Section I

Mathematical Literacy

Section I presents an overview of our approach to mathematical literacy, from two very different points of view. Chapter 1 is an informal look at the ways mathematics appears in the language we use to tell our stories, using a work of popular fiction as a vehicle to demonstrate ways to find math where you least expect it. Chapter 2 presents a more academic approach to the following questions: How do we make meaning from symbols, both letter and numbers? How do we help students who are struggling with that meaning-making? The conceptual information in Chapter 1 and 2 will provide necessary background for the hands-on material presented in Sections II and III.

Chapter 1

Exploring The Porous Boundary Between Doing Mathematics and Understanding Text

INTRODUCTION TO MATHEMATICAL LITERACY

In most people's minds, mathematics and reading occupy completely separate territory. Numbers are not letters. Radicals are not commas. The two disciplines don't just use different languages; they use different alphabets.

People are often drawn to one field of study or the other, considering themselves "math people" or "language people." Language people, in particular, often believe that they do not and cannot speak the language of mathematics. It is our position that they are misguided.

While numbers are not letters and radicals are not commas, they are all symbols. They all provide information for readers prepared to decode them. They all derive meaning from the way they are arranged on the page, in sentences and number sentences and paragraphs and proofs.

In this book, we hope to demonstrate ways to teach students to derive mathematics meaning from sources as varied as junk mail, blogs, picture books, and popular fiction. It is critical that students not be allowed to shut themselves off from mathematics by avoiding it. The truth is that they *can't* avoid it. They can certainly choose not to take a course in trigonometry, but they cannot afford to ignore the mathematics implicit in a credit card offer letter. With the information provided in this book, educators will be better equipped to send young people out into the world fully prepared to deal with bait-and-switch interest rates.

We would like to stress that mathematical literacy is not the same as mathematical proficiency. Yes, working math problems is an important part of a mathematics education, and we will be referencing standards, where appropriate, to tie this discussion of math literacy to the practical matter of math proficiency. In particular, we will be referencing the *Common Core State Standards for Mathematics*. Still, our goal is not just to teach students to work problems according to necessarily limited recommendations printed in accepted standards. We would like them to be able to discern the mathematics concealed in the bombardment of text they receive every day. Even more, we would like them to be able to use mathematics to interpret that information in a useful way.

The idea of being proficient in mathematics calls to mind the idea of being able to decipher problems and solve them quickly and correctly. But mathematics and thus mathematical literacy go far beyond practicing problems. They also go beyond being good at a skill set in order to solve classroom textbook activities and arrive at the correct answer—and then start all over again with a new problem.

Instead, students also need to be able to think and communicate in mathematical terms. To be able to code switch between the symbolic systems of language and math is paramount to mathematical literacy. To be able to think and apply mathematical knowledge in real-world situations outside of the traditional problem–solution format is even more important. This aligns with the Common Core standards' stated goal that students must be able to "access the knowledge and skills necessary in their post-school lives" (Common Core State Standards Initiative, 2010).

In a review of research in mathematical literacy, Wallace and Clark (2005) identified three reading stances linked to integrating literacy into the math classroom: Reading Problems, Reading Mathematics, and Reading Life.

The first, Reading Problems, is a standard approach to reading in the math classroom—students read a textbook's instructions to uncover necessary steps to solving a problem, and then they use their mathematical skills to discern an approach to solving the problem quickly and correctly, before moving on to similar problems. This is exemplified in traditional textbooks, which typically use text to explain how a specific type of problem is solved, and then use text again at the beginning of a problem set to give instructions: "Solve each of these equations for x." The student then leaves the world of language and solves twenty similar problems designed to facilitate practice of a single skill. In such situations, arriving at the correct answer is the overriding goal, which can result in missed opportunities to learn from the problem-solving process. Mathematical and reading skills are both exercised, but separately.

The second stance, Reading Mathematics, often involves repeated emphasis on reading problems designed to require students to derive mathematical meaning, bridging the real world with mathematical concepts that might have seemed rote and abstract when limited to problem solving alone. These problems tap into literacy skills, such as decoding text and inferring meaning, that go beyond simply manipulating numbers without understanding why. They require students to integrate language and mathematics, because the problems can't be solved any other way.

While important, the Reading Mathematics stance is still limited. Eventually, students must have the opportunity to read and reason outside the boundaries of problems and solutions. Sometimes there is no right answer. Sometimes there are many. Sometimes only one of those many "right" answers will work in a particular situation. A comprehensive grasp of mathematics can only come when learners recognize that mathematics exists outside the math textbook and that its purpose is to describe their world.

The third and last stance, Reading Life, encourages students to use real-life problems and texts (often called environmental print) to look critically at the intersection between math and everyday life. Plucking examples from the world-at-large gives teachers the opportunity to incorporate a social justice perspective that can be missing from the math classroom (Gutstein, 2006). For example, have you ever read the fine print on a lottery ticket? Do you completely understand the odds of holding a winning ticket? Do you think that lottery tickets are marketed in such a way as to play on the hopes of people who do not have the education to recognize their slim chances of winning?

How about a credit card solicitation? Could a person whose grasp of mathematics is limited to answering structured problems as presented in a textbook be reasonably expected to select the optimum credit card agreement, based on predetermined variables like late fees, transfer fees, and interest rates? Again, is a person who lacks the education to recognize the pitfalls in these agreements at a disadvantage? What about a person who comes from a family whose socioeconomic circumstances made holding a credit card impossible? Would a thorough grounding in mathematical literacy help such a person overcome that disadvantage? The real-world implications of the inability to recognize a bad business deal are a concrete example of the need to promote mathematics as a route to equity for all students.

Our goal in writing this book is to help educators give students a comfort with mathematics that will enable them to tackle these life skills. In the process, we hope that students will also learn to appreciate the elegant equations that describe the twist of a double helix or the vast space between stars, because mathematics explains our universe to us. It is embedded in everything around us. A thorough grounding in mathematics is an ornament to life.

YOUNG CHILDREN LOVE TO COUNT—WHERE DO WE LOSE THEM?

Many students first encounter difficulties in learning math in primary and middle school when trying to derive meaning from a mathematics textbook. This means that teachers working in grades 6 through 12—the target audience of this book—are dealing with young people who have already found themselves flummoxed by the thought of cracking open a math book. The vocabulary is unfamiliar. The pages are laid out differently than in, for example, a history book. There are examples and homework problems and textboxes that define terms and theories and axioms, and these things are presented in a way that can be visually dense—a lot of symbols and equations, accompanied by only a small amount of straightforward text.

When mathematics study progresses from simple arithmetic problems to "word problems" or "story problems," the rubber really hits the road, and that road is the interface between mathematics and language. If you have ever worked a word problem—even a very elementary one like "Johnny has seven apples, but Suzie took one away. Sam gave him three more. It takes six apples to make a pie. Does Johnny have enough to make a pie?"—then you have walked that math/language interface.

Looking at the example word problem about Johnny and his apples, consider the words "took" and "gave." They really don't sound like math words, but in the context of the problem, they tell you to subtract or add in order to get to the answer. Most people, including lifelong math haters, can solve Johnny's apple problem in their heads. The goal of this chapter is to highlight regular words that can signal to readers that a little math is coming their way. Additionally, we want to help you recognize those words, so that you can show your students how to recognize them as well.

ONE TEXT, MANY LESSONS IN MATH LITERACY

This book is an unusual collaboration between a well-credentialed reading specialist and professor of education and an award-winning novelist with a strong background in higher-level mathematics. In later chapters, we will bring in established writers, published in a variety of fields, to give first-person insights that will be useful in using their work in the mathematics classroom.

First, however, we want to examine a substantial passage of fiction in terms of mathematical concepts, and we have the unique opportunity to do this with the work of someone who walks that line between mathematics and language every day of her life—one of this book's co-authors, licensed chemical engineer and novelist Mary Anna Evans.

This will be the first of many times that we explore the intersection between language and vocabulary and mathematics in this book, in keeping with Common Core State Standards for Mathematics, which emphasize the necessity of being able to construct arguments and critique the reasoning of other people (Common Core Standards Initiative, 2010).

As the basis for this discussion, we have created a complete list of the words used in the prologue and first chapter of Ms. Evans' novel, *Artifacts* (2003). The original text of this passage is included in Appendix A for reference, and the word list is included in Appendix B. Throughout the chapter, this word list will serve as a living example of the concepts discussed, and suggestions for classroom discussion and activities will be provided. At the end of the chapter, we have provided a complete lesson plan, referenced to Common Core standards, that might be assigned based on this word list or on other similar lists.

Co-teaching with Computer Educators to Enhance Word Analysis Assignments

Word lists for passages longer than a few paragraphs are very tedious to compile. In order to analyze the long passage from *Artifacts* used in this chapter, the authors made use of an algorithm for creating concordances that required significant manipulation of the text. Involving computer instructors in this project, so that students learn to use the sorting power of word processors, spreadsheets, and database software to quickly prepare word lists for long and complex passages, would be a useful interdisciplinary collaboration. This collaboration is tied to the Common Core standards, which state succinctly to "use appropriate tools strategically" (Common Core Standards Initiative, 2010, p. 7). The labor-intensive process of sorting words is eliminated, and the entire act of creating word lists could be moved out of the math classroom and into the computer technology classroom, saving class time for the more pertinent task of interpreting the mathematics content of the passage being studied.

Artifacts is a mystery featuring an archaeologist, and it was not written with the intent of communicating math skills. It was written for the purpose of telling a good story. Yet mathematics teachers have found lessons in Cartesian coordinates and fractions and simultaneous equations buried in the book that they've been able to use to good effect in their classrooms.

We suggest that, at this point, you take a few minutes to read the text in Appendix A. This will give you a context for the discussion of the mathematics buried in a passage that was written solely to entertain. In all likelihood, the thought of mathematics will not cross your mind while reading.

When you're finished, return to this chapter for an analysis of the ordinal numbers and geometry and other mathematical principles you just absorbed. It is our hope that after you've read the discussion of mathematical language gleaned from this small part of one book, you will find yourself equipped to pluck math concepts out of any book *your* students read and enjoy.

This new skill can serve as the basis of a quick 10-minute activity, such as asking students to silently read a paragraph about adults playing roulette or about children playing a board game, and then asking them to discuss concepts of probability that affect such games. (Hint: A board game played with a spinner will proceed differently than when a player's moves are dictated by a pair of dice. A spinner will give equal odds of giving any number on the dial, whereas the dots on a pair of dice will add up to seven a lot more frequently than they will add up to two or twelve.)

The ability to recognize math in literature can also foster lessons with a greater scope that span the entire curriculum. A math teacher who is teaching a probability unit and who knows that her students are reading Shirley Jackson's *The Lottery* in their literature class can add depth to their experience in both classrooms by comparing the simple lottery in the story to complex state-run lotteries.

But it all starts with being able to look at a story and see the math in it, so go to Appendix A and read a story that's got a lot more math in it than you might think.

Ordinal Numbers, Cardinal Numbers, Numerals, and Other Words Expressing Quantity

We will begin our work with the mathematics in *Artifacts* with one of the most basic concepts in math: quantity. In linguistics, cardinal numbers are words that represent quantity. When an English speaker casually uses the word "number," the meaning generally refers to cardinal numbers—"one," "two," "three," and so on. Ordinal numbers represent the rank of a number in terms of order or position. Examples include "first," "second," and "third." Numerals, symbols that represent quantity like "1," "2," and "3," also appear frequently in fiction, often to represent dates. More general words expressing quantity are very common: "many," "most," and "less" come immediately to mind.

These are words that we all use every day, and the difference between "one" and "1" and "first" is not a difficult concept for most people, so we have chosen this for our first exploration of the list of words in the excerpt from the novel *Artifacts* that is presented in Appendix B.

In reviewing the Common Core standards for the middle and high school grades that are the focus of this book, we found little mention of such a basic element of mathematical understanding as the necessity for building a mathematical vocabulary, beyond general statements like "Make sense of problems and persevere in solving them."

The authors of this book would argue that students need language as a way for their minds to interface with everything in their world, including mathematics. We see the verbal concepts presented in this section, and in this chapter, as rock-bottom fundamentals that must be addressed long before a student tackles algebra or calculus. So bear with us as we start with the words, "one," "two," and "three," and go on from there to build a vocabulary that will interface with any branch of mathematics your students will encounter.

So what numbers did we find in the excerpt from *Artifacts*? Here are the numerals, ordinal numbers, and cardinal numbers we have gleaned from that word list:

> 1, 15, 1782, fifty, first, five, hundred, mid-twentieth, one, second, seventy, ten, thousands, twentieth, two

(Before we continue, does this list make you vaguely uncomfortable? It is presented in alphabetical order, but this bears no resemblance to numerical order. Most people, when looking at this list, will get the sense that something is just . . . wrong. This is an indicator that even people who consider themselves far more oriented toward language than numbers have absorbed mathematical concepts at a subconscious level.)

From an educator's point of view, we should probably first remember that the passage that we have read comes from a mystery novel. When you think about it, this is an awful lot of numbers to be found in a work that isn't about numbers or math. It's simply a story about an archaeologist uncovering a dead body. You will likely find this is true of any written work, after you develop the habit of looking for the presence of math concepts. Such concepts are ubiquitous. People count and measure and plan all the time. There is value in pointing out this fact in the math classroom, and simple exercises like this one are a concrete way to demonstrate the ubiquity of mathematics in daily life.

How might this list be used in the classroom? A quick and easy exercise would be to ask students to look at the list in the previous box and distinguish which words are ordinal numbers, which are cardinal numbers, and which are numerals. A more in-depth exercise would be to ask students to read the source passage silently in class and develop their own lists of such words, which can then be compared to the boxed list in a classroom discussion.

The discussion could turn lively. Students might find themselves returning to the text itself for context, since "second" can be an ordinal number or it can be a unit of time. Which is it in this passage? Context is a powerful thing, in both mathematical systems and language systems. The value of the numeral "2" is very different in the following contexts: 2,000,001 and 1,000,002. And the value of "second" is very different in the following two sentences:

"You only need to stay under water for a second."

"You only need to stay under water for a second minute."

And is "thousands" actually a cardinal number? It doesn't refer to a specific quantity, but could represent any number greater than or equal to two thousand. An advanced student could be set loose on more in-depth reading in linguistics to develop his or her own opinion of whether "thousands" is a cardinal number. The answer to this question is beside the point; the exploration is everything.

Now, let's return to the word list and compile some lists of words that are number related but aren't necessarily ordinal numbers, cardinal numbers, or numerals. Here is a list of some mathematics-related words we compiled from the *Artifacts* word list. Do you agree with the inclusion of some words that are only tangentially related to math, like "a," which communicates something about quantity, and "accompany," which could signal a need for addition if it were encountered in a word problem?

> a, absence, abundance, accompany, all, alone, an, and, another, both, couple, Dernier, each, enough, every, everybody, everyday, everything, exactly, except, few, fraction, gave, get, getting, gift, gifts, given, got, grabbed, half, half-breeds, half-Creek, half-day, including, insufficient, isolation, last, loss, lost, many, missing, more, most, much, nearly, nothing, once, only, plurality, remained, remains, several, some, take, taken

This might be a different list from one that you might compile from the same data source—for example, we have reserved geometry-related words for a later section—but this is a benefit when it comes to fostering classroom discussion.

Notice that some of these words, like "fraction," are clearly math oriented. Some of them, like "gave," are less clearly related, yet they do communicate some sense of quantity. Consider the word problem we discussed in an earlier section:

> Johnny has seven apples, but Suzie took one away. Sam gave him three more. It takes six apples to make a pie. Does Johnny have enough to make a pie?

Most students will recognize, when guided by the teacher, that the word "gave" is a clear indicator that solving this problem will require you to use addition. Giving students the ability to develop a problem-solving strategy based on the verbal description of a real-life problem is the most basic reason for all the activities in this chapter. After a student has dissected the language in the *Artifacts* excerpt and found math there, the task of deciphering a newspaper report on the latest cancer research should be far less daunting.

Looking back at the word list from *Artifacts*, the word "Dernier" isn't even in English, but reading the original passage reveals that "Dernier" is French for "last," which most people would perceive as representing a position in time or numerical order. Notice that context allows us to interpret words in languages that we might not even speak, and it allows us to infer mathematical meaning, as well. At this point, students could be instructed to re-read the passage and state which of those concepts—time or numerical order—was intended by the author when "Dernier" was used in that context.

It's almost inevitable that some readers will perceive mathematical meaning in certain words, while others will disagree. A student who is asked to defend the inclusion of "missing" on her list will be required to draw on verbal, logic, and mathematical skills simultaneously. In other words, she will be asked to dance along that boundary between the verbal world and the numerical world, and that's a good thing.

Geometry—The Shape of Things

Geometry is a branch of mathematics that requires different skills from the math courses that students have taken on their way to the geometry classroom. Geometry requires students to visualize shapes in two or three dimensions. It requires the student to be able to describe an object's position in space. It brings the concepts of measurement and distance into play.

The study of geometry is an opportunity to seize the attention of visually oriented kids who gravitate toward more right-brained activities. It is also an opportunity to teach students with a natural inclination toward math—the kids with a facile understanding of how to push symbols around on a page—what to do when it *isn't* obvious how to solve a problem. First, you draw a picture.

This simple act will often reveal what is known about a problem and what is not known, and how these two factors relate to each other. This may be the most important takeaway skill from the study of geometry, which is so central to the study of math that it is referenced at all levels of the Common Core standards.

So let's look at the word list from *Artifacts* for geometry-related words:

> about, above-ground, across, adjacent, against, angle, around, aside, atop, away, balance, base, behind, beneath, beside, between, bottom, broad, by, cavernous, center, crest, crossed, curve, deeper, direction, directions, displaced, distance, down, edge, empty, end, feet, foot, form, formed, forth, fragments, from, front, full, further, gap, grid, here, high, in, inch-long, inch-thick, into, large, largest, left, left-to-right, length, level, lifted, little, long, longer, middle, midst, narrow, next, off, on, onto, open, opened, over, palm-sized, paralleled, part, penetrate, pinpointed, place, point, puny, quarter-inch, radiating, recessed, rectangle, right, ring, row, shape, shaped, size, sizeable, small, solid, somewhere, spiral, spot, spots, square, stick-straight, surface, survey, surveyor's, tall, through, tiny, to, top, topmost, topped, topping, touch, touched, touching, toward, translated, under, underlain, unfolded, up, upper, upward, vertical, west, where, zone

Did you see other spatially oriented words that we missed? This is the beauty of open-ended discussions like those presented in this chapter. One student will get a different answer from a student sitting two seats away. And that's okay.

There are certainly some obvious geometry-related words in the list presented in the previous box: angle, paralleled, point, rectangle, shape, square, translated. Again, it's interesting to find these words in a novel, and this is something you might point out the next time a student asks you, "How are we going to use this in the *real* world?"

(Some things in an educator's life never change. We feel sure that Aristotle asked Plato this age-old question, and then Alexander the Great turned around and asked it of Aristotle. The purpose of this book is to give you some fascinating answers, so that you're ready the next time it pops up.)

What else can we glean from the word list? Many words give a sense of distance: displaced, distance, further, length, long, longer, quarter-inch. Others give an impression of locating an object in two- or three-dimensional space: behind, beneath, between, by, down, front, further, grid, here, inch-long, inch-thick, left, left-to-right, middle, midst, over, right, somewhere, top, topmost, toward, under, underlain, up, upper, upward, west.

Notice how many of these words are prepositions, which are a part of speech that often describes position. "Off the beaten path," "under the boardwalk," and "by the river" are all prepositional phrases that tell the reader where something is . . . and that's geometry.

"Surveyor" and "surveyor's" are of particular interest as geometry-related words, because a land surveyor makes a living through the use of geometrical principles. The notes of two archaeologists who are using the principles of surveying to set up their work site are a critical element of the plot of *Artifacts*, and an important geometric concept—translation—is a clue to the motive for two murders. Was the book written with geometry in mind? Of course not. It was created to entertain people. Can its storyline be used to teach math? Oh, absolutely.

A QUESTION OF TIME

In our culture, children are taught to tell time from an early age, and when we use the phrase "tell time," we don't merely mean understanding the symbols on a clock's face. (Now that most clocks are digital, this is not very complicated.) Implicit in the notion of "telling time" is an understanding of what time means.

If I am brushing my teeth at 7:22 AM, and my school bus comes at 7:30, is this a good thing? A meaningful answer to this question depends on the knowledge of how long it takes me to brush my teeth, whether I can walk right out the door when I'm finished, and how long it takes me to walk to the bus stop. A comprehensive understanding of time as it relates to daily life is one of the most valuable life skills. Chronic tardiness can result in unemployment, which will send the person involved straight to his or her bank account to assess whether there are enough funds in that account to pay the bills. (And bill paying is yet another math skill.)

The concept of time touches every aspect of life, so we would expect our ongoing analysis of the *Artifacts* word list to reveal many references to time, and it does. The list was gleaned from a book centered on archaeology and history, so it is especially rich in such references. It's worth noting that the Common Core standards for middle and high school grades make little mention of time, because concepts related to interpreting time as measured by a clock are taught in the lower grades. Yet Einstein's work in relativity dealt heavily with the concept of time, and the sciences frequently measure physical phenomena in terms of time. Velocity is defined as the distance traveled divided by the time elapsed. The unit of "hertz" expresses cycles per second. It seems appropriate to retain some focus on time in middle and high school grades, as students mature into an ability to comprehend the subtleties of time measurement.

Even beyond the list presented in the following box, an argument could be made that nearly every English verb could be considered time related. Consider the irregular verb, "to be," and some of its derivatives, "was," were," "is," "am," and "been." The verb form communicates very clearly whether the action being discussed takes place in the present or in the past. The form "will be," is one way that this verb can take the reader into the future. Similarly, an action verb like "run" can communicate the passage of time through its various tenses: "run," "ran," and "will run."

Review this word list and consider whether you agree with the words we've chosen to include.

after, age, already, always, ancient, as, awaited, awhile, before, centuries, century, contemporary, current, date, dated, day, day's, days, decades, early, finally, former, later, minute, moment, now, old, past, paused, primeval, recent, since, sometimes, sudden, time, times, tomorrow, Tuesday-night, when, while, whilst, will, year's, years, yet

What's the Likelihood of Finding Math in Your Pleasure Reading?

Other instances of math in day-to-day life include logic, probability, and finance. Almost everyone who claims a complete mathematical ineptitude can handle pocket change competently. When we pay for purchases in cash and make change, deciding the optimum number of quarters, nickels, dimes, and pennies to complete the transaction, we are doing algebra, whether the math-phobes of the world want to believe it or not. Virtually all of us have some sense of how likely it is that a given lottery ticket is a winner, whether we want to admit it or not. The ability to follow a line of reasoning like this, "If the phone rings and if I take the time to answer it, then I will be late for work," is a clear indication of an understanding of basic logic.

To round out this chapter, we have compiled a list of words for each of these disciplines—logic, probability, and finance. A similar list can be gleaned from almost any passage plucked randomly from a book, magazine, newspaper, or website. Asking students to look for those words in their out-of-class reading and then share examples with the class is a good way to sharpen their observation skills and their understanding of the presence of mathematics in their personal lives.

Logic: if, maybe, no, nor, not, or, sure, surely, then, therefore, unless, whether, which, why

Probability: likelihood, likely, probably, rarely, sample, samples, sampling, unlikely

Finance: interest, pay, paycheck

LOOKING FOR MATHEMATICAL TEXT IN YOUR STUDENTS' WORLD

Have we completely dissected the possibilities of our *Artifacts* word list yet? No, and that's a beautiful thing. One rewarding activity might be to discuss the mathematical concepts presented earlier and then send students to the original book excerpt and ask whether the narrative itself has a direct link to mathematics.

A long-term assignment might include instructions to read the entire novel, with different students asked to write papers on various mathematical angles that can be teased out of the narrative. For example, a considerable portion of the multiracial protagonist's genealogy can be inferred from the story. Is it possible to estimate the relative proportions of her heritage that originate in Africa, Europe, and the Americas? Another assignment might be a paper describing the methods that surveyors use to measure large areas of land.

Another paper still might stem from the notion of compounded interest, based on a scene late in the book describing the growth of an interest-bearing bank account that was established in the 1960s.

Less open-ended questions appropriate for classroom discussion might include the following:

Classroom Discussion Questions for *Artifacts*

1. When the word "foot" is mentioned, does it refer to the unit of measurement or to the part of Faye's body that is attached to her leg? When the word "second" is used, is Faye focused on the second in a list of objects she's counting, or is she thinking of a unit of time? Can we answer those questions from the list, out of context, or must we return to the original text and read the sentence from which the words came?
2. How does mathematics relate to the work life of a real archaeologist? Is measurement an important part of the profession? Counting? Statistics? Can an archaeologist survive without geometry? (Hint: If you dig up an artifact that turns out to be proof that Martians landed in 20,000 B.C.E., you're going to want to send a work crew back to the spot where you dug it up. It would behoove you to be able to find the spot.)

At the conclusion of this chapter, we have included a complete lesson plan based on a scene from *Artifacts* where the characters are all astonished at the size of an interest-bearing bank account that was established in the 1960s. Our intent is that this plan could serve as a model for developing your own classroom activities from whatever book is setting your students' imaginations aflame. Has this discussion prepared you to pluck mathematics out of other non–mathematics-related books? Do you think that these exercises will help your students learn to do the same thing?

When you think your students are ready to try their wings, take these exercises into new territory, trying assignments like these:

Helping Students Find Math in Unexpected Places

1. Provide a short reading passage of your choice, and ask your students to make their own lists of math-related words. Then ask the students to discuss their lists and defend their word choices.
2. Ask your students to write an essay on any subject, and then surprise them with an assignment to pluck math words out of their own prose. They will likely be shocked to see how naturally their mind handles mathematical concepts.

These projects can be integrated with teachers in other subject areas. Math teachers and English teachers can team up to investigate the mathematics buried in the texts that students are reading for English class. Computer technology teachers can assist with the database aspect of compiling a list of mathematical words from a reading passage.

When these projects are all in the past, your students will have internalized the concept of mathematics as presented in text, and they will carry this concept with them, whether they are reading a mathematics textbook, a business balance sheet, or the sales pitch for a life insurance policy. Their understanding of math will be better integrated with their understanding of verbal communication, and this integration is a good thing to have.

LESSON PLAN 1-1

INTERESTED IN GETTING RICH? AN INVESTIGATION OF SIMPLE INTEREST, COMPOUND INTEREST, AND EXPONENTIAL FUNCTIONS BASED ON *ARTIFACTS*

This lesson plan is designed to give a comprehensive approach to demonstrating that mathematics content can be found anywhere, even in popular fiction. The lesson plan includes guided questions that preteach vocabulary or concepts that will be reinforced and practiced throughout the reading assignment and further activities.

(*Continued*)

LESSON PLAN 1-1 (Continued)

Materials & Preparation: Worksheet, pencils, graph paper, *Artifacts*, calculators (optional)
Duration: 45–60 minutes or homework assignment

Applicable Common Core Standards:
Grade 6: Ratios and Proportional Relationships
Expression and Equations
Grade 7: Ratios and Proportional Relationships
The Number System
Expressions and Equations
Grade 8: Expressions and Equations
Geometry
High School:
Algebra: Seeing Structure in Expressions
Reasoning with Equations and Inequalities
Functions: Interpreting Functions
Linear, Quadratic, and Exponential Models
Modeling

Procedure: Spark students' interest by shouting phrases like, "No interest until January!" or "Buy now and get a special reduced interest rate!" in the same fashion that they have likely heard on radio and television commercials. Ask students for examples of times when they have heard the words "interest" and "rate" in advertisements, and then ask them to explain the concept of interest from the point of view of the consumer.

After a short discussion activating prior knowledge of the financial concepts of interest paid to individuals who have invested money and interest paid by individuals who have borrowed money, hand out the prereading questions. These questions will expose students to the interest formula, its variables, and how to find various information depending on what variables are defined. You may instruct students to work independently or in groups depending on ability level and strength in prerequisite skills, like solving multivariable equations. After allowing students time to discuss and work the problems, you may choose to have a short lesson or discussion to clear up any misconceptions about appreciation, interest, and rate.

Assign the excerpts from *Artifacts* and the postreading activity sheet for students to complete as they read. As students work through the plot of the story, they will uncover mathematical vocabulary related to the material taught in the prereading activity. Students can continue practicing and extend their knowledge of interest with the real-world scenarios discussed in the novel. By the time the students have completed the pre- and postreading activities, they will have a deeper understanding of the content as well as its real-world applications discussed in the novel.

Percentage, Interest, and Appreciation Prereading Questions

1. **True or false?** Interest is calculated based on the amount of the original investment, also known as the principal.

Answer: True

Simple annual interest is calculated with the following algebraic formula: $\mathbf{I = P \times R \times T}$, where **I** represents the amount of interest, **P** is the amount of the principal or original investment, **R** is the annual interest rate written in the form of a decimal, and **T** is the time in years that the money is invested.

For example, the interest on $50 invested at a 6% annual rate for a year is 50 × 0.06 × 1, or $3. To get the total amount of the investment at the end of the year, add the principal and the amount of interest earned, which would yield $53 for this problem.

Try this problem on your own: Sara invested $100 for 1 year. At the end of that year, she received $109. What annual rate of simple interest did she receive?

a. 0.09%

b. 0.90%

c. **9% – Correct answer. Interest of $9 equals Principle ($100) × Rate × Time (1 year). Solving the equation yields a rate of 9%.**

d. 90%

2. **Appreciation** is found by subtracting the original value of an asset from its appreciated value. For example, a piece of land that was bought for $10,000 and sold for $25,000 has appreciated by $15,000.

Try this problem on your own: A house was purchased in 1989 for $175,000. It was sold in 2004 for $235,000. What was the total amount of appreciation?

Answer: $235,000 − $175,000 = $60,000

Reading Passages

Excerpt from *Artifacts*, pages 58–60

"Thank you for taking the time to see me," Faye began. She didn't want to try [the senator's] patience and she'd never been one to beat around the bush anyway, so she plunged directly to the point. . . .

(*Continued*)

LESSON PLAN 1-1 (Continued)

"Seagreen Island is mine," she said. "Well, it should be. My great-great-grandfather purchased Last Isle in the 1850s, back when it was all one island. Shortly after he bought it, the great hurricane of 1856 carried away most of the island, along with a few hundred planters and their families and slaves. There was a resort there at the time."

"I've heard the story."

The adrenaline was getting to Faye. . . . "Most people haven't," she said. "If a bunch of rich Astors and Vanderbilts and Roosevelts had been swept off Cape Cod, it would be in the history books."

"There was a war coming on in 1856, and the victors do write the history books." . . . .

"Yeah, but if somebody had bothered to write about what happened on Last Isle," she rattled on, "my great-grandmother might never have lost her land." . . . .

"This dispute is older than I am. Why are you coming to me now?"

"Don't you see? Some of my land has been absorbed into the wildlife refuge. Let them keep it. It's not fair, but at least they're preserving it. Help me get Seagreen Island back . . . and no tacky tourists will ever tear up the place. Your voters will be happy, and God knows that will make you happy."

Excerpt from *Artifacts*, page 98

"Your problem is an interesting and important one," [the senator said]. If your claims prove true, then your family was defrauded of a piece of property whose value has appreciated significantly the past few years."

Excerpt from *Artifacts*, pages 284–285

After everyone drank to Abby, [the sheriff] said, "I guess you folks have heard about the bones that keep washing ashore on Seagreen Island."

"Are they Abby's?"

"Maybe. It'd be hard to prove. . . ."

"Do you have the arm bones? The right upper arm?" Douglass asked. . . . "Because I was there when she broke it . . . . I've got a picture of her in the cast."

"It's circumstantial," the sheriff said. "Reckon it'd be enough to get the trustee of Mr. Williford's estate to release the reward money at this late date?"

"I imagine so," Douglass responded, "considering that I'm the trustee."

Everybody looked at Faye, who didn't want to be crass and ask, "How much?" She stifled the question with a loud exhalation.

"Don't know how much, exactly," Douglass said. "I don't check regularly, because it just sits in the bank and earns interest. The reward was twenty thousand dollars in 1964. Reckon it would make a regular person rich, but you're gonna blow it on this house. Reckon it'll just make you comfortable."

Faye wondered how long it would be before she could breathe again.

Postreading Questions Based on the Excerpts from *Artifacts*

3. You were told in the excerpt that Faye's property on Seagreen Island has appreciated in value significantly. Assume the island—valuable beachfront property—was worth $120,000 in 1856. If it was worth $720,000 when Faye's family was defrauded of it in 1932, how much had it appreciated during that time?

Answer: $720,000—$120,000 = $600,000

4. If the current value of Seagreen Island is $12,000,000, how much has its price appreciated since 1932? And how much has its price appreciated since 1856?

Answer: Since 1932: $12,000,000 − $720,000 = $11,280,000

Since 1856: $12,000,000 − $120,000 = $11,880,000

5. As we learned in Question 1, simple interest is calculated by applying the interest rate to the original amount invested. Simple interest on $1,000 invested at 10% will be $100 per year. Think about this for a minute. That means, after the first year, there will be a $1,100 in your account, but you will only be earning interest on the original $1,000. You would be much happier if you earned interest on your whole account—$1,100—during the second year, wouldn't you? This is called **compounding**.

- Let's try going further with the idea of compounding. If you earn 10% interest on $1,100 during the second year, how much money will be in your account?

Answer: $1,100 + ($1,100 × 0.10) = $1,210

- If you earn 10% interest again in the third year, how much interest will you earn?

Answer: $1,210 + ($1,210 × 0.10) = $1,331

You can see that calculating compounded interest year by year over a period of 100 years would get very old. Fortunately, there is a formula to calculate compounded interest over any period of time: $\mathbf{A = P(1 + r)^n}$, where **A** is the amount of money accumulated at the end of the investment period, **r** is the interest rate expressed as a decimal, and **n** is the number of years in the investment period.

You may use a calculator for this problem:

- Using the formula provided above, calculate the amount of money in an account after 20 years, if the initial deposit was $300 and the interest is 8%, compounded annually.

Answer: $\$300(1.08)^{20} = \$1{,}398$

(Continued)

LESSON PLAN 1-1 (Continued)

6. **You may use a calculator for this problem.**
   When Douglass tells Faye that she will receive the $20,000 reward, plus the interest it has earned since 1964, she finds that she is unable to breathe. *Artifacts* was published in 2003, 39 years after the money was invested in 1964. If we assume an average interest rate during those 39 years of 5%, compounded annually, how much money will she receive? Why can't Faye breathe?

Answer: $\$20{,}000(1.05)^{39} = \$134{,}095$. If Faye was lucky, the interest rate was more than 5%. Invested at 10%, the total would be $822,895. Would you have expected the number to be so big?

Let's hope Douglass invested that money well!

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Chapter 2

Reading Strategies: Making Meaning of Mathematical Text

INTRODUCTION

Supporting student reading isn't such a difficult task when we remember that reading is about making meaning—about being strategic in approaching text that is difficult to understand at first glance. When students are asked what to do when they are trying to read something they don't understand, students of all ages will often immediately say something like, "Sound it out." In other words, they are describing the strategy they use to decode a word they don't recognize. But what should readers do when they encounter an entire passage that doesn't make sense to them?

Reading is an active process (Goodman, 1996); readers have to work to make meaning, by making connections, asking questions, and visualizing what they are reading (Harvey & Goudvis, 2000). Effective readers monitor their comprehension and know when they need to make adjustments (Rhodes & Shanklin, 1993). How many times have you gotten to the bottom of a page and realized you had no idea of what you read? What did you do? Go back?

Struggling readers don't always realize that reading is supposed to make meaning, and when they realize that they don't understand what they just read, they often simply keep going, because they think that reading is just about decoding the words. Teaching students to monitor their comprehension can help them to be more strategic readers.

HOW DOES READING MATHEMATICS DIFFER FROM OTHER KINDS OF READING?

Historically, most reading done in the mathematics classroom involves technical prose, such as in a textbook (Alvermann & Moore, 1991). Reading a textbook is different from reading other types of texts, posing specific obstacles to making meaning (Koebler & Grouws, 1992).

In writing this chapter, we reviewed a textbook currently in use for a sixth-grade mathematics course. We opened the book randomly, finding a two-page layout on variables and expressions. The concepts presented on those two pages included variables, constants, and algebraic expressions. The procedure for evaluating algebraic expressions with one missing variable was presented, followed by the procedure for evaluating algebraic expressions with two missing variables. Students were also instructed in methods for translating between symbols of multiplication and division. Even for an adult experienced in mathematical concepts and procedures, this is a lot of material for two pages.

On the two pages following that two-page layout, students were given guided practice, independent practice, and standardized testing preparation questions. There were over thirty questions with several multistep problems that required students to use skills that included completing tables and reading graphs. This density of text is necessary when the volume of material presented in a year-long mathematics class is considered, but it is difficult for anyone to read and interpret. Simply looking at all those numbers, letters, and symbols is daunting enough to make a struggling reader want to close the book.

How can we help that struggling reader?

Because of the volume of material that must be presented, chapters must include many new terms and concepts in a short amount of space, which requires very complex text structures. For example, students are often conditioned to find the topic sentence at the beginning of a paragraph, but in a math textbook, the topic sentence may be in the middle or at the end of the paragraph, after several supporting statements, a newly defined word, and a formula that consists of three letters, four numbers, and two symbols.

Mathematics textbooks often present new ideas multimodally, where words, symbols, and abbreviations are all used to represent words and concepts. Words can have multiple meanings and can represent concepts, operations, or relationships, which adds yet another meaning-making layer. A still more confusing phenomenon is the redefining of sight words the student already knows, giving them new mathematical meanings that bear little resemblance to familiar concepts (Hancewicz, Heuer, Metsisto, & Tuttle, 2005; Hiebert et al., 2003; Smith, 2002).

For example, consider these words:

point

model

formula

mean

set

Taken out of context, even a math teacher might hesitate when asked what they mean (and even the word "mean" can have a different meaning for mathematicians than for others). An adult might know that babies drink something called "formula" because the manufacturers have historically promoted it as being mixed according to a formula that was designed to make sure that nutrients were available in the proportions most conducive to health. But do adolescent students have this prior knowledge? The following concatenation of numbers and letters looks nothing like the white powder that parents mix with water to feed their infants:

$$a^2 + b^2 = c^2$$

Similarly, anyone who thoroughly grasps what a mathematician means by the words "set," "model," and "point" could explain the relationships between the mathematical concepts and the colloquial uses of the words. But a middle schooler who has only encountered these words in casual conversation is likely to be flummoxed, particularly if that middle schooler's vocabulary skills aren't strong in general. It's important for a math teacher to remember to take time when teaching new material, early on, to teach the vocabulary that is specific to mathematics.

If reading a math textbook poses such problems, why would we even suggest adding reading material to a math classroom? We suggest it because a carefully selected novel or trade nonfiction book can tie the abstract world of mathematics to the world that students inhabit and understand. It can give them the prior knowledge to know that "modeling" does not necessarily mean posing for pictures. It can bring the theoretical world to life.

HOW CAN WE HELP STUDENTS MAKE MEANING FROM MATHEMATICAL TEXT?

The meaning of a word, paragraph, chapter, or book is inextricably linked to the reader's prior knowledge of the material being presented. Even a chunk of text as simple as this—C-A-T—is meaningless if you've never seen a cat or if you don't speak English.

This prior knowledge, sometimes referred to by the term "schema" (Alvermann, Phelps, & Gillis, 2010; Goodman, 1996), allows the reader to easily access related information, assimilate new information, determine the importance of new information, and much more. Making meaning from a math textbook that is full of information for which the reader has little or no prior knowledge is an opportunity for failure.

Television producers show awareness of the importance of prior knowledge when a show opens with scenes reviewing the action in previous shows. Otherwise, viewers who missed a show or, worse, have never seen an episode of that program can be left behind.

In a more complex example, imagine yourself watching a trailer for a movie. There are random scenes of exciting events—explosions, helicopter crashes, beautiful people running for their lives—but no information that can help you piece together the plot. And then the image of something specific and familiar, maybe the theft of a unique artwork, triggers a memory, and you have a little a-ha moment. You realize that you have read the book on which this movie is based, and now everything makes sense.

A common metaphor for the storage and access of knowledge in the brain is the filing cabinet model—this is how one of this book's authors, Dr. Faith Wallace, was first introduced to the concept of prior knowledge. She remembers viewing a transparency of the brain with a file cabinet over it, which imagines the brain as a grouping of file folders where information is stored. These folders are labeled in such a way as to facilitate information retrieval, and then they themselves are stored in drawers and cabinets, which are also organized by topic.

Perhaps a better metaphor for the current generation is to think of the brain as being similar to the Internet. Information is accessed through connections that are much like hyperlinks linking far-flung storage devices. Using this model, it is reasonable to think of prior knowledge as being the search engine that helps us navigate those links. When you "type" a C and an A into your prior knowledge search engine, it begins offering you options, just as your Internet search engine does, asking you whether you're looking for "cat" or "California" or "catastrophe." Without the help of that prior knowledge, you would spend a lot more time sifting through your knowledge and memory for the tidbit of information you're seeking.

Relating this model of the brain to reading and meaning making, the speed with which you get to the meaning of something you are reading just might have to do with how much prior knowledge you have about the topic. How quickly can you relate new material to something you already understand?

Classroom time is, by necessity, dedicated both to storing new information in the databases between your students' ears and to building links that will help them find that information later, when they need it. This is the knowledge base that they will take with them when they leave your classroom. And perhaps the day will come when one of your students is mixing powdered formula for his baby's bottle and the hyperlink between his world and the things he learned in your math class lights up.

"So *that's* why they call this stuff formula!"

Prior Knowledge: An Experiment

Read the following passage and be prepared to answer some questions about its content:

> So this was Excellent? Lots of traps on this one, I thought as I walked. From the line I saw the tunnel to the triple and the weave to the see-saw – talk about discrimination! Maybe I will lead out. Planning ahead, I would probably have to pivot or flip to get to the table. After that maybe a front cross to the double and rear cross to the walk? That serpentine will cause problems getting her to scramble. Better hit my contact—running or target? I think this should be easier since we are at 20 now—Preferred. No MACH in our future no matter what happens today, but at least we can double Q.

Now, what do you think this passage is about? Did reading this passage give you some degree of sympathy for students from different cultures who try to make sense of the proceedings in an American classroom, especially students who are not native English speakers?

Were you able to read to decode the passage and make meaning? Or were you left scratching your head wondering what that was about?

For fun, one of this book's authors, Dr. Faith Wallace, wrote this passage and gave it to the other author, Mary Anna Evans, and asked Mary Anna to tell her what it meant. Here are some quotes from Mary Anna's response:

> "Maybe it was about something to do with flying – a MACH is referenced and other military words like contact, pivot, and flip."
>
> "By the capitalization of Excellent, I infer that the narrator is looking at something that can be named, possibly a place."
>
> "There is a military or paramilitary sound to much of the language, and I'm guessing that this passage comes from a science fiction or military fiction work."

Please know that, even though Mary Anna had no idea what Faith was writing about, she was still using some degree of prior knowledge, because she knows that Faith reads a great deal of science fiction. What she forgot, however, is that Faith also trains dogs for agility trials. The passage is written from the point of view of a trainer preparing to begin a competition.

Faith also tested her premise in the classroom, asking students to interpret the passage. When asked what activity the narrator was describing, she got guesses that ranged from gymnastics to video gaming to snowboarding. None of the students was able to define "scramble" in the context of the passage, yet nearly all the students were able to infer that "20" and "Preferred" had the same meaning, based on the second to last sentence.

So what would it have taken to give students the prior knowledge needed to understand the passage? A crash course in dog agility? No. It could have been done with nothing more than a quick viewing of a 1-minute video of a dog running an agility course, with narration that points out that when the dog runs up and over an A-frame–shaped obstacle, she is "scrambling."

Before giving any reading assignment, it is essential to consider whether students' prior knowledge is sufficient for them to understand it. Questions designed to test understanding must be carefully framed. Remember that Faith's students were able to infer the meaning of "Preferred" without ever understanding what the passage was really about. But without a careful assessment of the reading assignment before it is given and, if necessary, a prereading discussion of unfamiliar concepts and vocabulary that will be encountered in the passage, any class time spent reading can be utterly wasted.

Helping students with meaning making starts early, long before they encounter the text they will be reading. Your care in choosing and evaluating any text to be read in the classroom will be inherent in any later use of that text. For this reason, it is essential not to rely on a book review, or even on the recommendations in this book. You must read material to be assigned personally, in order to assess appropriateness for *your* students. Nobody knows your readers better than you.

While reading, think about where your students might struggle, focusing on text structure, vocabulary, and prior knowledge. Are there too many new vocabulary words? Is the sentence structure confusing?

Then develop lesson plans with this evaluation in mind. Do you need to preteach math concepts or vocabulary on which this new text builds?

USING READING ACTIVITIES TO ENHANCE STUDENT UNDERSTANDING

Before beginning reading, students benefit by being reminded of their goals, and those goals aren't always the same. Most reading material can be read with one of the following three goals in mind (Wallace, Clark, & Cherry, 2006):

How come?

What if?

So what?

Nonfiction text is often read with the motivation to find out facts—in other words, to answer the question, "How come?" Fiction explores imaginary worlds that are rife with the possibilities implicit in the question, "What if?" And the special type of text called "environmental print"—the barrage of text that flows into our lives through sources like the Internet, the mailbox, billboards, and instruction manuals—typically addresses the question of "So what?" We have included an entire chapter devoted to each of these types of reading material in this book (Chapters 3 through 8) to help you explore those questions with your students.

Among the structured activities presented in the academic literature that pertain to the importance of prior knowledge to literacy is the comprehension guide proposed by Harold Herber (1978). A variation on Herber's comprehension guide, the Three-Level Comprehension Guide described by Alvermann et al. (2010) in *Content Area Reading and Literacy*: *Succeeding in Today's Diverse Classrooms*, is the basis for the example project at the end of this section, based on Lewis Carroll's classic novel, *Alice's Adventures in Wonderland*. Such comprehension guides are excellent tools to help students apply prior knowledge to the texts they read.

At the end of this chapter, we have included three sample lesson plans designed as structured activities used to explore important elements of meaning making, such as prior knowledge, vocabulary, and questioning. First, however, we want to discuss one activity, the comprehension guide to *Alice's Adventures in Wonderland*, within the context of the chapter. It is based on a classic work of fiction familiar to nearly everyone, yet one that is not likely to be the first story one thinks of when one thinks, "mathematics." It is simply and quickly presented, yet it follows Alvermann's Three-Level Comprehension Guide, discussed earlier, by asking three questions aimed at progressive levels of understanding—literal, interpretive, and applied—and it addresses these important concepts from Common Core standards (Common Core State Standards Initiative, 2010).

Common Core Standards:

Grade 6: Expressions and Equations

  Ratios and Proportions

Grade 7: Expressions and Equations

  Ratios and Proportions

  The Number System

Grade 8: Expressions and Equations

  Functions

High School:

Algebra: Reasoning with Equalities and Inequalities

  Modeling

This exercise uses a familiar work of fiction to introduce a less familiar mathematical concept—ratio—and asks the student to continually compare what he or she reads and computes against the prior knowledge that will make it

Reading Guide: *Alice in Wonderland*

Comprehension Guide

Read the following excerpt from *Alice's Adventures in Wonderland* and follow the instructions below:

> This time Alice waited patiently until it chose to speak again. In a minute or two the Caterpillar took the hookah out of its mouth and yawned once or twice, and shook itself. Then it got down off the mushroom, and crawled away in the grass, merely remarking as it went,
>
> "One side will make you grow taller, and the other side will make you grow shorter."
>
> "One side of WHAT? The other side of WHAT?" thought Alice to herself.
>
> "Of the mushroom," said the Caterpillar, just as if she had asked it aloud; and in another moment it was out of sight. . . .
>
> It was so long since she had been anything near the right size, that it felt quite strange at first; but she got used to it in a few minutes, and began talking to herself, as usual. "Come, there's half my plan done now! How puzzling all these changes are! I'm never sure what I'm going to be, from one minute to another! However, I've got back to my right size: the next thing is, to get into that beautiful garden – how IS that to be done, I wonder?" As she said this, she came suddenly upon an open place, with a little house in it about four feet high.
>
> "Whoever lives there," thought Alice, "it'll never do to come upon them THIS size: why, I should frighten them out of their wits!"
>
> So she began nibbling at the righthand bit again, and did not venture to go near the house till she had brought herself down to nine inches high.
>
> (Lewis Carroll, 1916, pp. 28–30)

1. Literal. Place a checkmark next to the statement (or statements) you think says the same thing the author says.

__________ a. Alice is able to make herself normal-sized after the caterpillar tells her the secret of the mushroom.

__________ b. The mushroom Alice is carrying has the power to make her larger or smaller.

__________ c. Alice can control her size by the side of the mushroom she nibbles.

Answer: All of these statements are true, based on Carroll's text.

2. Interpretive. Check the statement (or statements) you think the author implies. Some thinking is required! Be ready to support your answers.

__________ a. Alice can control her size by the amount of mushroom she eats.

__________ b. Alice, at normal size, is too big for a house that is 4 feet tall.

__________ c. Alice's normal size is bigger than 9 inches.

Answer: All of these statements are true, based on Carroll's text.

Reality check: Using your prior knowledge, think about the size of a house and the size of a little girl. Does it make sense that a normal-sized girl is too big for a 4-foot-tall house? Does it make sense that she is taller than 9 inches?

3. Applied. Answer the following questions

a. If Alice is 9 inches tall and the house is 4 feet tall, what is the ratio of her height to the height of the house?

Answer: 9:48, or 3:16

b. If one bite of mushroom always increased Alice's height by 8 inches, can you write an algebraic formula to describe that relationship? Let H represent her new height in inches, let h represent her old height in inches, and let b equal the number of bites taken.

Answer: $H = h + 8b$

c. If Alice's normal height is 4 feet tall, use the ratio you calculated in question a to calculate the height of a house that is proportionate to the one described in the book.

Answer: $4/x = 3/16$**. Solving for** x **yields 64/3, or 21 1/3 feet**

d. Does this sound like a reasonable height for a house? How tall do you think your house is? How tall are you? What is the ratio of your height to your house's height?

easier to recognize whether the answer obtained from working a math problem is plausible. And did you notice Alice's confusion over the Caterpillar's meaning?

> "One side will make you grow taller, and the other side will make you grow shorter."
>
> "One side of WHAT? The other side of WHAT?" thought Alice to herself.
>
> "Of the mushroom," said the Caterpillar, just as if she had asked it aloud . . .

Even in Wonderland—or maybe especially in Wonderland—meaning making can be impossible without the necessary prior knowledge.

LESSON PLAN 2-1

A MIRROR TO NATURE ANTICIPATION GUIDE

In an anticipation guide, questions given before a reading assignment are geared toward preteaching concepts, vocabulary, or opening discussion for further instruction on a concept. Prereading activities are often paired with a postreading activity so that students can practice concepts, revisit their original answers, and possibly form new opinions.

Materials: *A Mirror to Nature* by Jane Yolen, printed Anticipation Guide

Duration: 25 minutes: 5 minutes prereading, 10 minutes reading, and 10 minutes postreading and discussion

(*Continued*)

LESSON PLAN 2-1 (Continued)

Applicable Common Core Standards:
Grade 7: Geometry
Grade 8: Geometry
High School:
  Modeling
Geometry: Congruence
  Similarity, Right Triangles, & Trigonometry
  Modeling with Geometry

Procedure: Students have prior knowledge of reflections, but most do not realize the different types of reflection and their mathematical meanings. This activity is geared toward opening their minds to reflections across different axes, as well as making them aware of ways that their prior knowledge of reflections translates into mathematics. Hand out the Anticipation Guide, and instruct students to complete the prereading questions. Use student answers as an opportunity to discuss their prior knowledge, allowing students to respond to each others' answers

Pass out the book *A Mirror to Nature*, and instruct students to begin reading it and looking at the pictures. The book plays on prior knowledge of reflections in water and can be used to demonstrate the various lines of reflection and symmetry. Accompanying the photographs showing reflection are short poems about the photograph's subject.

As students complete the reading, you may instruct them to move to the postreading questions and possibly to re-answer some of their prereading questions. If students want to change their answers, they may do so, leaving the original answers for comparison. The contrast between answers will serve as a learning tool when they realize how their knowledge has developed and changed after reading the story, and again after the content is discussed in class. This short time spent manifesting their prior knowledge will leave students ripe for extending their understanding of transformations in further lessons.

A Mirror to Nature Prereading Questions

1. What is a reflection?
2. What do you think a line of reflection is?
3. Do you think there can be different kinds of lines of reflection?

A Mirror to Nature Postreading Questions

1. What are some similarities between reflections that we see in everyday life and the reflections of shapes that we study in mathematics texts?
2. What is a line of reflection?
3. What are some lines of reflection you observed while reading this book or passage?

LESSON PLAN 2-2

THE UNIVERSAL BOOK OF MATHEMATICS VOCABULARY SQUARE

This is a vocabulary-building exercise that gives multiple representations of the concept behind each vocabulary word in a form that can easily be used and reused as a flashcard.

Materials: Standard size note cards, *The Universal Book of Mathematics* by David Darling or any math dictionary

Duration: 5-10 minutes per vocabulary card

Applicable Common Core Standards

Can be used for vocabulary associated with any of the Common Core standards

Procedure: Pass out note cards so that students have one for each word to be studied. Instruct the students to fold the note card into fourths and label each section, as illustrated in the following example. On the reverse side of the note card, the student should write the vocabulary word only. This allows the note cards to be used as flashcards later. Have multiple dictionaries, such as *The Universal Book of Mathematics*, or student math books available so students can look up the definitions of words they will be studying.

Students should create vocabulary squares as they learn new words in class. Set aside 10 minutes at the end of a lesson for students to create the cards for any new vocabulary. Definitions should be taken from your lesson, the textbook, or a math dictionary. Including the formal definition is important, both for standardized test preparation and for teaching students to formulate their own examples from technical writing sources. The "My Own Words"

(*Continued*)

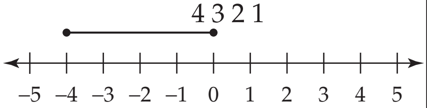
LESSON PLAN 2-2 (Continued)

section requires students to create their own version of the word's definition. This part is crucial to helping students understand what the word means in a way that makes sense to them.

When teaching the example, you may choose to use a word found in your students' notes or from their textbook, or even from standardized test practice questions. The picture/memory cue gives students either a visual clue or verbal reminder. Pictures are particularly valuable for students who are more visually oriented, but if the vocabulary word is not well suited to illustration, a verbal cue or an anecdote that was used when you taught the word would suffice.

By the end of a unit, students will have several vocabulary flashcards with many representations, giving both a wide understanding of that unit's vocabulary and a convenient means for reviewing it.

Vocabulary Square Example for "Absolute Value"

Definition The value of a number without regard to its sign; the distance from zero measured along the real number line	**My Own Words** How far a number is away from zero Always a positive number
Example Which of the following values of x make the equation $\lvert -x + 10 \rvert = 8$ true? **a.** 18 **b.** −2 **c.** 2 **d.** −18 **Answer: a**	**Picture or Memory Cue** $\lvert -4 - 4 \rvert$ Place Away from Zero 

LESSON PLAN 2-3

THE RED BLAZER GIRLS MULTICOLUMN JOURNAL

The purpose of this triple-entry journal activity is to provide a means for organizing important quotes from a reading, their mathematical significance, and their relevance to the story. It may be adapted to fit any book or topic.

Materials: *The Red Blazer Girls: The Ring of Rocamadour* by Michael Beil, triple-entry journal template

Duration: 1-2 weeks depending on reading level and class time given

Applicable Common Core Standards:
Grade 8: Geometry
High School:
Algebra: Reasoning with Equations and Inequalities
Trigonometric Functions
Modeling

Procedure: Assign blocks of chapters from the *The Red Blazer Girls: The Ring of Rocamadour* for students to read and collect clues. You may have students read in or outside of class. Clues should include mathematical concepts that will help them solve the mystery of the ring. (The bolded information in the "Clues" column is included to help the students comprehend key material. This feature of the triple-entry journal gives teachers the opportunity to develop their own hints or suggestions to guide student inquiry.)

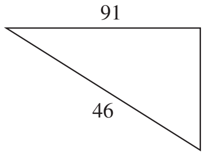
Set goals throughout reading, having students piece together the mathematics collected in the "Clues" column and demonstrate their skills in the "Solve It" and "Explain" columns. (See example below.) The "Solve It" column gives students an allotted location to substitute variables, combine pieces of information, and solve mathematical problems posed throughout the reading. In addition, the "Explain It" column requires students to make meaning of the mathematics they are doing by narrating their procedures and describing how they decided which procedures were appropriate to use. It is important to mention that students do not have to read the entire novel. Equally productive goals can be achieved by reading excerpts.

By the end of the activity, students will have demonstrated their understanding of a wide variety of mathematical concepts. You may supplement or expand on concepts discussed in the book with short lessons and explanations intended to bolster students' reading comprehension.

Page	Clues	Solve It	Explain
60	(i) + (ii) = (iii) (iv) − (v) = (vi)		
86	**(i)** = x **(ii)** = $3y$ **(iii)** = $612 \div d$; where d = distance (rounded to the nearest whole foot)	(i) + (ii) = (iii) $x + 3y = 612 \div d$	I substituted the value of each variable into the equation.

(*Continued*)

LESSON PLAN 2-3 (Continued)

Page	Clues	Solve It	Explain
132	$A = 91$ $B = 46$ **Use the clues collected in Chapters 1 to 19 to complete the first equation.**	 $A^2 + B^2 = C^2$ $(91)^2 + (46)^2 = C^2$ $8{,}281 + 2{,}116 = C^2$ $10{,}397 \div C^2$ $\mathbf{C} = 101.97$ $\boldsymbol{d} = 102$ $x + 3y = 612 \div 102$ $x + 3y = 6$	The values that Sophie measured were for each leg of the right triangle. I substituted the values in for A and B in the Pythagorean theorem. Then I added the values, which left me with $B^2 = 10{,}397$ Then I took the square root of both sides on the calculator, which gave me 101.97. The clue says to round to the nearest whole foot, which is 102. The only like terms to combine are $612 \div 102$. I got $d = 102$ from the work done in the previous entry.

Modifications for Middle School: The heavy use of systems of equations in this text lends it to high school use; however, the middle grades can capitalize on the Pythagorean Theorem and linear equations in the text; simply omit the steps involved in solving the system. The story provides commentary and solutions, so students with less math background will not suffer from lack of comprehension.

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