31.77	31.79	33.25	38.71	49.91	61.0	50.9
1473	1200	30.18	31.97	32.30	32.32	33.78	39.88	51.29	64.9	51.9
1673	1400	30.51	32.40	32.73	32.76	34.19	40.90	52.34		

Mean Molar Heat Capacities of Gases Between 25 and T°C at 1 atm Pressure or Less (English Units:  $c_p = \text{btu/lb mol} \cdot \text{°F}$ )

T(°C)	H <sub>2</sub>	N <sub>2</sub>	CO	Air	O <sub>2</sub>	NO	H <sub>2</sub> O	CO <sub>2</sub>	HCl	Cl <sub>2</sub>	CH <sub>4</sub>	SO <sub>2</sub>	C <sub>2</sub> H <sub>4</sub>	SO <sub>3</sub>	C <sub>2</sub> H <sub>6</sub>
25	6.894	6.961	6.965	6.972	7.017	7.134	8.024	8.884	6.96	8.12	8.55	9.54	10.45	12.11	12.63
100	6.924	6.972	6.983	6.996	7.083	7.144	8.084	9.251	6.97	8.24	8.98	9.85	11.35	12.84	13.76
200	6.957	6.996	7.017	7.021	7.181	7.224	8.177	9.701	6.98	8.37	9.62	10.25	12.53	13.74	15.27
300	6.970	7.036	7.070	7.073	7.293	7.252	8.215	10.108	7.00	8.48	10.29	10.62	13.65	14.54	16.72
400	6.982	7.089	7.136	7.152	7.406	7.301	8.409	10.462	7.02	8.55	10.97	10.94	14.67	15.22	18.11
500	6.995	7.159	7.210	7.225	7.515	7.389	8.539	10.776	7.06	8.61	11.65	11.22	15.60	15.82	19.39
600	7.011	7.229	7.289	7.299	7.616	7.470	8.678	11.053	7.10	8.66	12.27	11.45	16.45	16.33	20.58
700	7.032	7.298	7.365	7.374	7.706	7.549	8.816	11.303	7.15	8.70	12.90	11.66	17.22	16.77	21.68
800	7.060	7.369	7.443	7.447	7.792	7.630	8.963	11.53	7.21	8.73	13.48	11.84	17.95	17.17	22.72
900	7.076	7.443	7.521	7.520	7.874	7.708	9.109	11.74	7.27	8.77	14.04	12.01	18.63	17.52	23.69
1000	7.128	7.507	7.587	7.593	7.941	7.773	9.246	11.92	7.33	8.80	14.56	12.15	19.23	17.86	24.56
1100	7.169	7.574	7.653	7.660	8.009	7.839	9.389	12.10	7.39	8.82	15.04	12.28	19.81	18.17	25.40
1200	7.209	7.635	7.714	7.719	8.068	7.898	9.524	12.25	7.45	8.94	15.49	12.39	20.33	18.44	26.15
1300	7.252	7.692	7.772	7.778	8.123	7.952	9.66	12.39							
1400	7.288	7.738	7.818	7.824	8.166	7.994	9.77	12.50							
1500	7.326	7.786	7.866	7.873	8.203	8.039	9.89	12.69							
1600	7.386	7.844	7.922	7.929	8.269	8.092	9.95	12.75							
1700	7.421	7.879	7.958	7.965	8.305	8.124	10.13	12.70							
1800	7.467	7.924	8.001	8.010	8.349	8.164	10.24	12.94							
1900	7.505	7.957	8.033	8.043	8.383	8.192	10.34	13.01							
2000	7.548	7.994	8.069	8.081	8.423	8.225	10.43	13.10							
2100	7.588	8.028	8.101	8.115	8.460	8.255	10.52	13.17							
2200	7.624	8.054	8.127	8.144	8.491	8.277	10.61	13.24							

Source: O. A. Hougen, K. W. Watson, and R. A. Ragatz, *Chemical Process Principles*, Part I, 2nd ed. New York: John Wiley & Sons, Inc., 1954. With permission.

Substituting the known values,

$$\text{heat required} = (3.0)(29.68)(673 - 298) = 33\,390 \text{ J}$$

For case (b), the  $c_{pm}$  at 1123 K (obtained by linear interpolation between 1073 and 1173 K) is 31.00 J/g mol · K:

$$\text{heat required} = (3.0)(31.00)(1123 - 298) = 76\,725 \text{ J}$$

For case (c), there is no mean heat capacity for the interval 673–1123 K. However, we can use the heat required to heat the gas from 298 to 673 K in case (a) and subtract it from case (b), which includes the heat to go from 298 to 673 K plus 673 to 1123 K:

$$\begin{aligned} \text{heat required (673–1123 K)} &= \text{heat required (298–1123 K)} \\ &\quad - \text{heat required (298–673)} \quad \text{(1.6-5)} \end{aligned}$$

Substituting the proper values into Eq. (1.6-5),

$$\text{heat required} = 76\,725 - 33\,390 = 43\,335 \text{ J}$$

On heating a gas mixture, the total heat required is determined by first calculating the heat required for each individual component and then adding the results to obtain the total.

The heat capacities of solids and liquids are also functions of temperature and independent of pressure. Data are given in Appendix A.2, Physical Properties of Water; A.3, Physical Properties of Inorganic and Organic Compounds; and A.4, Physical Properties of Foods and Biological Materials. More data are available in (P1).

### **EXAMPLE 1.6-2. Heating of Milk**

Rich cows' milk (4536 kg/h) at 4.4°C is being heated in a heat exchanger to 54.4°C by hot water. How much heat is needed?

**Solution:** From Appendix A.4, the average heat capacity of rich cows' milk is 3.85 kJ/kg · K. Temperature rise  $\Delta T = (54.4 - 4.4)^\circ\text{C} = 50 \text{ K}$ .

$$\text{heat required} = (4536 \text{ kg/h})(3.85 \text{ kJ/kg} \cdot \text{K})(1/3600 \text{ h/s})(50 \text{ K}) = 242.5 \text{ kW}$$

The enthalpy,  $H$ , of a substance in J/kg represents the sum of the internal energy plus the pressure–volume term. For no reaction and a constant-pressure process with a change in temperature, the heat change as computed from Eq. (1.6-4) is the difference in enthalpy,  $\Delta H$ , of the substance relative to a given temperature or base point. In other units,  $H = \text{btu/lb}_m$  or cal/g.

## **1.6C Latent Heat and Steam Tables**

Whenever a substance undergoes a change of phase, relatively large amounts of heat change are involved at a constant temperature. For example, ice at 0°C and 1 atm pressure can absorb 6013.4 kJ/kg mol. This enthalpy change is called the *latent heat of fusion*. Data for other compounds are available in various handbooks (P1, W1).

When a liquid phase vaporizes to a vapor phase under its vapor pressure at constant temperature, an amount of heat called the *latent heat of vaporization* must be added. Tabulations

of latent heats of vaporization are given in various handbooks. For water at 25°C and a pressure of 23.75 mm Hg, the latent heat is 44 020 kJ/kg mol, and at 25°C and 760 mm Hg, 44 045 kJ/kg mol. Hence, the effect of pressure can be neglected in engineering calculations. However, there is a large effect of temperature on the latent heat of water. Also, the effect of pressure on the heat capacity of liquid water is small and can be neglected.

Since water is a very common chemical, the thermodynamic properties of it have been compiled in steam tables and are given in Appendix A.2 in SI and in English units.

**EXAMPLE 1.6-3. Use of Steam Tables**

Find the enthalpy change (i.e., how much heat must be added) for each of the following cases using SI and English units:

- (a) Heating 1 kg (lb<sub>m</sub>) water from 21.11°C (70°F) to 60°C (140°F) at 101.325 kPa (1 atm) pressure.
- (b) Heating 1 kg (lb<sub>m</sub>) water from 21.11°C (70°F) to 115.6°C (240°F) and vaporizing at 172.2 kPa (24.97 psia).
- (c) Vaporizing 1 kg (lb<sub>m</sub>) water at 115.6°C (240°F) and 172.2 kPa (24.97 psia).

**Solution:** For part (a), the effect of pressure on the enthalpy of liquid water is negligible. From Appendix A.2,

$$\begin{aligned}
 H \text{ at } 21.11^\circ\text{C} &= 88.60 \text{ kJ/kg} \quad \text{or} \quad \text{at } 70^\circ\text{F} = 38.09 \text{ btu/lb}_m \\
 H \text{ at } 60^\circ\text{C} &= 251.13 \text{ kJ/kg} \quad \text{or} \quad \text{at } 140^\circ\text{F} = 107.96 \text{ btu/lb}_m \\
 \text{change in } H = \Delta H &= 251.13 - 88.60 = 162.53 \text{ kJ/kg} \\
 &= 107.96 - 38.09 = 69.87 \text{ btu/lb}_m
 \end{aligned}$$

In part (b), the enthalpy at 115.6°C (240°F) and 172.2 kPa (24.97 psia) of the saturated vapor is 2699.9 kJ/kg or 1160.7 btu/lb<sub>m</sub>.

$$\begin{aligned}
 \text{change in } H = \Delta H &= 2699.9 - 88.60 = 2611.3 \text{ kJ/kg} \\
 &= 1160.7 - 38.09 = 1122.6 \text{ btu/lb}_m
 \end{aligned}$$

The latent heat of water at 115.6°C (240°F) in part (c) is

$$\begin{aligned}
 2699.9 - 484.9 &= 2215.0 \text{ kJ/kg} \\
 1160.7 - 208.44 &= 952.26 \text{ btu/lb}_m
 \end{aligned}$$

**1.6D Heat of Reaction**

When chemical reactions occur, heat effects always accompany these reactions. This area where energy changes occur is often called *thermochemistry*. For example, when HCl is neutralized with NaOH, heat is given off and the reaction is exothermic. Heat is absorbed in an endothermic reaction. This heat of reaction is dependent on the chemical nature of each reacting material and product and on their physical states.

For purposes of organizing data, we define a standard heat of reaction  $\Delta H^0$  as the change in enthalpy when 1 kg mol reacts under a pressure of 101.325 kPa at a temperature of 298 K (25°C). For example, for the reaction



the  $\Delta H^0$  is  $-285.840 \times 10^3$  kJ/kg mol or  $-68.317$  kcal/g mol. The reaction is exothermic and the value is negative since the reaction loses enthalpy. In this case, the  $H_2$  gas reacts with the  $O_2$  gas to give liquid water, all at 298 K (25°C).

Special names are given to  $\Delta H^0$  depending upon the type of reaction. When the product is formed from the elements, as in Eq. (1.6-6), we call the  $\Delta H^0$  the *heat of formation* of the product water,  $\Delta H_f^0$ . For the combustion of  $CH_4$  to form  $CO_2$  and  $H_2O$ , we call it *heat of combustion*,  $\Delta H_c^0$ . Data are given in Appendix A.3 for various values of  $\Delta H_c^0$ .

**EXAMPLE 1.6-4. Combustion of Carbon**

A total of 10.0 g mol of carbon graphite is burned in a calorimeter held at 298 K and 1 atm. The combustion is incomplete, and 90% of the C goes to  $CO_2$  and 10% to CO. What is the total enthalpy change in kJ and kcal?

**Solution:** From Appendix A.3 the  $\Delta H_c^0$  for carbon going to  $CO_2$  is  $-393.513 \times 10^3$  kJ/kg mol or  $-94.0518$  kcal/g mol, and for carbon going to CO it is  $-110.523 \times 10^3$  kJ/kg mol or  $-26.4157$  kcal/g mol. Since 9 mol  $CO_2$  and 1 mol CO are formed,

$$\begin{aligned} \text{total } \Delta H &= 9(-393.513) + 1(-110.523) = -3652 \text{ kJ} \\ &= 9(-94.0518) + 1(-26.4157) = -872.9 \text{ kcal} \end{aligned}$$

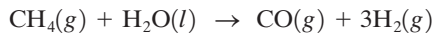
If a table of heats of formation,  $\Delta H_f^0$ , of compounds is available, the standard heat of the reaction,  $\Delta H^0$ , can be calculated by

$$\Delta H^0 = \sum \Delta H_f^0(\text{products}) - \sum \Delta H_f^0(\text{reactants}) \quad (1.6-7)$$

In Appendix A.3, a short table of some values of  $\Delta H_f^0$  is given. Other data are also available (H1, P1, S1).

**EXAMPLE 1.6-5. Reaction of Methane**

For the following reaction of 1 kg mol of  $CH_4$  at 101.32 kPa and 298 K,



calculate the standard heat of reaction  $\Delta H^0$  at 298 K in kJ.

**Solution:** From Appendix A.3, the following standard heats of formation are obtained at 298 K:

	$\Delta H_f^0$ (kJ/kg mol)
$CH_4(g)$	$-74.848 \times 10^3$
$H_2O(l)$	$-285.840 \times 10^3$
$CO(g)$	$-110.523 \times 10^3$
$H_2(g)$	0

Note that the  $\Delta H_f^0$  of all elements is, by definition, zero. Substituting into Eq. (1.6-7),

$$\begin{aligned}\Delta H^0 &= [-110.523 \times 10^3 - 3(0)] - (-74.848 \times 10^3 - 285.840 \times 10^3) \\ &= +250.165 \times 10^3 \text{ kJ/kg mol} \quad (\text{endothermic})\end{aligned}$$

## 1.7 CONSERVATION OF ENERGY AND HEAT BALANCES

### 1.7A Conservation of Energy

In making material balances we used the law of conservation of mass, which states that the mass entering is equal to the mass leaving plus the mass left in the process. In a similar manner, we can state the *law of conservation of energy*, which says that all energy entering a process is equal to that leaving plus that left in the process. In this section elementary heat balances will be made. More elaborate energy balances will be considered in Sections 2.7 and 5.6.

Energy can appear in many forms. Some of the common forms are enthalpy, electrical energy, chemical energy (in terms of  $\Delta H$  reaction), kinetic energy, potential energy, work, and heat inflow.

In many cases in process engineering, which often takes place at constant pressure, electrical energy, kinetic energy, potential energy, and work either are not present or can be neglected. Then only the enthalpy of the materials (at constant pressure), the standard chemical reaction energy ( $\Delta H^0$ ) at 25°C, and the heat added or removed must be taken into account in the energy balance. This is generally called a *heat balance*.

### 1.7B Heat Balances

In making a heat balance at steady state we use methods similar to those used in making a material balance. The energy or heat coming into a process in the inlet materials plus any net energy added to the process are equal to the energy leaving in the materials. Expressed mathematically,

$$\sum H_R + (-\Delta H_{298}^0) + q = \sum H_p \quad (1.7-1)$$

where  $\sum H_R$  is the sum of enthalpies of all materials entering the reaction process relative to the reference state for the standard heat of reaction at 298 K and 101.32 kPa. If the inlet temperature is above 298 K, this sum will be positive.  $\Delta H_{298}^0$  = standard heat of the reaction at 298 K and 101.32 kPa. The reaction contributes heat to the process, so the negative of  $\Delta H_{298}^0$  is taken to be positive input heat for an exothermic reaction. Also,  $q$  = net energy or heat added to the system. If heat leaves the system, this item will be negative.  $\sum H_p$  = sum of enthalpies of all leaving materials referred to the standard reference state at 298 K (25°C).

Note that if the materials coming into a process are below 298 K,  $\sum H_R$  will be negative. Care must be taken not to confuse the signs of the items in Eq. (1.7-1). If no chemical reaction occurs, then simple heating, cooling, or phase change is occurring. Use of Eq. (1.7-1) will be illustrated by several examples. For convenience it is common practice to call the terms on the left-hand side of Eq. (1.7-1) input items, and those on the right, output items.

#### **EXAMPLE 1.7-1. Heating of Fermentation Medium**

A liquid fermentation medium at 30°C is pumped at a rate of 2000 kg/h through a heater, where it is heated to 70°C under pressure. The waste heat water used to

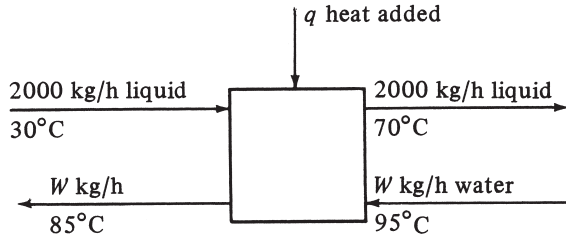


FIGURE 1.7-1. Process flow diagram for Example 1.7-1.

heat this medium enters at  $95^{\circ}\text{C}$  and leaves at  $85^{\circ}\text{C}$ . The average heat capacity of the fermentation medium is  $4.06 \text{ kJ/kg}\cdot\text{K}$ , and that for water is  $4.21 \text{ kJ/kg}\cdot\text{K}$  (Appendix A.2). The fermentation stream and the wastewater stream are separated by a metal surface through which heat is transferred and do not physically mix with each other. Make a complete heat balance on the system. Calculate the water flow and the amount of heat added to the fermentation medium assuming no heat losses. The process flow is given in Fig. 1.7-1.

**Solution:** It is convenient to use the standard reference state of  $298 \text{ K}$  ( $25^{\circ}\text{C}$ ) as the datum to calculate the various enthalpies. From Eq. (1.7-1) the input items are as follows:

*Input items.*  $\sum H_R$  of the enthalpies of the two streams relative to  $298 \text{ K}$  ( $25^{\circ}\text{C}$ ) (note that  $\Delta t = 30 - 25^{\circ}\text{C} = 5^{\circ}\text{C} = 5 \text{ K}$ ):

$$\begin{aligned} H(\text{liquid}) &= (2000 \text{ kg/h})(4.06 \text{ kJ/kg}\cdot\text{K})(5 \text{ K}) \\ &= 4.060 \times 10^4 \text{ kJ/h} \end{aligned}$$

$$H(\text{water}) = W(4.21)(95 - 25) = 2.947 \times 10^2 W \text{ kJ/h} \quad (W = \text{kg/h})$$

$$(-\Delta H_{298}^0) = 0 \quad (\text{since there is no chemical reaction})$$

$$q = 0 \quad (\text{there are no heat losses or additions})$$

*Output items.*  $\sum H_P$  of the two streams relative to  $298 \text{ K}$  ( $25^{\circ}\text{C}$ ):

$$H(\text{liquid}) = 2000(4.06)(70 - 25) = 3.65 \times 10^5 \text{ kJ/h}$$

$$H(\text{water}) = W(4.21)(85 - 25) = 2.526 \times 10^2 W \text{ kJ/h}$$

Equating input to output in Eq. (1.7-1) and solving for  $W$ ,

$$4.060 \times 10^4 + 2.947 \times 10^2 W = 3.654 \times 10^5 + 2.526 \times 10^2 W$$

$$W = 7720 \text{ kg/h water flow}$$

The amount of heat added to the fermentation medium is simply the difference of the outlet and inlet liquid enthalpies:

$$\begin{aligned} H(\text{outlet liquid}) - H(\text{inlet liquid}) &= 3.654 \times 10^5 - 4.060 \times 10^4 \\ &= 3.248 \times 10^5 \text{ kJ/h (90.25 kW)} \end{aligned}$$

Note in this example that since the heat capacities were assumed constant, a simpler balance could have been written as follows:

heat gained by liquid = heat lost by water

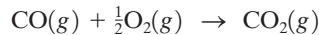
$$2000(4.06)(70 - 30) = W(4.21)(95 - 85)$$

Then, solving,  $W = 7720$  kg/h. This simple balance works well when  $c_p$  is constant. However, when  $c_p$  varies with temperature and the material is a gas,  $c_{pm}$  values are only available between 298 K (25°C) and  $t$  K, and the simple method cannot be used without obtaining new  $c_{pm}$  values over different temperature ranges.

**EXAMPLE 1.7-2 Heat and Material Balance in Combustion**

The waste gas from a process of 1000 g mol/h of CO at 473 K is burned at 1 atm pressure in a furnace using air at 373 K. The combustion is complete and 90% excess air is used. The flue gas leaves the furnace at 1273 K. Calculate the heat removed in the furnace.

**Solution:** First, the process flow diagram is drawn in Fig. 1.7-2, and then a material balance is made:



$$\Delta H_{298}^0 = -282.989 \times 10^3 \text{ kJ/kg mol}$$

(from Appendix A.3)

$$\text{mol CO} = 1000 \text{ g mol/h} = \text{moles CO}_2$$

$$= 1.00 \text{ kg mol/h}$$

$$\text{mol O}_2 \text{ theoretically required} = \frac{1}{2}(1.00) = 0.500 \text{ kg mol/h}$$

$$\text{mol O}_2 \text{ actually added} = 0.500(1.9) = 0.950 \text{ kg mol/h}$$

$$\text{mol N}_2 \text{ added} = 0.950 \frac{0.79}{0.21} = 3.570 \text{ kg mol/h}$$

$$\text{air added} = 0.950 + 3.570 = 4.520 \text{ kg mol/h} = A$$

$$\text{O}_2 \text{ in outlet flue gas} = \text{added} - \text{used}$$

$$= 0.950 - 0.500 = 0.450 \text{ kg mol/h}$$

$$\text{CO}_2 \text{ in outlet flue gas} = 1.00 \text{ kg mol/h}$$

$$\text{N}_2 \text{ in outlet flue gas} = 3.570 \text{ kg mol/h}$$

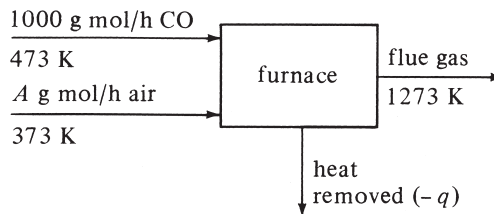


FIGURE 1.7-2. Process flow diagram for Example 1.7-2.

For the heat balance relative to the standard state at 298 K, we follow Eq. (1.7-1).

*Input items*

$$H(\text{CO}) = 1.00(c_{pm})(473 - 298) = 1.00(29.38)(473 - 298) = 5142 \text{ kJ/h}$$

(The  $c_{pm}$  of CO of 29.38 kJ/kg mol · K between 298 and 473 K is obtained from Table 1.6-1.)

$$H(\text{air}) = 4.520(c_{pm})(373 - 298) = 4.520(29.29)(373 - 298) = 9929 \text{ kJ/h}$$

$$q = \text{heat added, kJ/h}$$

(This will give a negative value here, indicating that heat was removed.)

$$-\Delta H_{298}^0 = -(-282.989 \times 10^3 \text{ kJ/kg mol})(1.00 \text{ kg mol/h}) = 282\,990 \text{ kJ/h}$$

*Output items*

$$H(\text{CO}_2) = 1.00(c_{pm})(1273 - 298) = 1.00(49.91)(1273 - 298) = 48\,660 \text{ kJ/h}$$

$$H(\text{O}_2) = 0.450(c_{pm})(1273 - 298) = 0.450(33.25)(1273 - 298) = 14\,590 \text{ kJ/h}$$

$$H(\text{N}_2) = 3.570(c_{pm})(1273 - 298) = 3.570(31.43)(1273 - 298) = 109\,400 \text{ kJ/h}$$

Equating input to output and solving for  $q$ ,

$$5142 + 9929 + q + 282\,990 = 48\,660 + 14\,590 + 109\,400$$

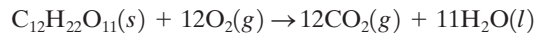
$$q = -125\,411 \text{ kJ/h}$$

Hence, heat is removed:  $-34\,837 \text{ W}$ .

Often when chemical reactions occur in the process and the heat capacities vary with temperature, the solution in a heat balance can be trial and error if the final temperature is the unknown.

### **EXAMPLE 1.7-3. Oxidation of Lactose**

In many biochemical processes, lactose is used as a nutrient, which is oxidized as follows:



The heat of combustion  $\Delta H_c^0$  in Appendix A.3 at 25°C is  $-5648.8 \times 10^3 \text{ J/g mol}$ . Calculate the heat of complete oxidation (combustion) at 37°C, which is the temperature of many biochemical reactions. The  $c_{pm}$  of solid lactose is 1.20 J/g · K, and the molecular weight is 342.3 g mass/g mol.

**Solution:** This can be treated as an ordinary heat-balance problem. First, the process flow diagram is drawn in Fig. 1.7-3. Next, the datum temperature of 25°C is selected and the input and output enthalpies calculated. The temperature difference  $\Delta t = (37 - 25)^\circ\text{C} = (37 - 25) \text{ K}$ .



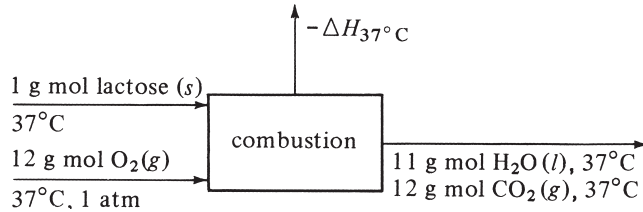


FIGURE 1.7-3. Process flow diagram for Example 1.7-3.

*Input items*

$$H(\text{lactose}) = (342.3 \text{ g}) \left( c_{pm} \frac{\text{J}}{\text{g} \cdot \text{K}} \right) (37 - 25) \text{ K} = 342.3(1.20)(37 - 25) \\ = 4929 \text{ J}$$

$$H(\text{O}_2 \text{ gas}) = (12 \text{ g mol}) \left( c_{pm} \frac{\text{J}}{\text{g mol} \cdot \text{K}} \right) (37 - 25) \text{ K} \\ = 12(29.38)(37 - 25) = 4230 \text{ J}$$

(The  $c_{pm}$  of  $\text{O}_2$  was obtained from Table 1.6-1.)

$$-\Delta H_{25}^0 = -(-5648.8 \times 10^3)$$

*Output items*

$$H(\text{H}_2\text{O liquid}) = 11(18.02 \text{ g}) \left( c_{pm} \frac{\text{J}}{\text{g} \cdot \text{K}} \right) (37 - 25) \text{ K} \\ = 11(18.02)(4.18)(37 - 25) = 9943 \text{ J}$$

(The  $c_{pm}$  of liquid water was obtained from Appendix A.2.)

$$H(\text{CO}_2 \text{ gas}) = (12 \text{ g mol}) \left( c_{pm} \frac{\text{J}}{\text{g mol} \cdot \text{K}} \right) (37 - 25) \text{ K} \\ = 12(37.45)(37 - 25) = 5393 \text{ J}$$

(The  $c_{pm}$  of  $\text{CO}_2$  is obtained from Table 1.6-1.)

$\Delta H_{37^\circ\text{C}}$ :

Setting input = output and solving,

$$4929 + 4230 + 5648.8 \times 10^3 = 9943 + 5393 - \Delta H_{37^\circ\text{C}}$$

$$\Delta H_{37^\circ\text{C}} = -5642.6 \times 10^3 \text{ J/g mol} = \Delta H_{310\text{K}}$$

## 1.8 NUMERICAL METHODS FOR INTEGRATION

### 1.8A Introduction and Graphical Integration

Often the mathematical function  $f(x)$  to be integrated is too complex and we are not able to integrate it analytically. Or in some cases the function is one that has been obtained from

experimental data, and no mathematical equation is available to represent the data so that they can be integrated analytically. In these cases, we can use either numerical or graphical integration.

Integration of a function  $f(x)$  between the limits  $x = a$  to  $x = b$  can be represented by

$$\int_{x=a}^{x=b} f(x) dx \quad (1.8-1)$$

By graphically plotting  $f(x)$  versus  $x$ , the area under the curve is equal to the value of the integral.

### 1.8B Numerical Integration and Simpson's Rule

Often it is desirable or necessary to perform a numerical integration by computing the value of a definite integral from a set of numerical values of the integrand  $f(x)$ . This, of course, can be done graphically, but in most cases numerical methods suitable for the digital computer are desired.

The integral to be evaluated is Eq. (1.8-1), where the interval is  $b - a$ . The most generally used numerical method is the parabolic rule, often called *Simpson's rule*. This method divides the total interval  $b - a$  into an even number of subintervals  $m$ , where

$$m = \frac{b - a}{h} \quad (1.8-2)$$

The value of  $h$ , a constant, is the spacing in  $x$  used. Then, approximating  $f(x)$  by a parabola on each subinterval, Simpson's rule is

$$\int_{x=a}^{x=b} f(x) dx = \frac{h}{3} [f_0 + 4(f_1 + f_3 + f_5 + \dots + f_{m-1}) + 2(f_2 + f_4 + f_6 + \dots + f_{m-2}) + f_m] \quad (1.8-3)$$

where  $f_0$  is the value of  $f(x)$  at  $x = a$ ;  $f_1$  the value of  $f(x)$  at  $x = x_1, \dots$ ;  $f_m$  the value of  $f(x)$  at  $x = b$ . The reader should note that  $m$  must be an even number and the increments evenly spaced. This method is well suited for digital computation with a spreadsheet. Spreadsheets have numerical calculation methods built into their programs. Simpson's rule is a widely used numerical integration method.

In some cases the available experimental data for  $f(x)$  are not at equally spaced increments of  $x$ . Then the numerical integration can be performed using the sum of the single-interval rectangles (trapezoidal rule) for the value of the interval. This is much less accurate than Simpson's rule. The trapezoidal-rule method becomes more accurate as the interval becomes smaller.

Sometimes the experimental data for  $f(x)$  are spaced at large and/or irregular increments of  $x$ . These data can be smoothed by fitting a polynomial, exponential, logarithmic, or some other function to the data, which often can be integrated analytically. If the function is relatively complex, then it can be numerically integrated using Simpson's rule. All of this can be done using Excel and a spreadsheet.

## PROBLEMS

**1.2-1. Temperature of a Chemical Process.** The temperature of a chemical reaction was found to be 353.2 K. What is the temperature in °F, °C, and °R?

**Ans.** 176°F, 80°C, 636°R

**1.2-2. Temperature for Smokehouse Processing of Meat.** In smokehouse processing of sausage meat, a final temperature of 155°F inside the sausage is often used. Calculate this temperature in °C, K, and °R.

**1.3-1. Molecular Weight of Air.** For purposes of most engineering calculations, air is assumed to be composed of 21 mol % oxygen and 79 mol % nitrogen. Calculate the average molecular weight.

**Ans.** 28.9 g mass/g mol, lb mass/lb mol, or kg mass/kg mol

**1.3-2. Oxidation of CO and Mole Units.** The gas CO is being oxidized by O<sub>2</sub> to form CO<sub>2</sub>. How many kg of CO<sub>2</sub> will be formed from 56 kg of CO? Also, calculate the kg of O<sub>2</sub> theoretically needed for this reaction. (*Hint:* First write the balanced chemical equation to obtain the mol O<sub>2</sub> needed for 1.0 kg mol CO. Then calculate the kg mol of CO in 56 kg CO.)

**Ans.** 88.0 kg CO<sub>2</sub>, 32.0 kg O<sub>2</sub>

**1.3-3. Composition of a Gas Mixture.** A gaseous mixture contains 20 g of N<sub>2</sub>, 83 g of O<sub>2</sub>, and 45 g of CO<sub>2</sub>. Calculate the composition in mole fraction and the average molecular weight of the mixture.

**Ans.** Average mol wt = 34.2 g mass/g mol, 34.2 kg mass/kg mol

**1.3-4. Composition of a Protein Solution.** A liquid solution contains 1.15 wt % of a protein, 0.27 wt % KCl, and the remainder water. The average molecular weight of the protein by gel permeation is 525000 g mass/g mol. Calculate the mole fraction of each component in solution.

**1.3-5. Concentration of NaCl Solution.** An aqueous solution of NaCl has a concentration of 24.0 wt % NaCl with a density of 1.178 g/cm<sup>3</sup> at 25°C. Calculate the following.

(a) Mole fraction of NaCl and water.

(b) Concentration of NaCl as g mol/liter, lb<sub>m</sub>/ft<sup>3</sup>, lb<sub>m</sub>/gal, and kg/m<sup>3</sup>.

**1.4-1. Conversion of Pressure Measurements in Freeze Drying.** In the experimental measurement of freeze drying of beef, an absolute pressure of 2.4 mm Hg was held in the chamber. Convert this pressure to atm, in. of water at 4°C, μm of Hg, and Pa. (*Hint:* See Appendix A.1 for conversion factors.)

**Ans.**  $3.16 \times 10^{-3}$  atm, 1.285 in. H<sub>2</sub>O, 2400 μm Hg, 320 Pa

**1.4-2. Compression and Cooling of Nitrogen Gas.** A volume of 65.0 ft<sup>3</sup> of N<sub>2</sub> gas at 90°F and 29.0 psig is compressed to 75 psig and cooled to 65°F. Calculate the final volume in ft<sup>3</sup> and the final density in lb<sub>m</sub>/ft<sup>3</sup>. [*Hint:* Be sure to convert all pressures to psia first and then to atm. Substitute original conditions into Eq. (1.4-1) to obtain *n*, lb mol.]

**1.4-3. Gas Composition and Volume.** A gas mixture of 0.13 g mol NH<sub>3</sub>, 1.27 g mol N<sub>2</sub>, and 0.025 g mol H<sub>2</sub>O vapor is contained at a total pressure of 830 mm Hg and 323 K. Calculate the following.

(a) Mole fraction of each component.

(b) Partial pressure of each component in mm Hg.

(c) Total volume of mixture in m<sup>3</sup> and ft<sup>3</sup>.

**1.4-4. Evaporation of a Heat-Sensitive Organic Liquid.** An organic liquid is being evaporated from a liquid solution containing a few percent nonvolatile dissolved solids. Since it is heat-sensitive and may discolor at high temperatures, it will be evaporated

under vacuum. If the lowest absolute pressure that can be obtained in the apparatus is 12.0 mm Hg, what will be the temperature of evaporation in K? It will be assumed that the small amount of solids does not affect the vapor pressure, which is given as follows:

$$\log P_A = -2250 \left( \frac{1}{T} \right) + 9.05$$

where  $P_A$  is in mm Hg and  $T$  in K.

**Ans.**  $T = 282.3 \text{ K}$  or  $9.1^\circ\text{C}$

**1.5-1. Evaporation of Cane Sugar Solutions.** An evaporator is used to concentrate cane sugar solutions. A feed of 10 000 kg/d of a solution containing 38 wt % sugar is evaporated, producing a 74 wt % solution. Calculate the weight of solution produced and amount of water removed.

**Ans.** 5135 kg/d of 74 wt % solution, 4865 kg/d water

**1.5-2. Processing of Fish Meal.** Fish are processed into fish meal and used as a supplementary protein food. In the processing the oil is first extracted to produce wet fish cake containing 80 wt % water and 20 wt % bone-dry cake. This wet cake feed is dried in rotary-drum dryers to give a “dry” fish cake product containing 40 wt % water. Finally, the product is finely ground and packed. Calculate the kg/h of wet cake feed needed to produce 1000 kg/h of “dry” fish cake product.

**Ans.** 3000 kg/h wet cake feed

**1.5-3. Drying of Lumber.** A batch of 100 kg of wet lumber containing 11 wt % moisture is dried to a water content of 6.38 kg water/1.0 kg bone-dry lumber. What is the weight of “dried” lumber and the amount of water removed?

**1.5-4. Processing of Paper Pulp.** A wet paper pulp contains 68 wt % water. After the pulp was dried, it was found that 55% of the original water in the wet pulp was removed. Calculate the composition of the “dried” pulp and its weight for a feed of 1000 kg/min of wet pulp.

**1.5-5. Production of Jam from Crushed Fruit in Two Stages.** In a process producing jam (C1), crushed fruit containing 14 wt % soluble solids is mixed in a mixer with sugar (1.22 kg sugar/1.00 kg crushed fruit) and pectin (0.0025 kg pectin/1.00 kg crushed fruit). The resultant mixture is then evaporated in a kettle to produce a jam containing 67 wt % soluble solids. For a feed of 1000 kg crushed fruit, calculate the kg mixture from the mixer, kg water evaporated, and kg jam produced.

**Ans.** 2222.5 kg mixture, 189 kg water, 2033.5 kg jam

**1.5-6. Drying of Cassava (Tapioca) Root.** Tapioca flour is used in many countries for bread and similar products. The flour is made by drying coarse granules of the cassava root containing 66 wt % moisture to 5% moisture and then grinding to produce a flour. How many kg of granules must be dried and how much water removed to produce 5000 kg/h of flour?

**1.5-7. Processing of Soybeans in Three Stages.** A feed of 10 000 kg of soybeans is processed in a sequence of three stages or steps (E1). The feed contains 35 wt % protein, 27.1 wt % carbohydrate, 9.4 wt % fiber and ash, 10.5 wt % moisture, and 18.0 wt % oil. In the first stage the beans are crushed and pressed to remove oil, giving an expressed-oil stream and a stream of pressed beans containing 6% oil. Assume no loss of other constituents with the oil stream. In the second step the pressed beans are extracted with hexane to produce an extracted-meal stream containing 0.5 wt % oil and a hexane–oil stream. Assume no hexane in the extracted meal. Finally, in the last step the extracted meal is dried to give a dried meal of 8 wt % moisture. Calculate:

- (a) Kg of pressed beans from the first stage.
- (b) Kg of extracted meal from stage 2.
- (c) Kg of final dried meal and the wt % protein in the dried meal.

**Ans.** (a) 8723 kg; (b) 8241 kg; (c) 7816 kg, 44.8 wt % protein

**1.5-8. Recycle in a Dryer.** A solid material containing 15.0 wt % moisture is dried so that it contains 7.0 wt % water by blowing fresh warm air mixed with recycled air over the solid in the dryer. The inlet fresh air has a humidity of 0.01 kg water/kg dry air, the air from the drier that is recycled has a humidity of 0.1 kg water/kg dry air, and the mixed air to the dryer, 0.03 kg water/kg dry air. For a feed of 100 kg solid/h fed to the dryer, calculate the kg dry air/h in the fresh air, the kg dry air/h in the recycled air, and the kg/h of “dried” product.

**Ans.** 95.6 kg/h dry air in fresh air, 27.3 kg/h dry air in recycled air, and 91.4 kg/h “dried” product

**1.5-9. Crystallization and Recycle.** It is desired to produce 1000 kg/h of  $\text{Na}_3\text{PO}_4 \cdot 12\text{H}_2\text{O}$  crystals from a feed solution containing 5.6 wt %  $\text{Na}_3\text{PO}_4$  and traces of impurity. The original solution is first evaporated in an evaporator to a 35 wt %  $\text{Na}_3\text{PO}_4$  solution and then cooled to 293 K in a crystallizer, where the hydrated crystals and a mother-liquor solution are removed. One out of every 10 kg of mother liquor is discarded to waste to get rid of the impurities, and the remaining mother liquor is recycled to the evaporator. The solubility of  $\text{Na}_3\text{PO}_4$  at 293 K is 9.91 wt %. Calculate the kg/h of feed solution and kg/h of water evaporated.

**Ans.** 7771 kg/h feed, 6739 kg/h water

**1.5-10. Evaporation and Bypass in Orange Juice Concentration.** In a process for concentrating 1000 kg of freshly extracted orange juice (C1) containing 12.5 wt % solids, the juice is strained, yielding 800 kg of strained juice and 200 kg of pulpy juice. The strained juice is concentrated in a vacuum evaporator to give an evaporated juice of 58% solids. The 200 kg of pulpy juice is bypassed around the evaporator and mixed with the evaporated juice in a mixer to improve the flavor. This final concentrated juice contains 42 wt % solids. Calculate the concentration of solids in the strained juice, the kg of final concentrated juice, and the concentration of solids in the pulpy juice bypassed. (*Hint:* First, make a total balance and then a solids balance on the overall process. Next, make a balance on the evaporator. Finally, make a balance on the mixer.)

**Ans.** 34.2 wt % solids in pulpy juice

**1.5-11. Manufacture of Acetylene.** For the making of 6000 ft<sup>3</sup> of acetylene ( $\text{CHCH}$ ) gas at 70°F and 750 mm Hg, solid calcium carbide ( $\text{CaC}_2$ ) which contains 97 wt %  $\text{CaC}_2$  and 3 wt % solid inerts is used along with water. The reaction is



The final lime slurry contains water, solid inerts, and  $\text{Ca}(\text{OH})_2$  lime. In this slurry the total wt % solids of inerts plus  $\text{Ca}(\text{OH})_2$  is 20%. How many lb of water must be added and how many lb of final lime slurry is produced? [*Hint:* Use a basis of 6000 ft<sup>3</sup> and convert to lb mol. This gives 15.30 lb mol  $\text{C}_2\text{H}_2$ , 15.30 lb mol  $\text{Ca}(\text{OH})_2$ , and 15.30 lb mol  $\text{CaC}_2$  added. Convert lb mol  $\text{CaC}_2$  feed to lb and calculate lb inerts added. The total lb solids in the slurry is then the sum of the  $\text{Ca}(\text{OH})_2$  plus inerts. In calculating the water added, remember that some is consumed in the reaction.]

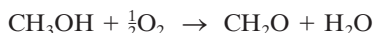
**Ans.** 5200 lb water added (2359 kg), 5815 lb lime slurry (2638 kg)

**1.5-12. Combustion of Solid Fuel.** A fuel analyzes 74.0 wt % C and 12.0% ash (inert). Air is added to burn the fuel, producing a flue gas of 12.4%  $\text{CO}_2$ , 1.2%  $\text{CO}$ , 5.7%  $\text{O}_2$ , and

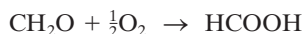
80.7% N<sub>2</sub>. Calculate the kg of fuel used for 100 kg mol of outlet flue gas and the kg mol of air used. (*Hint*: First calculate the mol O<sub>2</sub> added in the air, using the fact that the N<sub>2</sub> in the flue gas equals the N<sub>2</sub> added in the air. Then make a carbon balance to obtain the total moles of C added.)

**1.5-13. Burning of Coke.** A furnace burns a coke containing 81.0 wt % C, 0.8% H, and the rest inert ash. The furnace uses 60% excess air (air over and above that needed to burn all C to CO<sub>2</sub> and H to H<sub>2</sub>O). Calculate the moles of all components in the flue gas if only 95% of the carbon goes to CO<sub>2</sub> and 5% to CO.

**1.5-14. Production of Formaldehyde.** Formaldehyde (CH<sub>2</sub>O) is made by the catalytic oxidation of pure methanol vapor and air in a reactor. The moles from this reactor are 63.1 N<sub>2</sub>, 13.4 O<sub>2</sub>, 5.9 H<sub>2</sub>O, 4.1 CH<sub>2</sub>O, 12.3 CH<sub>3</sub>OH, and 1.2 HCOOH. The reaction is



A side reaction occurring is



Calculate the mol methanol feed, mol air feed, and percent conversion of methanol to formaldehyde.

**Ans.** 17.6 mol CH<sub>3</sub>OH, 79.8 mol air, 23.3% conversion

**1.6-1. Heating of CO<sub>2</sub> Gas.** A total of 250 g of CO<sub>2</sub> gas at 373 K is heated to 623 K at 101.32 kPa total pressure. Calculate the amount of heat needed in cal, btu, and kJ.

**Ans.** 15 050 cal, 59.7 btu, 62.98 kJ

**1.6-2. Heating a Gas Mixture.** A mixture of 25 lb mol N<sub>2</sub> and 75 lb mol CH<sub>4</sub> is being heated from 400°F to 800°F at 1 atm pressure. Calculate the total amount of heat needed in btu.

**1.6-3. Final Temperature in Heating Applesauce.** A mixture of 454 kg of applesauce at 10°C is heated in a heat exchanger by adding 121 300 kJ. Calculate the outlet temperature of the applesauce. (*Hint*: In Appendix A.4 a heat capacity for applesauce is given at 32.8°C. Assume that this is constant and use this as the average  $c_{pm}$ .)

**Ans.** 76.5°C

**1.6-4. Use of Steam Tables.** Using the steam tables, determine the enthalpy change for 1 lb water for each of the following cases:

(a) Heating liquid water from 40°F to 240°F at 30 psia. (Note that the effect of total pressure on the enthalpy of liquid water can be neglected.)

(b) Heating liquid water from 40°F to 240°F and vaporizing at 240°F and 24.97 psia.

(c) Cooling and condensing a saturated vapor at 212°F and 1 atm abs to a liquid at 60°F.

(d) Condensing a saturated vapor at 212°F and 1 atm abs.

**Ans.** (a) 200.42 btu/lb<sub>m</sub>; (b) 1152.7 btu/lb<sub>m</sub>; (c) -1122.4 btu/lb<sub>m</sub>;  
(d) -970.3 btu/lb<sub>m</sub>, -2256.9 kJ/kg

**1.6-5. Heating and Vaporization Using Steam Tables.** A flow rate of 1000 kg/h of water at 21.1°C is heated to 110°C when the total pressure is 244.2 kPa in the first stage of a process. In the second stage at the same pressure the water is heated further, until it is all vaporized at its boiling point. Calculate the total enthalpy change in the first stage and in both stages.

**1.6-6. Combustion of CH<sub>4</sub> and H<sub>2</sub>.** For 100 g mol of a gas mixture of 75 mol % CH<sub>4</sub> and 25% H<sub>2</sub>, calculate the total heat of combustion of the mixture at 298 K and 101.32 kPa, assuming that combustion is complete.

**1.6-7. Heat of Reaction from Heats of Formation.** For the reaction



calculate the heat of reaction,  $\Delta H$ , at 298 K and 101.32 kPa for 4 g mol of NH<sub>3</sub> reacting.

**Ans.**  $\Delta H$ , heat of reaction = -904.7 kJ

**1.7-1. Heat Balance and Cooling of Milk.** In the processing of rich cows' milk, 4540 kg/h of milk is cooled from 60°C to 4.44°C by a refrigerant. Calculate the heat removed from the milk.

**Ans.** Heat removed = 269.6 kW

**1.7-2. Heating of Oil by Air.** A flow of 2200 lb<sub>m</sub>/h of hydrocarbon oil at 100°F enters a heat exchanger, where it is heated to 150°F by hot air. The hot air enters at 300°F and is to leave at 200°F. Calculate the total lb mol air/h needed. The mean heat capacity of the oil is 0.45 btu/lb<sub>m</sub> · °F.

**Ans.** 70.1 lb mol air/h, 31.8 kg mol/h

**1.7-3. Combustion of Methane in a Furnace.** A gas stream of 10 000 kg mol/h of CH<sub>4</sub> at 101.32 kPa and 373 K is burned in a furnace using air at 313 K. The combustion is complete and 50% excess air is used. The flue gas leaves the furnace at 673 K. Calculate the heat removed in the furnace. (*Hint:* Use a datum of 298 K and liquid water at 298 K. The input items will be the following: the enthalpy of CH<sub>4</sub> at 373 K referred to 298 K; the enthalpy of the air at 313 K referred to 298 K;  $-\Delta H_c^0$ , the heat of combustion of CH<sub>4</sub> at 298 K, which is referred to liquid water; and  $q$ , the heat added. The output items will include: the enthalpies of CO<sub>2</sub>, O<sub>2</sub>, N<sub>2</sub>, and H<sub>2</sub>O gases at 673 K referred to 298 K; and the latent heat of H<sub>2</sub>O vapor at 298 K and 101.32 kPa from Appendix A.2. It is necessary to include this latent heat since the basis of the calculation and of the  $\Delta H_c^0$  is liquid water.)

**1.7-4. Preheating Air by Steam for Use in a Dryer.** An air stream at 32.2°C is to be used in a dryer and is first preheated in a steam heater, where it is heated to 65.5°C. The air flow is 1000 kg mol/h. The steam enters the heater saturated at 148.9°C, is condensed and cooled, and leaves as a liquid at 137.8°C. Calculate the amount of steam used in kg/h.

**Ans.** 450 kg steam/h

**1.7-5. Cooling of Cans of Potato Soup After Thermal Processing.** A total of 1500 cans of potato soup undergo thermal processing in a retort at 240°F. The cans are then cooled to 100°F in the retort before being removed from the retort by cooling water, which enters at 75°F and leaves at 85°F. Calculate the lb of cooling water needed. Each can of soup contains 1.0 lb of liquid soup, and the empty metal can weighs 0.16 lb. The mean heat capacity of the soup is 0.94 btu/lb<sub>m</sub> · °F and that of the metal can is 0.12 btu/lb<sub>m</sub> · °F. A metal rack or basket which is used to hold the cans in the retort weighs 350 lb and has a heat capacity of 0.12 btu/lb<sub>m</sub> · °F. Assume that the metal rack is cooled from 240°F to 85°F, the temperature of the outlet water. The amount of heat removed from the retort walls in cooling from 240 to 100°F is 10 000 btu. Radiation loss from the retort during cooling is estimated as 5000 btu.

**Ans.** 21 320 lb water, 9670 kg

**1.8-1. Numerical Integration Using Simpson's Method.** The following experimental data for  $y = f(x)$  were obtained:

$x$	$f(x)$	$x$	$f(x)$
0	100	0.4	53
0.1	75	0.5	60
0.2	60.5	0.6	72.5
0.3	53.5		

Determine the integral using Simpson's method:

$$A = \int_{x=0}^{x=0.6} f(x) dx$$

**Ans.**  $A = 38.45$

**1.8-2. Numerical Integration to Obtain Wastewater Flow.** The rate of flow of wastewater in an open channel has been measured and the following data obtained:

<i>Time</i> (min)	<i>Flow</i> (m <sup>3</sup> /min)	<i>Time</i> (min)	<i>Flow</i> (m <sup>3</sup> /min)
0	655	70	800
10	705	80	725
20	780	90	670
30	830	100	640
40	870	110	620
50	890	120	610
60	870		

Determine the total flow in m<sup>3</sup> for 120 min by numerical integration.

**Ans.** 92 350 m<sup>3</sup>

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