CHAPTER THREE

STRESS AND DEFORMATION ANALYSIS

THE BIG PICTURE — Stress and Deformation Analysis

Discussion Map

- As a designer, you are responsible for ensuring the safety of the components and systems you design.
- You must apply your prior knowledge of the principles of strength of materials.

Discover

How could consumer products and machines fail?

Describe some product failures you have seen.

This chapter presents a brief review of the fundamentals of stress analysis. It will help you design products that do not fail, and it will prepare you for other topics later in this book.

A designer is responsible for ensuring the safety of the components and systems that he or she designs. Many factors affect safety, but one of the most critical aspects of design safety is that the level of stress to which a machine component is subjected must be safe under reasonably foreseeable conditions. This principle implies, of course, that nothing actually breaks. Safety may also be compromised if components are permitted to deflect excessively, even though nothing breaks.

You have already studied the principles of strength of materials to learn the fundamentals of stress analysis. Thus, at this point, you should be competent to analyze load-carrying members for stress and deflection due to direct tensile and compressive loads, direct shear, torsional shear, and bending.

Think, now, about consumer products and machines with which you are familiar, and try to explain how they *could fail*. Of course, we do not expect them to fail, because most such products are well designed. But some do fail. Can you recall any? How did they fail? What were the operating conditions when they failed? What was the material of the components that failed? Can you visualize and describe the kinds of loads that were placed on the components that failed? Were they subjected to bending, tension, compression, shear, or torsion? Could there have been more than one type of stress acting at the same time? Are there evidences of accidental overloads? Should such loads have been anticipated by the designer? Could the failure be due to the manufacture of the product rather than its design?

Talk about product and machine failures with your associates and your instructor. Consider parts of your car, home appliances, lawn maintenance equipment, or equipment where you have worked. If possible, bring failed components to the meetings with your associates, and discuss the components and their failure.

Most of this book emphasizes developing special methods to analyze and design machine elements. These methods are all based on the fundamentals of stress analysis, and it is assumed that you have completed a course in strength of materials. This chapter presents a brief review of the fundamentals. (See References 2–4.)

YOU ARE THE DESIGNER

You are the designer of a utility crane that might be used in an automotive repair facility, in a manufacturing plant, or on a mobile unit such as a truck bed. Its function is to raise heavy loads.

A schematic layout of one possible configuration of the crane is shown in Figure 3–1. It is comprised of four primary load-carrying members, labeled 1, 2, 3, and 4. These members are connected to each other with pin-type joints at A, B, C, D, E, and F. The load is applied to the end of the horizontal boom, member 3. Anchor points for the crane are provided at joints A and B that carry the loads from the crane to a rigid structure. Note that this is a simplified view of the crane showing only the primary structural components and the forces in the plane of the applied load. The crane would also need stabilizing members in the plane perpendicular to the drawing.

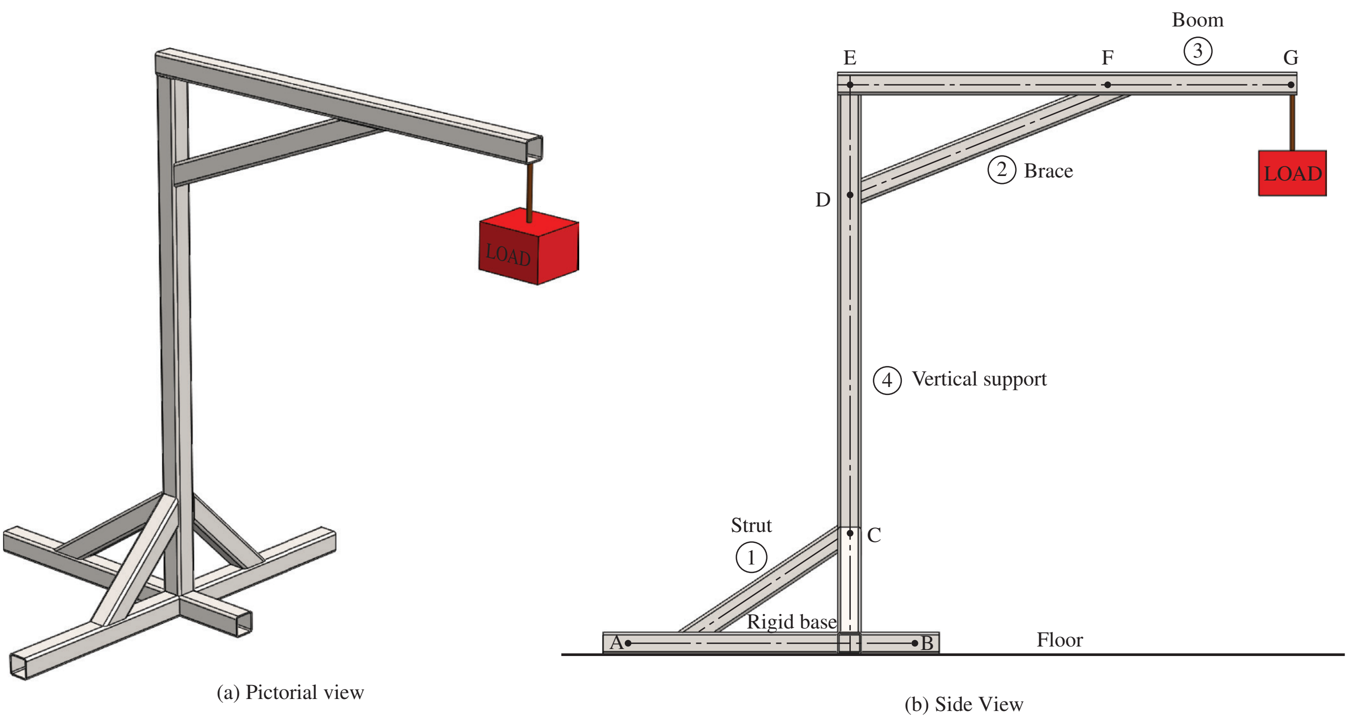


FIGURE 3–1 Schematic layout of a crane

You will need to analyze the kinds of forces that are exerted on each of the load-carrying members before you can design them. This calls for the use of the principles of statics in which you should have already gained competence. The following discussion provides a review of some of the key principles you will need in this course.

Your work as a designer proceeds as follows:

1. Analyze the forces that are exerted on each load-carrying member using the principles of statics.
2. Identify the kinds of stresses that each member is subjected to by the applied forces.
3. Propose the general shape of each load-carrying member and the material from which each is to be made.
4. Complete the stress analysis for each member to determine its final dimensions.

Let's work through steps 1 and 2 now as a review of statics. You will improve your ability to do steps 3 and 4 as you perform several practice problems in this chapter and in Chapters 4 and 5 by reviewing strength of materials and adding competencies that build on that foundation.

Force Analysis

One approach to the force analysis is outlined here.

1. Consider the entire crane structure as a free-body with the applied force acting at point G and the reactions acting at support points A and B. See Figure 3–2, which shows these forces and important dimensions of the crane structure.
2. Break the structure apart so that each member is represented as a free-body diagram, showing all forces acting at each joint. See the result in Figure 3–3.
3. Analyze the magnitudes and directions of all forces.

Comments are given here to summarize the methods used in the static analysis and to report results. You should work through the details of the analysis yourself or with colleagues to ensure that you can perform such calculations. All of the forces are directly proportional to the applied force F. We will show the results with an assumed value of $F = 10.0$ kN (approximately 2250 lb).

Step 1: The pin joints at A and B can provide support in any direction. We show the x and y components of the reactions in Figure 3–2. Then, proceed as follows:

1. Sum moments about B to find $R_{Ay} = 2.667\ F = 26.67$ kN
2. Sum forces in the vertical direction to find $R_{By} = 3.667\ F = 36.67$ kN.

At this point, we need to recognize that the strut AC is pin-connected at each end and carries loads only at its ends. Therefore, it is a *two-force member,* and the direction of the total force, R_A, acts along the member itself. Then R_{Ay} and R_{Ax} are the rectangular components of R_A as shown in the lower left of Figure 3–2. We can then say that

$$\tan(33.7°) = R_{Ay}/R_{Ax}$$

and then

$$R_{Ax} = R_{Ay}/\tan(33.7°) = 26.67\text{ kN}/\tan(33.7°) = 40.0\text{ kN}$$

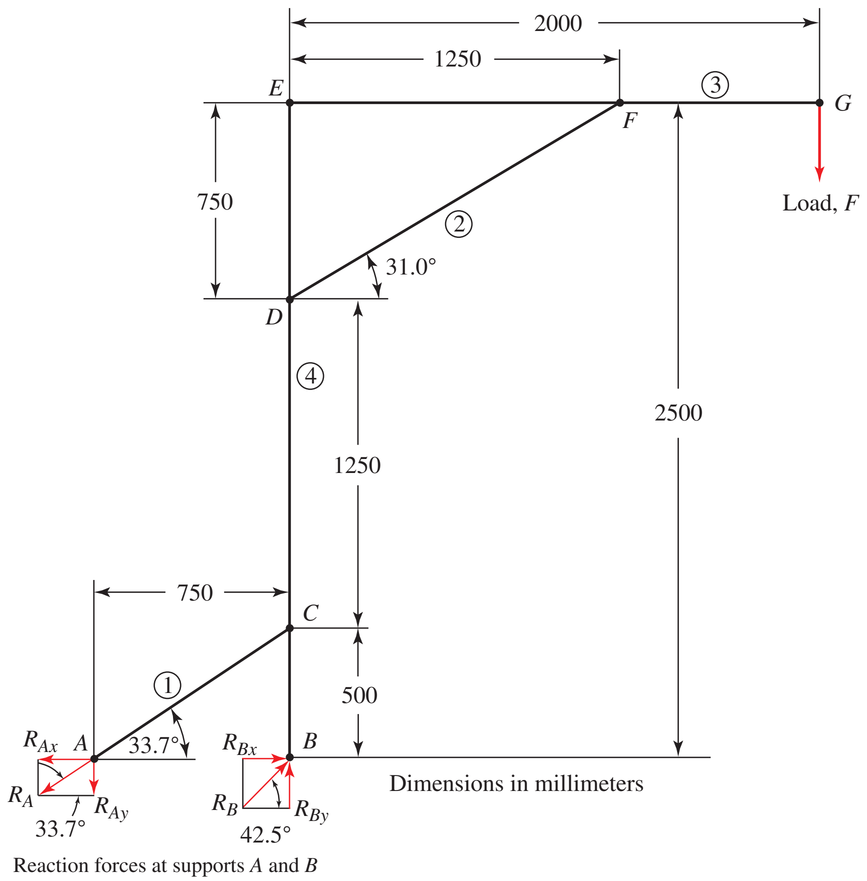


FIGURE 3–2 Free-body diagram of complete crane structure

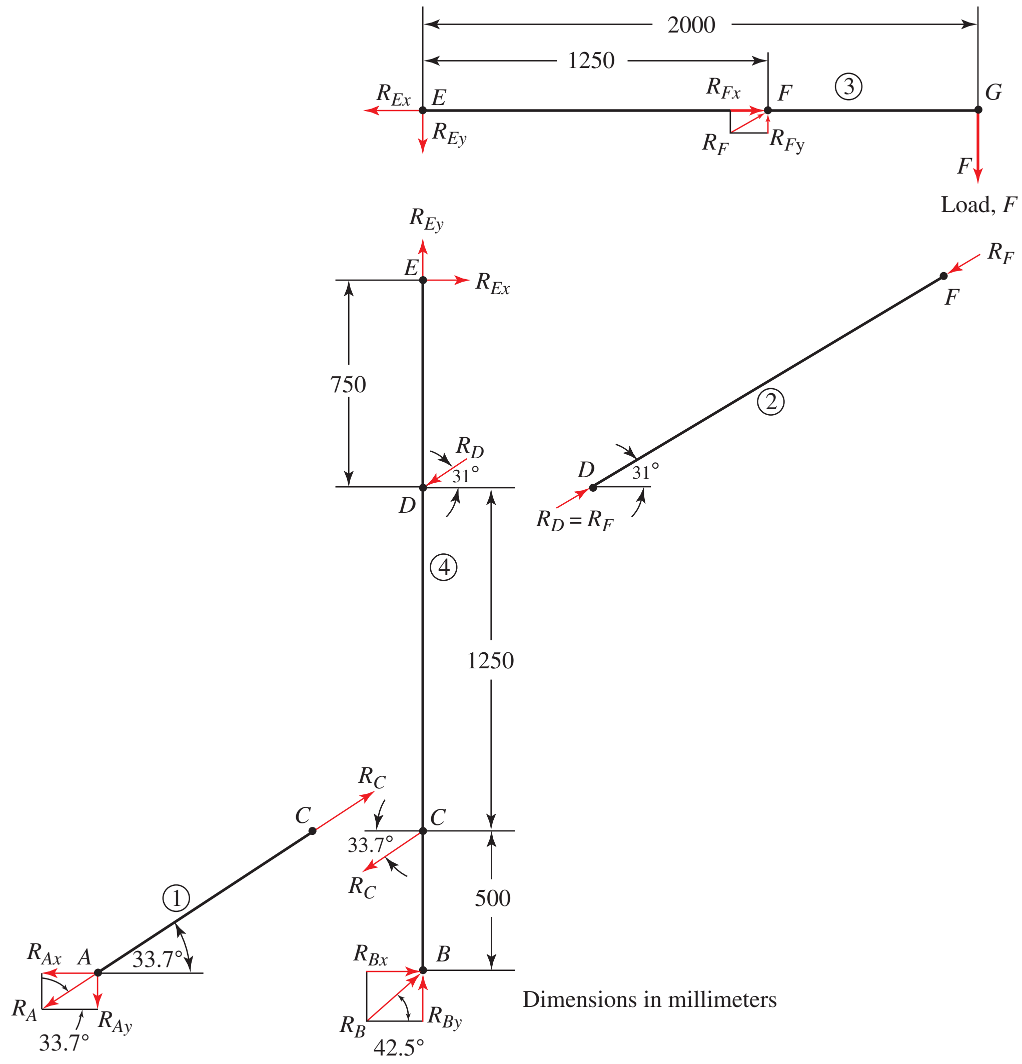


FIGURE 3–3 Free-body diagrams of each component of the crane

The total force, R_A, can be computed from the Pythagorean theorem,

$$R_A = \sqrt{R_{Ax}^2 + R_{Ay}^2} = \sqrt{(40.0)^2 + (26.67)^2} = 48.07 \text{ kN}$$

This force acts along the strut *AC*, at an angle of 33.7° above the horizontal, and it is the force that tends to shear the pin in joint *A*. The force at *C* on the strut *AC* is also 48.07 kN acting upward to the right to balance R_A on the two-force member as shown in Figure 3–3. Member *AC* is therefore in pure tension.

We can now use the sum of the forces in the horizontal direction on the entire structure to show that $R_{Ax} = R_{Bx} = 40.0$ kN.

The resultant of R_{Bx} and R_{By} is 54.3 kN acting at an angle of 42.5° above the horizontal, and it is the total shearing force on the pin in joint *B*. See the diagram in the lower right of Figure 3–2.

Step 2: The set of free–body diagrams is shown in Figure 3–3.

Step 3: Now consider the free-body diagrams of all of the members in Figure 3–3. We have already discussed member 1, recognizing it as a two–force member in tension carrying forces R_A and R_C equal to 48.07 kN. The reaction to R_C acts on the vertical member 4.

Now note that member 2 is also a two-force member, but it is in compression rather than tension. Therefore, we know that the forces on points *D* and *F* are equal and that they act in line with member 2, 31.0° with respect to the horizontal. The reactions to these forces, then, act at point *D* on the vertical support, member 4, and at point *F* on the horizontal boom, member 3. We can find the value of R_F by considering the free-body diagram of member 3. You should be able to verify the following results using the methods already demonstrated.

$$R_{Fy} = 1.600\ F = (1.600)(10.0 \text{ kN}) = 16.00 \text{ kN}$$
$$R_{Fx} = 2.667\ F = (2.667)(10.0 \text{ kN}) = 26.67 \text{ kN}$$
$$R_F = 3.110\ F = (3.110)(10.0 \text{ kN}) = 31.10 \text{ kN}$$
$$R_{Ey} = 0.600\ F = (0.600)(10.0 \text{ kN}) = 6.00 \text{ kN}$$
$$R_{Ex} = 2.667\ F = (2.667)(10.0 \text{ kN}) = 26.67 \text{ kN}$$
$$R_E = 2.733\ F = (2.733)(10.0 \text{ kN}) = 27.33 \text{ kN}$$

Now all forces on the vertical member 4 are known from earlier analyses using the principle of action-reaction at each joint.

Types of Stresses on Each Member

Consider again the free-body diagrams in Figure 3–3 to visualize the kinds of stresses that are created in each member. This will lead to the use of particular kinds of stress analysis as the design process is completed. Members 3 and 4 carry forces perpendicular to their long axes and, therefore, they act as beams in bending. Figure 3–4 shows these members with the additional shearing force and bending moment diagrams. You should have learned to prepare such

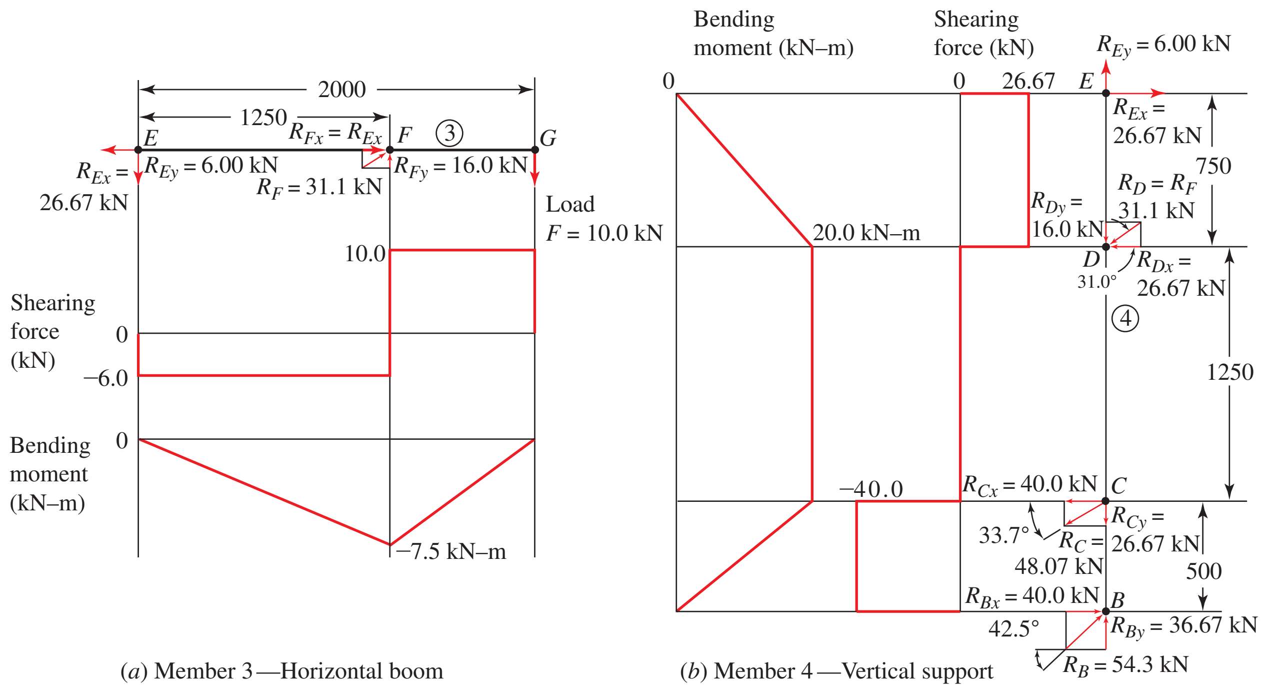


FIGURE 3–4 Shearing force and bending moment diagrams for members 3 and 4

diagrams in the prerequisite study of strength of materials. The following is a summary of the kinds of stresses in each member.

Member 1: The strut is in pure tension.

Member 2: The brace is in pure compression. Column buckling should be checked.

Member 3: The boom acts as a beam in bending. The right end between *F* and *G* is subjected to bending stress and vertical shear stress. Between *E* and *F* there is bending and shear combined with an axial tensile stress.

Member 4: The vertical support experiences a complex set of stresses depending on the segment being considered as described here.

Between *E* and *D*: Combined bending stress, vertical shear stress, and axial tension.

Between *D* and *C*: Combined bending stress and axial compression.

Between *C* and *B*: Combined bending stress, vertical shear stress, and axial compression.

Pin Joints: The connections between members at each joint must be designed to resist the total reaction force acting at each, computed in the earlier analysis. In general, each connection will likely include a cylindrical pin connecting two parts. The pin will typically be in direct shear. ■

3–1 OBJECTIVES OF THIS CHAPTER

After completing this chapter, you will:

1. Have reviewed the principles of stress and deformation analysis for several kinds of stresses, including the following:

   Normal stresses due to direct tension and compression forces

   Shear stress due to direct shear force

   Shear stress due to torsional load for both circular and non-circular sections

   Shear stress in beams due to bending

   Normal stress in beams due to bending

2. Be able to interpret the nature of the stress at a point by drawing the *stress element* at any point in a load-carrying member for a variety of types of loads.
3. Have reviewed the importance of the *flexural center* of a beam cross section with regard to the alignment of loads on beams.
4. Have reviewed beam-deflection formulas.
5. Be able to analyze beam-loading patterns that produce abrupt changes in the magnitude of the bending moment in the beam.
6. Be able to use the principle of superposition to analyze machine elements that are subjected to loading patterns that produce combined stresses.
7. Be able to properly apply stress concentration factors in stress analyses.

3–2 PHILOSOPHY OF A SAFE DESIGN

In this book, every design approach will ensure that the stress level is below yield in ductile materials, automatically ensuring that the part will not break under a static load. For brittle materials, we will ensure that the stress levels are well below the ultimate tensile strength. We will also analyze deflection where it is critical to safety or performance of a part.

Two other failure modes that apply to machine members are fatigue and wear. *Fatigue* is the response of a part subjected to repeated loads (see Chapter 5). *Wear* is discussed within the chapters devoted to the machine elements, such as gears, bearings, and chains, for which it is a major concern.

3–3 REPRESENTING STRESSES ON A STRESS ELEMENT

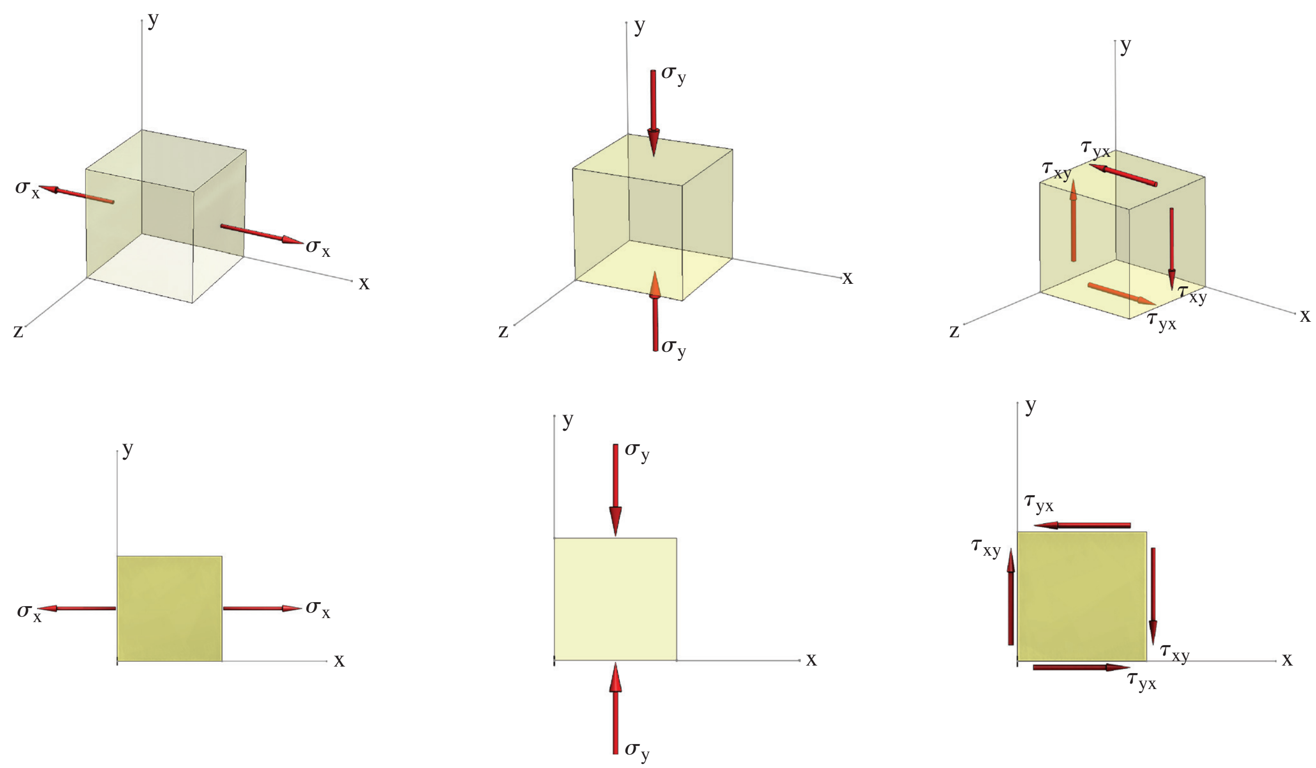
One major goal of stress analysis is to determine *the point* within a load-carrying member that is subjected to the highest stress level. You should develop the ability to visualize a *stress element*, a single, infinitesimally small cube from the member in a highly stressed area, and to show vectors that represent the kind of stresses that exist on that element. The orientation of the stress element is critical, and it must be aligned with specified axes on the member, typically called x, y, and z.

Figure 3–5 shows three examples of stress elements with two basic fundamental kinds of stress: Normal (tensile and compressive) and shear. Both the complete three-dimensional cube and the simplified, two-dimensional square forms for the stress elements are shown. The square is one face of the cube in a selected plane. The sides of the square represent the projections of the faces of the cube that are perpendicular to the selected plane. It is recommended that you visualize the cube form first and then represent a square stress element showing stresses on a particular plane of interest in a given problem. In some problems with more general states of stress, two or three square stress elements may be required to depict the complete stress condition.

Tensile and compressive stresses, called *normal stresses*, are shown acting perpendicular to opposite faces of the stress element. Tensile stresses tend to pull on the element, whereas compressive stresses tend to crush it.

Shear stresses are created by direct shear, vertical shear in beams, or torsion. In each case, the action on an element subjected to shear is a tendency to *cut* the element by exerting a stress downward on one face while simultaneously exerting a stress upward on the opposite, parallel face. This action is that of a simple pair of shears or scissors. But note that if only one pair of shear stresses acts on a stress element, it will not be in equilibrium. Rather, it will tend to spin because the pair of shear stresses forms a couple. To produce equilibrium, a second pair of shear stresses on the other two faces of the element must exist, acting in a direction that opposes the first pair.

In summary, shear stresses on an element will always be shown as two pairs of equal stresses acting on (parallel to) the four sides of the element. Figure 3–5(c) shows an example.



(*a*) Direct Tension (*b*) Direct Compression (*c*) Pure Shear

FIGURE 3–5 Stress elements for normal and shear stresses

Sign Convention for Shear Stresses

This book adopts the following convention:

> ***Positive shear stresses tend to rotate the element in a clockwise direction.***
>
> ***Negative shear stresses tend to rotate the element in a counterclockwise direction.***

A double subscript notation is used to denote shear stresses in a plane. For example, in Figure 3–5(c), drawn for the x–y plane, the pair of shear stresses, τ_{xy}, indicates a shear stress acting on the element face that is perpendicular to the x-axis and parallel to the y-axis. Then τ_{yx} acts on the face that is perpendicular to the y-axis and parallel to the x-axis. In this example, τ_{xy} is positive and τ_{yx} is negative.

3–4 NORMAL STRESSES DUE TO DIRECT AXIAL LOAD

Stress can be defined as the internal resistance offered by a unit area of a material to an externally applied load. *Normal stresses* (σ) are either *tensile* (positive) or *compressive* (negative).

For a load-carrying member in which the external load is uniformly distributed across the cross-sectional area of the member, the magnitude of the stress can be calculated from the direct stress formula:

➩ Direct Tensile or Compressive Stress

$$\sigma = \text{force/area} = F/A \qquad (3\text{–}1)$$

The units for stress are always *force per unit area*, as is evident from Equation (3–1). Common units in the U.S. Customary system and the SI metric system follow.

U.S. Customary Units	*SI Metric Units*
$\text{lb/in}^2 = \text{psi}$	$\text{N/m}^2 = \text{pascal} = \text{Pa}$
$\text{kips/in}^2 = \text{ksi}$	$\text{N/mm}^2 = \text{megapascal}$
Note: 1.0 kip = 1000 lb	$= 10^6\ \text{Pa} = \text{MPa}$
1.0 ksi = 1000 psi	

The conditions on the use of Equation (3–1) are as follows:

1. The load-carrying member must be straight.
2. The line of action of the load must pass through the centroid of the cross section of the member.
3. The member must be of uniform cross section near where the stress is being computed.
4. The material must be homogeneous and isotropic.
5. In the case of compression members, the member must be short to prevent buckling. The conditions under which buckling is expected are discussed in Chapter 6.

Example Problem 3–1 A tensile force of 9500 N is applied to a 12-mm-diameter round bar, as shown in Figure 3–6. Compute the direct tensile stress in the bar.

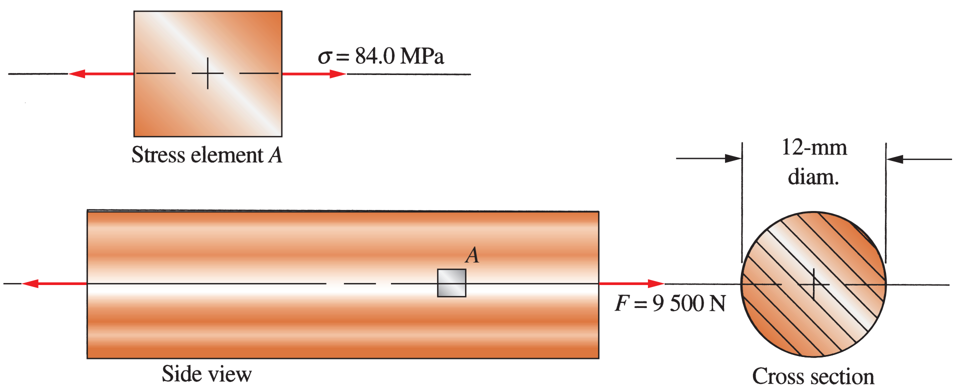


FIGURE 3–6 Tensile stress in a round bar

Solution

Objective: Compute the tensile stress in the round bar.

Given: Force = F = 9500 N; diameter = D = 12 mm.

Analysis: Use the direct tensile stress formula, Equation (3–1): $\sigma = F/A$. Compute the cross-sectional area from $A = \pi D^2/4$.

Results:

$$A = \pi D^2/4 = \pi(12\ \text{mm})^2/4 = 113\ \text{mm}^2$$

$$\sigma = F/A = (9500\ \text{N})/(113\ \text{mm}^2) = 84.0\ \text{N/mm}^2 = 84.0\ \text{MPa}$$

Comment: The results are shown on stress element A in Figure 3–6, which can be taken to be anywhere within the bar because, ideally, the stress is uniform on any cross section. The cube form of the element is as shown in Figure 3–5 (a).

Example Problem 3–2 For the round bar subjected to the tensile load shown in Figure 3–6, compute the total deformation if the original length of the bar is 3600 mm. The bar is made from a steel having a modulus of elasticity of 207 GPa.

Solution

Objective Compute the deformation of the bar.

Given Force = F = 9500 N; diameter = D = 12 mm.
Length = L = 3600 mm; E = 207 GPa

Analysis From Example Problem 3–1, we found that σ = 84.0 MPa. Use Equation (3–3).

Results

$$\delta = \frac{\sigma L}{E} = \frac{(84.0 \times 10^6\ \text{N/m}^2)(3600\ \text{mm})}{(207 \times 10^9\ \text{N/m}^2)} = 1.46\ \text{mm}$$

3–5 DEFORMATION UNDER DIRECT AXIAL LOAD

The following formula computes the stretch due to a direct axial tensile load or the shortening due to a direct axial compressive load:

$$\delta = FL/EA \quad (3\text{–}2)$$

➩ Deformation Due to Direct Axial Load

where δ = total deformation of the member carrying the axial load
F = direct axial load
L = original total length of the member
E = modulus of elasticity of the material
A = cross-sectional area of the member

Noting that $\sigma = F/A$, we can also compute the deformation from

$$\delta = \sigma L/E \quad (3\text{–}3)$$

3–6 SHEAR STRESS DUE TO DIRECT SHEAR LOAD

Direct shear stress occurs when the applied force tends to cut through the member as scissors or shears do or when a punch and a die are used to punch a slug of material from a sheet. Another important example of direct shear in machine design is the tendency for a key to be sheared off at the section between the shaft and the hub of a machine element when transmitting torque. Figure 3–7 shows the action.

The method of computing direct shear stress is similar to that used for computing direct tensile stress because the applied force is assumed to be uniformly distributed across the cross section of the part that is resisting the force. But the kind of stress is *shear stress* rather than *normal stress*. The symbol used for shear stress is the Greek letter tau (τ). The formula for direct shear stress can thus be written

➩ Direct Shear Stress

$$\tau = \text{shearing force/area in shear} = F/A_s \quad (3\text{–}4)$$

This stress is more properly called the *average shearing stress*, but we will make the simplifying assumption that the stress is uniformly distributed across the shear area.

3–7 TORSIONAL LOAD—TORQUE, ROTATIONAL SPEED, AND POWER

The relationship among the power (P), the rotational speed (n), and the torque (T) in a shaft is described by the equation

➩ Power–Torque–Speed Relationship

$$T = P/n \quad (3\text{–}5)$$

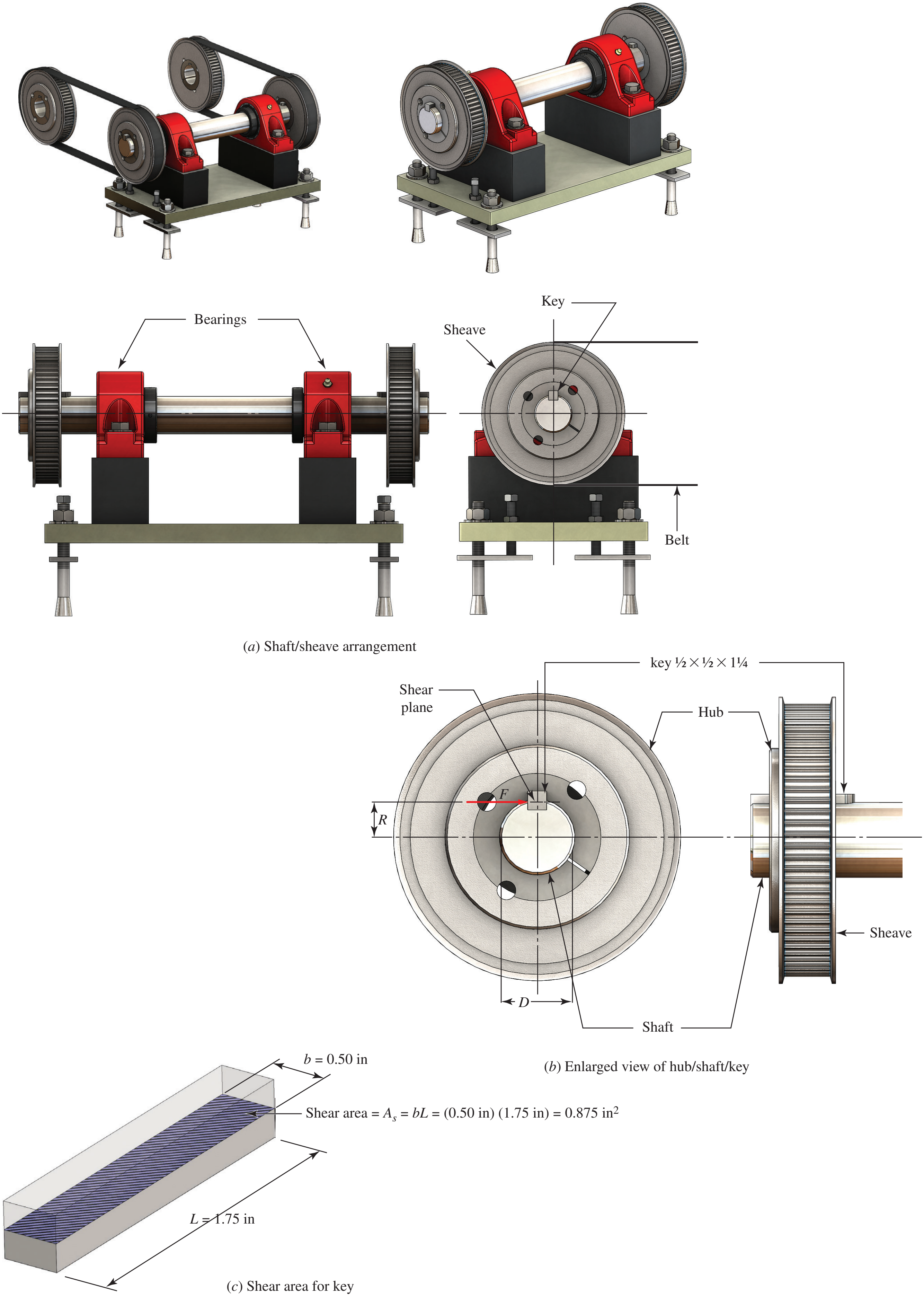
In SI units, power is expressed in the unit of *watt* (W) or its equivalent, *newton meter per second* (N · m/s), and the rotational speed is in *radians per second* (rad/s).

In the U.S. Customary Unit System, power is typically expressed as *horsepower*, equal to 550 ft · lb/s. The typical unit for rotational speed is rpm, or revolutions per minute. But the most convenient unit for torque is the pound-inch (lb · in). Considering all of these quantities and making the necessary conversions of units, we use the following formula to compute the torque (in lb · in) in a shaft carrying a certain power P (in hp) while rotating at a speed of n rpm.

➩ *P–T–n* Relationship for U.S. Customary Units

$$T = 63\,000\ P/n \quad (3\text{–}6)$$

The resulting torque will be in lb · in. You should verify the value of the constant, 63 000.



(*a*) Shaft/sheave arrangement

(*b*) Enlarged view of hub/shaft/key

(*c*) Shear area for key

FIGURE 3–7 Direct shear on a key

Example Problem 3–3

Figure 3–7 shows a shaft carrying two sprockets for synchronous belt drives that are keyed to the shaft. Figure 3–7 (b) shows that a force F is transmitted from the shaft to the hub of the sprocket through a square key. The shaft has a diameter of 2.25 in and transmits a torque of 14 063 lb·in. The key has a square cross section, 0.50 in on a side, and a length of 1.75 in. Compute the force on the key and the shear stress caused by this force.

Solution

Objective: Compute the force on the key and the shear stress.

Given: Layout of shaft, key, and hub shown in Figure 3–7.
Torque $= T = 14\,063$ lb·in; key dimensions $= 0.5$ in $\times$ 0.5 in $\times$ 1.75 in.
Shaft diameter $= D = 2.25$ in; radius $= R = D/2 = 1.125$ in.

Analysis: Torque $T =$ force $F \times$ radius R. Then $F = T/R$.
Use equation (3–4) to compute shearing stress: $\tau = F/A_s$.
Shear area is the cross section of the key at the interface between the shaft and the hub: $A_s = bL$.

Results:
$F = T/R = (14\,063 \text{ lb·in})/(1.125 \text{ in}) = 12\,500 \text{ lb}$
$A_s = bL = (0.50 \text{ in})(1.75 \text{ in}) = 0.875 \text{ in}^2$
$\tau = F/A = (12\,500 \text{ lb})/(0.875 \text{ in}^2) = 14\,300 \text{ lb/in}^2$

Comment: This level of shearing stress will be uniform on all parts of the cross section of the key.

Example Problem 3–4

Compute the torque on a shaft transmitting 750 W of power while rotating at 183 rad/s. (*Note:* This is equivalent to the output of a 1.0-hp, 4-pole electric motor, operating at its rated speed of 1750 rpm. See Chapter 21.)

Solution

Objective: Compute the torque T on the shaft.

Given: Power $= P = 750$ W $= 750$ N·m/s.
Rotational speed $= n = 183$ rad/s.

Analysis: Use Equation (3–5).

Results:
$T = P/n = (750 \text{ N·m/s})/(183 \text{ rad/s})$
$T = 4.10 \text{ N·m/rad} = 4.10 \text{ N·m}$

Comments: In such calculations, the unit of N·m/rad is dimensionally correct, and some advocate its use. Most, however, consider the radian to be dimensionless, and thus torque is expressed in N·m or other familiar units of force times distance.

Example Problem 3–5

Compute the torque on a shaft transmitting 1.0 hp while rotating at 1750 rpm. Note that these conditions are approximately the same as those for which the torque was computed in Example Problem 3–4 using SI units.

Solution

Objective: Compute the torque on the shaft.

Given: $P = 1.0$ hp; $n = 1750$ rpm.

Analysis: Use Equation (3–6).

Results: $T = 63\,000\,P/n = [63\,000(1.0)]/1750 = 36.0$ lb·in

3–8 SHEAR STRESS DUE TO TORSIONAL LOAD

When a *torque*, or twisting moment, is applied to a member, it tends to deform by twisting, causing a rotation of one part of the member relative to another. Such twisting causes a shear stress in the member. For a small element of the member, the nature of the stress is the same as that experienced under direct shear stress. However, in *torsional shear*, the distribution of stress is not uniform across the cross section.

The most frequent case of torsional shear in machine design is that of a round circular shaft transmitting power. Chapter 12 covers shaft design.

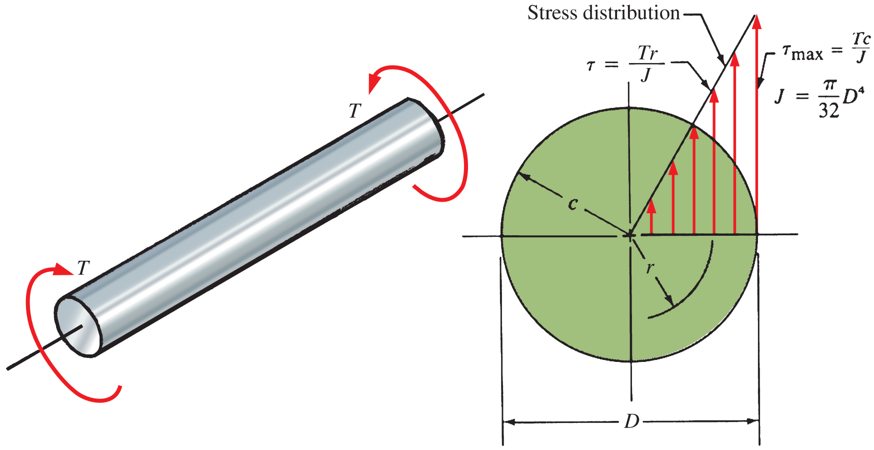


FIGURE 3–8 Stress distribution in a solid shaft

Torsional Shear Stress Formula

When subjected to a torque, the outer surface of a solid round shaft experiences the greatest shearing strain and therefore the largest torsional shear stress. See Figure 3–8. The value of the maximum torsional shear stress is found from

➩ Maximum Torsional Shear Stress in a Circular Shaft

$$\tau_{max} = Tc/J \qquad (3\text{–}7)$$

where c = radius of the shaft to its outside surface
J = polar moment of inertia

See Appendix 1 for formulas for J.

If it is desired to compute the torsional shear stress at some point inside the shaft, the more general formula is used:

➩ General Formula for Torsional Shear Stress

$$\tau = Tr/J \qquad (3\text{–}8)$$

where r = radial distance from the center of the shaft to the point of interest

Figure 3–8 shows graphically that this equation is based on the linear variation of the torsional shear stress from zero at the center of the shaft to the maximum value at the outer surface.

Equations (3–7) and (3–8) apply also to hollow shafts (Figure 3–9 shows the distribution of shear stress). Again note that the maximum shear stress occurs at the outer surface. Also note that the entire cross section carries a relatively high stress level. As a result, the hollow shaft is more efficient. Notice that the material near the center of the solid shaft is not highly stressed.

For design, it is convenient to define the *polar section modulus*, Z_p:

➩ Polar Section Modulus

$$Z_p = J/c \qquad (3\text{–}9)$$

Then the equation for the maximum torsional shear stress is

$$\tau_{max} = T/Z_p \qquad (3\text{–}10)$$

Formulas for the polar section modulus are also given in Appendix 1. This form of the torsional shear stress equation is useful for design problems because the polar section modulus is the only term related to the geometry of the cross section.

Example Problem 3–6 Compute the maximum torsional shear stress in a shaft having a diameter of 10 mm when it carries a torque of 4.10 N·m.

Solution

Objective: Compute the torsional shear stress in the shaft.

Given: Torque = T = 4.10 N·m; shaft diameter = D = 10 mm.
c = radius of the shaft = $D/2$ = 5.0 mm.

Analysiçs: Use Equation (3–7) to compute the torsional shear stress: $\tau_{max} = Tc/J$. J is the polar moment of inertia for the shaft: $J = \pi D^4/32$ (see Appendix 1).

Results:
$$J = \pi D^4/32 = [(\pi)(10\text{ mm})^4]/32 = 982\text{ mm}^4$$
$$\tau_{max} = \frac{(4.10\text{ N}\cdot\text{m})(5.0\text{ mm})}{982\text{ mm}^4}\frac{10^3\text{ mm}}{\text{m}} = 20.9\text{ N/mm}^2 = 20.9\text{ MPa}$$

Comment: The maximum torsional shear stress occurs at the outside surface of the shaft around its entire circumference.

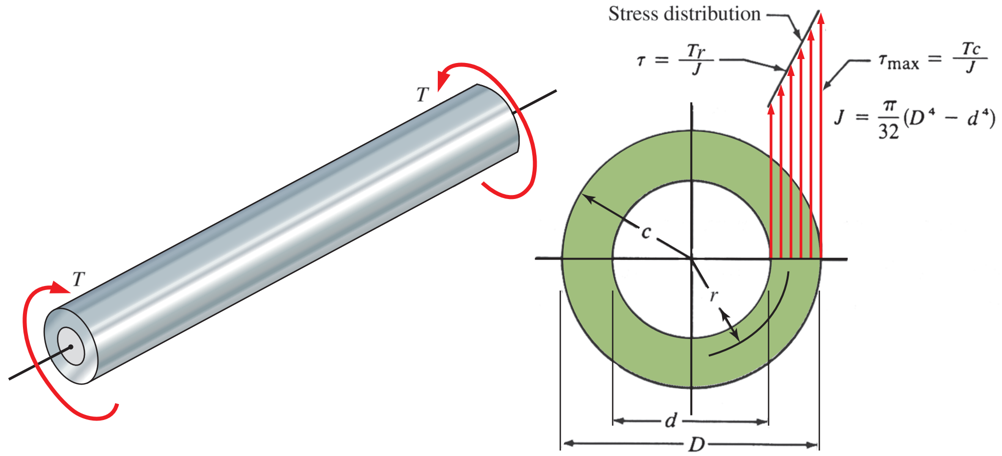


FIGURE 3–9 Stress distribution in a hollow shaft

3–9 TORSIONAL DEFORMATION

When a shaft is subjected to a torque, it undergoes a twisting in which one cross section is rotated relative to other cross sections in the shaft. The angle of twist is computed from

➩ Torsional Deformation

$$\theta = TL/GJ \tag{3–11}$$

where θ = angle of twist (radians)
L = length of the shaft over which the angle of twist is being computed
G = modulus of elasticity of the shaft material in *shear*

Example Problem 3–7 Compute the angle of twist of a 10-mm-diameter shaft carrying 4.10 N·m of torque if it is 250 mm long and made of steel with G = 80 GPa. Express the result in both radians and degrees.

Solution

Objective: Compute the angle of twist in the shaft.

Given: Torque = T = 4.10 N·m; length = L = 250 mm.
Shaft diameter = D = 10 mm; G = 80 GPa.

Analysis: Use Equation (3–11). For consistency, let $T = 4.10 \times 10^3$ N·mm and $G = 80 \times 10^3$ N/mm^2. From Example Problem 3–6, $J = 982$ mm^4.

Results:

$$\theta = \frac{TL}{GJ} = \frac{(4.10 \times 10^3 \text{ N·mm})(250 \text{ mm})}{(80 \times 10^3 \text{ N/mm}^2)(982 \text{ mm}^4)} = 0.013 \text{ rad}$$

Using π rad = 180°,

$$\theta = (0.013 \text{ rad})(180°/\pi \text{ rad}) = 0.75°$$

Comment: Over the length of 250 mm, the shaft twists 0.75°.

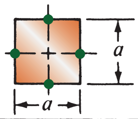
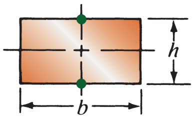
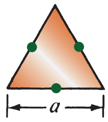
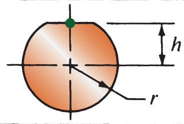
3–10 TORSION IN MEMBERS HAVING NON-CIRCULAR CROSS SECTIONS

The behavior of members having noncircular cross sections when subjected to torsion is radically different from that for members having circular cross sections. However, the factors of most use in machine design are the maximum stress and the total angle of twist for such members. The formulas for these factors can be expressed in similar forms to the formulas used for members of circular cross section (solid and hollow round shafts).

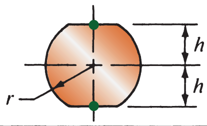
The following two formulas can be used:

➩ Torsional Shear Stress

$$\tau_{max} = T/Q \tag{3–12}$$

Cross-sectional shape		K = for use in $\theta = TL/GK$ Q = for use in $\tau = T/Q$	Colored dot (●) denotes location of τ_{max}
Square		$K = 0.141a^4$ $Q = 0.208a^3$	τ_{max} at midpoint of each side
Rectangle		$K = bh^3\left[\frac{1}{3} - 0.21\frac{h}{b}\left(1 - \frac{(h/b)^4}{12}\right)\right]$ $Q = \frac{bh^2}{[3 + 1.8(h/b)]}$	(Approximate; within ≈ 5%) τ_{max} at midpoint of long sides
Triangle (equilateral)		$K = 0.0217a^4$ $Q = 0.050a^3$	
Shaft with One Flat		$K = C_1r^4$ $Q = C_2r^3$	see table below

h/r	0	0.2	0.4	0.6	0.8	1.0
C_1	0.30	0.51	0.78	1.06	1.37	1.57
C_2	0.35	0.51	0.70	0.92	1.18	1.57

Shaft with Two Flats		$K = C_3r^4$ $Q = C_4r^3$	see table below

h/r	0.5	0.6	0.7	0.8	0.9	1.0
C_3	0.44	0.67	0.93	1.19	1.39	1.57
C_4	0.47	0.60	0.81	1.02	1.25	1.57

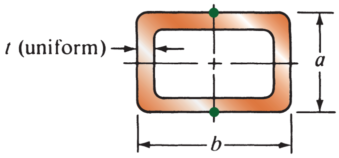
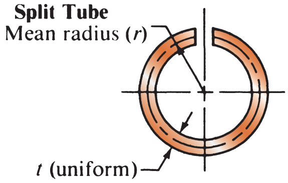
Hollow Rectangle		$K = \frac{2t(a-t)^2(b-t)^2}{(a+b-2t)}$ $Q = 2t(a-t)(b-t)$	Gives average stress; good approximation of maximum stress if t is small—thin-walled tube Inner corners should have generous fillets
Split Tube		$K = 2\pi rt^3/3$ $Q = \frac{4\pi^2r^2t^2}{(6\pi r + 1.8t)}$	t must be small—thin-walled tube

FIGURE 3–10 Methods for determining values for K and Q for several types of cross sections

➩ Deflection for Noncircular Sections

$$\theta = TL/GK \tag{3–13}$$

Note that Equations (3–12) and (3–13) are similar to Equations (3–10) and (3–11), with the substitution of Q for Z_p and K for J. Refer Figure 3–10 for the methods of determining the values for K and Q for several types of cross sections useful in machine design. These values are appropriate only if the ends of the member are free to deform. If either end is fixed, as by welding to a solid structure, the resulting stress and angular twist are quite different. (See References 1–3, 6, and 7.)

Example Problem 3–8 A 2.50-in-diameter shaft carrying a chain sprocket has one end milled in the form of a square to permit the use of a hand crank. The square is 1.75 in on a side. Compute the maximum shear stress on the square part of the shaft when a torque of 15 000 lb·in is applied.

Also, if the length of the square part is 8.00 in, compute the angle of twist over this part. The shaft material is steel with $G = 11.5 \times 10^6$ psi.

Solution

Objective: Compute the maximum shear stress and the angle of twist in the shaft.

Given: Torque = T = 15 000 lb·in; length = L = 8.00 in.
The shaft is square; thus, a = 1.75 in.
$G = 11.5 \times 10^6$ psi.

Analysis Figure 3–10 shows the methods for calculating the values for Q and K for use in Equations (3–12) and (3–13).

Results $Q = 0.208a^3 = (0.208)(1.75\ \text{in})^3 = 1.115\ \text{in}^3$
$K = 0.141a^4 = (0.141)(1.75\ \text{in})^4 = 1.322\ \text{in}^4$
Now the stress and the deflection can be computed.

$$\tau_{max} = \frac{T}{Q} = \frac{15\,000\ \text{lb}\cdot\text{in}}{(1.115\ \text{in}^3)} = 13\,460\ \text{psi}$$

$$\theta = \frac{TL}{GK} = \frac{(15\,000\ \text{lb}\cdot\text{in})(8.00\ \text{in})}{(11.5 \times 10^6\ \text{lb/in}^2)(1.322\ \text{in}^4)} = 0.0079\ \text{rad}$$

Convert the angle of twist to degrees:

$$\theta = (0.0079\ \text{rad})(180°/\pi\ \text{rad}) = 0.452°$$

Comments Over the length of 8.00 in, the square part of the shaft twists 0.452°. The maximum shear stress is 13 460 psi, and it occurs at the midpoint of each side as shown in Figure 3–10.

3–11 TORSION IN CLOSED, THIN-WALLED TUBES

A general approach for closed, thin-walled tubes of virtually any shape uses Equations (3–12) and (3–13) with special methods of evaluating K and Q. Figure 3–11 shows such a tube having a constant wall thickness. The values of K and Q are

$$K = 4A^2t/U \tag{3–14}$$

$$Q = 2tA \tag{3–15}$$

where A = area enclosed by the median boundary (indicated by the dashed line in Figure 3–11)
t = wall thickness (which must be uniform and thin)
U = length of the median boundary

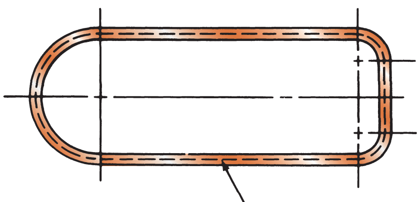


FIGURE 3–11 Closed, thin-walled tube with a constant wall thickness

The shear stress computed by this approach is the *average stress* in the tube wall. However, if the wall thickness t is small (a thin wall), the stress is nearly uniform throughout the wall, and this approach will yield a close approximation of the maximum stress. For the analysis of tubular sections having nonuniform wall thickness, see References 1–3, 6, and 7.

To design a member to resist torsion only, or torsion and bending combined, it is advisable to select hollow tubes, either round or rectangular, or some other closed shape. They possess good efficiency both in bending and in torsion.

3–12 TORSION IN OPEN, THIN-WALLED TUBES

The term *open tube* refers to a shape that appears to be tubular but is not completely closed. For example, some tubing is manufactured by starting with a thin, flat strip of steel that is roll-formed into the desired shape (circular, rectangular, square, and so on). Then the seam is welded along the entire length of the tube. It is interesting to compare the properties of the cross section of such a tube before and after it is welded. The following example problem illustrates the comparison for a particular size of circular tubing.

Example Problem 3–9 Figure 3–12 shows a tube before [Part (b)] and after [Part (a)] the seam is welded. Compare the stiffness and the strength of each shape.

Solution Objective Compare the torsional stiffness and the strength of the closed tube of Figure 3–12(a) with those of the open-seam (split) tube shown in Figure 3–12(b).

Given The tube shapes are shown in Figure 3–12. Both have the same length, diameter, and wall thickness, and both are made from the same material.

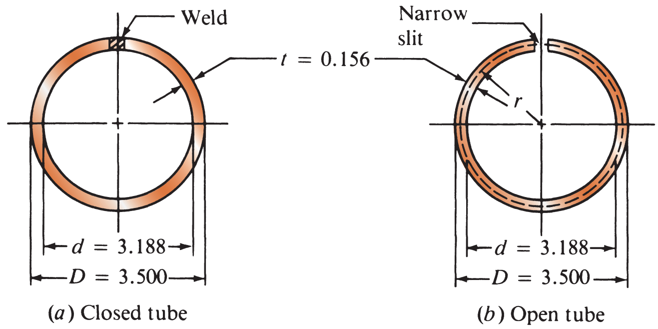


FIGURE 3–12 Comparison of closed and open tubes

Analysis Equation (3–13) gives the angle of twist for a noncircular member and shows that the angle is inversely proportional to the value of K. Similarly, Equation (3–11) shows that the angle of twist for a hollow circular tube is inversely proportional to the polar moment of inertia J. All other terms in the two equations are the same for each design. Therefore, the ratio of θ_{open} to θ_{closed} is equal to the ratio J/K. From Appendix 1, we find

$$J = \pi(D^4 - d^4)/32$$

From Figure 3–10, we find

$$K = 2\pi r t^3/3$$

Using similar logic, Equations (3–12) and (3–10) show that the maximum torsional shear stress is inversely proportional to Q and Z_p for the open and closed tubes, respectively. Then we can compare the strengths of the two forms by computing the ratio Z_p/Q. By Equation (3–9), we find that

$$Zp = J/c = J/(D/2)$$

The equation for Q for the split tube is listed in Figure 3–10.

Results We make the comparison of torsional stiffness by computing the ratio J/K. For the closed, hollow tube,

$$J = \pi(D^4 - d^4)/32$$
$$J = \pi(3.500^4 - 3.188^4)/32 = 4.592 \text{ in}^4$$

For the open tube before the slit is welded, from Figure 3–10,

$$K = 2\pi r t^3/3$$
$$K = [(2)(\pi)(1.672)(0.156)^3]/3 = 0.0133 \text{ in}^4$$
$$\text{Ratio} = J/K = 4.592/0.0133 = 345$$

Then we make the comparison of the strengths of the two forms by computing the ratio Z_p/Q.

The value of J has already been computed to be 4.592 in^4. Then

$$Z_p = J/c = J/(D/2) = (4.592 \text{ in}^4)/[(3.500 \text{ in})/2] = 2.624 \text{ in}^3$$

For the open tube,

$$Q = \frac{4\pi^2 r^2 t^2}{(6\pi r + 1.8t)} = \frac{4\pi^2(1.672 \text{ in})^2(0.156 \text{ in})^2}{[6\pi(1.672 \text{ in}) + 1.8(0.156 \text{ in})]} = 0.0845 \text{ in}^3$$

Then the strength comparison is

$$\text{Ratio} = Z_p/Q = 2.624/0.0845 = 31.1$$

Comments Thus, for a given applied torque, the slit tube would twist 345 times as much as the closed tube. The stress in the slit tube would be 31.1 times higher than in the closed tube. Also note that if the material for the tube is thin, it will likely buckle at a relatively low stress level, and the tube will collapse suddenly. This comparison shows the dramatic superiority of the closed form of a hollow section to an open form. A similar comparison could be made for shapes other than circular.

3–13 SHEAR STRESS DUE TO BENDING

A beam carrying loads transverse to its axis will experience shearing forces, denoted by V. In the analysis of beams, it is usual to compute the variation in shearing force across the entire length of the beam and to draw the *shearing force diagram*. Then the resulting vertical shearing stress can be computed from

⇨ Vertical Shearing Stress in Beams

$$\tau = VQ/It \tag{3–16}$$

where I = rectangular moment of inertia of the cross section of the beam

t = thickness of the section at the place where the shearing stress is to be computed

Q = *first moment*, with respect to the overall centroidal axis, *of the area* of that part of the cross section that lies away from the axis where the shearing stress is to be computed.

To calculate the value of Q, we define it by the following equation,

⇨ First Moment of the Area

$$Q = A_p\bar{y} \tag{3–17}$$

where A_p = that part of the area of the section above the place where the stress is to be computed

$\bar{y}$ = distance from the neutral axis of the section to the centroid of the area A_p

In some books or references, and in earlier editions of this book, Q was called the *statical moment*. Here we will use the term *first moment of the area*.

For most section shapes, the maximum vertical shearing stress occurs at the centroidal axis. Specifically, if the thickness is not less at a place away from the centroidal axis, then it is assured that the maximum vertical shearing stress occurs at the centroidal axis.

Figure 3–13 shows three examples of how Q is computed in typical beam cross sections. In each, the maximum vertical shearing stress occurs at the neutral axis.

Note that the vertical shearing stress is equal to the *horizontal shearing stress* because any element of material subjected to a shear stress on one face must have a shear stress of the same magnitude on the adjacent face

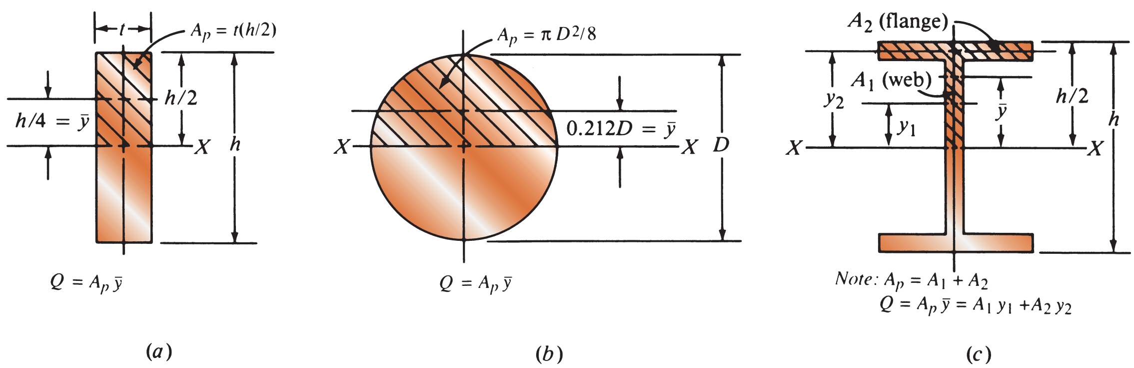


FIGURE 3–13 Illustrations of A_p and $\bar{y}$ used to compute Q for three shapes

Example Problem 3–10 Figure 3–14 shows a simply supported beam carrying two concentrated loads. The shearing force diagram is shown, along with the rectangular shape and size of the cross section of the beam. The stress distribution is parabolic, with the maximum stress occurring at the neutral axis. Use Equation (3–16) to compute the maximum shearing stress in the beam.

Solution

Objective: Compute the maximum shearing stress τ in the beam in Figure 3–14.

Given: The beam shape is rectangular: h = 8.00 in; t = 2.00 in.
Maximum shearing force = V = 1000 lb at all points between A and B.

Analysis: Use Equation (3–16) to compute τ. V and t are given. From Appendix 1,

$$I = th^3/12$$

The value of the first moment of the area Q can be computed from Equation (3–17). For the rectangular cross section shown in Figure 3–13(a), $A_p = t(h/2)$ and $\bar{y} = h/4$. Then

$$Q = A_p\bar{y} = (th/2)(h/4) = th^2/8$$

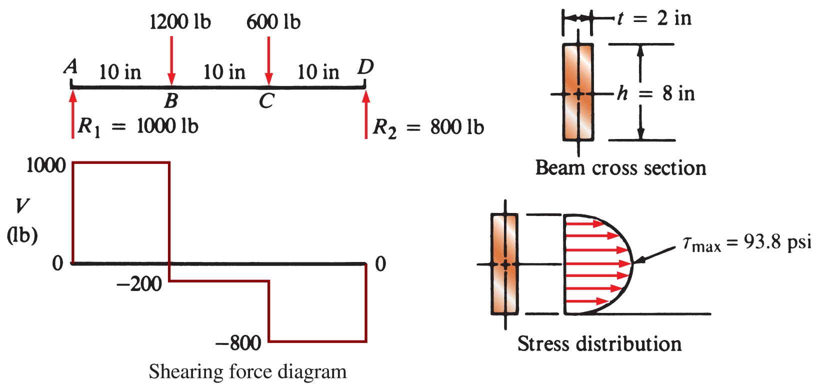


FIGURE 3–14 Shearing force diagram and vertical shearing stress for beam

Results $I = th^3/12 = (2.0\text{ in})(8.0\text{ in})^3/12 = 85.3\text{ in}^4$

$Q = A_p\bar{y} = th^2/8 = (2.0\text{ in})(8.0\text{ in})^2/8 = 16.0\text{ in}^3$

Then the maximum shearing stress is

$$\tau = \frac{VQ}{It} = \frac{(1000\text{ lb})(16.0\text{ in}^3)}{(85.3\text{ in}^4)(2.0\text{ in})} = 93.8\text{ lb/in}^2 = 93.8\text{ psi}$$

Comments The maximum shearing stress of 93.8 psi occurs at the neutral axis of the rectangular section as shown in Figure 3–14. The stress distribution within the cross section is generally parabolic, ending with zero shearing stress at the top and bottom surfaces. This is the nature of the shearing stress everywhere between the left support at A and the point of application of the 1200-lb load at B. The maximum shearing stress at any other point in the beam is proportional to the magnitude of the vertical shearing force at the point of interest.

for the element to be in equilibrium. Figure 3–15 shows this phenomenon.

In most beams, the magnitude of the vertical shearing stress is quite small compared with the bending stress (see the following section). For this reason, it is frequently not computed at all. Those cases where it is of importance include the following:

1. When the material of the beam has a relatively low shear strength (such as wood).
2. When the bending moment is zero or small (and thus the bending stress is small), for example, at the ends of simply supported beams and for short beams.
3. When the thickness of the section carrying the shearing force is small, as in sections made from rolled sheet, some extruded shapes, and the web of rolled structural shapes such as wide-flange beams.

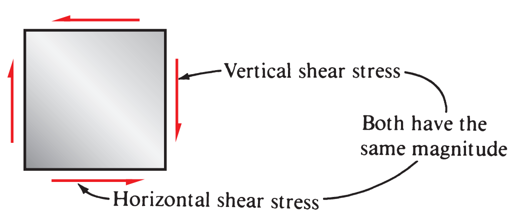


FIGURE 3–15 Shear stresses on an element

3–14 SHEAR STRESS DUE TO BENDING – SPECIAL SHEAR STRESS FORMULAS

Equation (3–16) can be cumbersome because of the need to evaluate the first moment of the area Q. Several commonly used cross sections have special, easy-to-use formulas for the maximum vertical shearing stress:

➩ τ_{max} for Rectangle

$$\tau_{max} = 3V/2A \text{ (exact)} \qquad (3\text{–}18)$$

where A = total cross-sectional area of the beam

➩ τ_{max} for Circle

$$\tau_{max} = 4V/3A \text{ (exact)} \qquad (3\text{–}19)$$

➩ τ_{max} for I-Shape

$$\tau_{max} \simeq V/th \text{ (approximate: about 15\% low)} \qquad (3\text{–}20)$$

where t = web thickness

h = height of the web (e.g., a wide-flange beam)

➩ τ_{max} for Thin-walled Tube

$$\tau_{max} \simeq 2V/A \text{ (approximate: a little high)} \qquad (3\text{–}21)$$

In all of these cases, the maximum shearing stress occurs at the neutral axis.

Example Problem 3–11 Compute the maximum shearing stress in the beam described in Example Problem 3–10 using the special shearing stress formula for a rectangular section.

Solution

Objective: Compute the maximum shearing stress τ in the beam in Figure 3–14.

Given: The data are the same as stated in Example Problem 3–10 and as shown in Figure 3–14.

Analysis: Use Equation (3–18) to compute $\tau = 3V/2A$. For the rectangle, $A = th$.

Results:

$$\tau_{max} = \frac{3V}{2A} = \frac{3(1000 \text{ lb})}{2[(2.0 \text{ in})(8.0 \text{ in})]} = 93.8 \text{ psi}$$

Comment: This result is the same as that obtained for Example Problem 3–10, as expected.

3–15 NORMAL STRESS DUE TO BENDING

A *beam* is a member that carries loads transverse to its axis. Such loads produce bending moments in the beam, which result in the development of bending stresses. Bending stresses are *normal stresses*, that is, either tensile or compressive. The maximum bending stress in a beam cross section will occur in the part farthest from the neutral axis of the section. At that point, the *flexure formula* gives the stress:

➪ Flexure Formula for Maximum Bending Stress

$$\sigma = Mc/I \tag{3–22}$$

where M = magnitude of the bending moment at the section

I = moment of inertia of the cross section with respect to its neutral axis

c = distance from the neutral axis to the outermost fiber of the beam cross section

The magnitude of the bending stress varies linearly within the cross section from a value of zero at the neutral axis, to the maximum tensile stress on one side of the neutral axis, and to the maximum compressive stress on the other side. Figure 3–16 shows a typical stress distribution in a beam cross section. Note that the stress distribution is independent of the shape of the cross section.

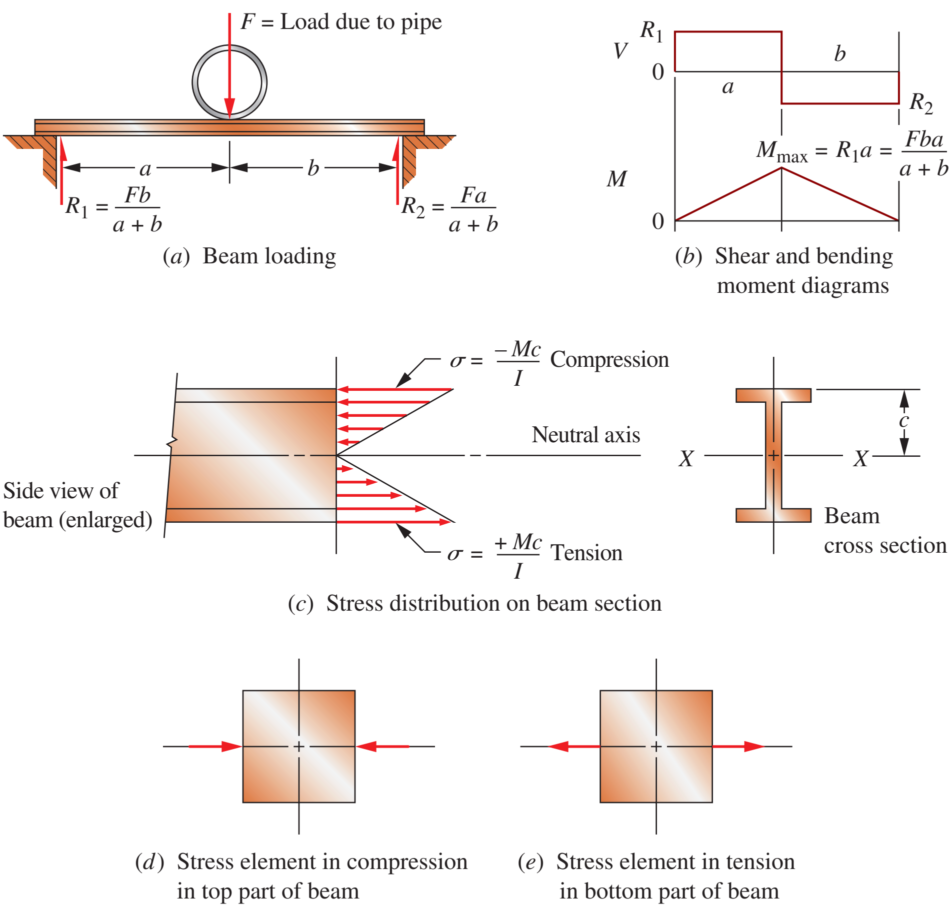


FIGURE 3–16 Typical bending stress distribution in a beam cross section

Note that *positive bending* occurs when the deflected shape of the beam is concave upward, resulting in compression on the upper part of the cross section and tension on the lower part. Conversely, *negative bending* causes the beam to be concave downward.

The flexure formula was developed subject to the following conditions:

1. The beam must be in pure bending. Shearing stresses must be zero or negligible. No axial loads are present.
2. The beam must not twist or be subjected to a torsional load.
3. The material of the beam must obey Hooke's law.
4. The modulus of elasticity of the material must be the same in both tension and compression.
5. The beam is initially straight and has a constant cross section.
6. Any plane cross section of the beam remains plane during bending.
7. No part of the beam shape fails because of local buckling or wrinkling.

If condition 1 is not strictly met, you can continue the analysis by using the method of combined stresses presented in Chapter 4. In most practical beams, which are long relative to their height, shear stresses are sufficiently small as to be negligible. Furthermore, the maximum bending stress occurs at the outermost fibers of the beam section, where the shear stress is in fact zero. A beam with varying cross section, which would violate condition 5, can be analyzed by the use of stress concentration factors discussed later in this chapter.

For design, it is convenient to define the term *section modulus*, S, as

$$S = I/c \tag{3–23}$$

The flexure formula then becomes

➪ Flexure Formula

$$\sigma = M/S \tag{3–24}$$

Since I and c are geometrical properties of the cross section of the beam, S is also. Then, in design, it is usual to define a design stress, σ_d, and, with the bending moment known, solve for S:

➪ Required Section Modulus

$$S = M/\sigma_d \tag{3–25}$$

This results in the required value of the section modulus. From this, the required dimensions of the beam cross section can be determined.

Example Problem 3–12

For the beam shown in Figure 3–16, the load F due to the pipe is 12 000 lb. The distances are $a = 4$ ft and $b = 6$ ft. Determine the required section modulus for the beam to limit the stress due to bending to 30 000 psi, the recommended design stress for a typical structural steel in static bending. Then specify the lightest suitable steel beam.

Solution

Objective: Compute the required section modulus S for the beam in Figure 3–16.

Given: The layout and the loading pattern are shown in Figure 3–16.

Lengths: Overall length $= L = 10$ ft; $a = 4$ ft; $b = 6$ ft.
Load $= F = 12\,000$ lb.
Design stress $= \sigma_d = 30\,000$ psi.

Analysis: Use Equation (3–25) to compute the required section modulus S. Compute the maximum bending moment that occurs at the point of application of the load using the formula shown in Part (b) of Figure 3–16.

Results:

$$M_{max} = R_1\,a = \frac{Fba}{a+b} = \frac{(12\,000\text{ lb})(6\text{ ft})(4\text{ ft})}{(6\text{ ft} + 4\text{ ft})} = 28\,800\text{ lb}\cdot\text{ft}$$

$$S = \frac{M}{\sigma_d} = \frac{28\,800\text{ lb}\cdot\text{ft}}{30\,000\text{ lb/in}^2}\,\frac{12\text{ in}}{\text{ft}} = 11.5\text{ in}^3$$

Comments: A steel beam section can now be selected from Tables A15–9 and A15–10 that has at least this value for the section modulus. The lightest section, typically preferred, is the W8×15 wide-flange shape with $S = 11.8\text{ in}^3$.

3–16 BEAMS WITH CONCENTRATED BENDING MOMENTS

Figures 3–16 and 3–17 show beams loaded only with concentrated forces or distributed loads. For such loading in any combination, the moment diagram is continuous. That is, there are no points of abrupt change in the value of the bending moment. Many machine elements such as cranks, levers, helical gears, and brackets carry loads whose line of action is offset from the centroidal axis of the beam in such a way that a concentrated moment is exerted on the beam.

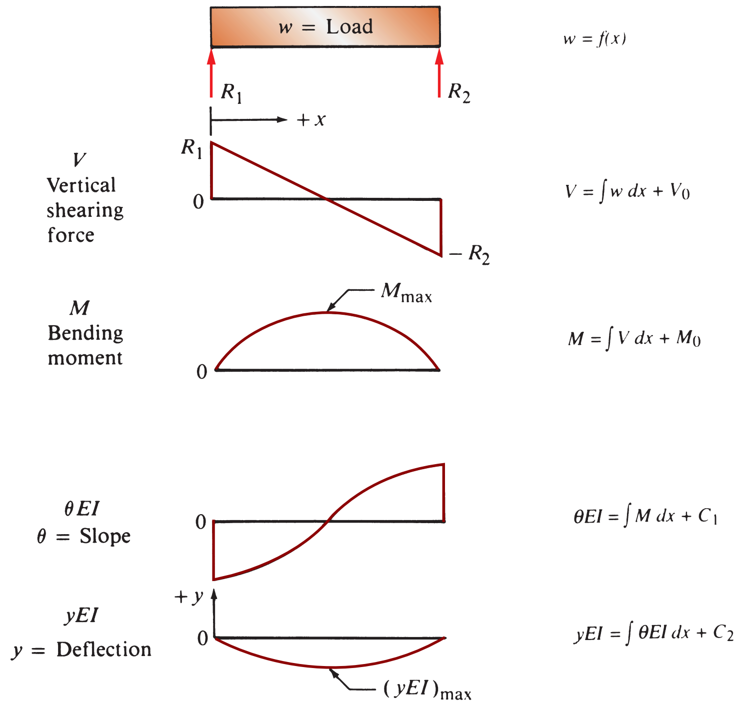


FIGURE 3–17 Relationships of load, vertical shearing force, bending moment, slope of deflected beam shape, and actual deflection curve of a beam

Figures 3–18, 3–19, and 3–20 show three different examples where concentrated moments are created on machine elements. The bell crank in Figure 3–18 pivots around point O and is used to transfer an applied force to a different line of action. Each arm behaves similar to a cantilever beam, bending with respect to an axis through the pivot. For analysis, we can isolate an arm by making an imaginary cut through the pivot and showing the reaction force at the pivot pin and the internal moment in the arm. The shearing force and bending moment diagrams included in Figure 3–18 show the results, and Example Problem 3–13 gives the details of the analysis. Note the similarity to a cantilever beam with the internal concentrated moment at the pivot reacting to the force, F_2, acting at the end of the arm.

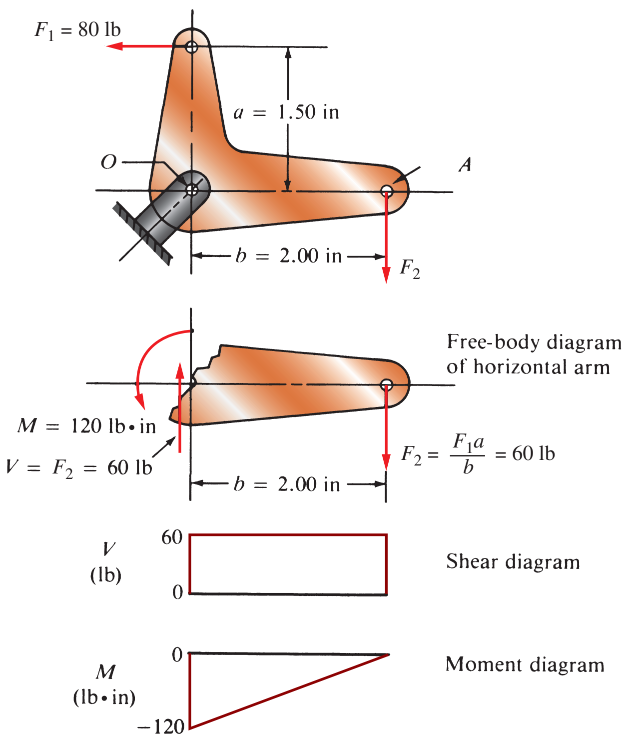


FIGURE 3–18 Bending moment in a bell crank

Figure 3–19 shows a print head for an impact-type printer in which the applied force, F, is offset from the neutral axis of the print head itself. Thus the force creates a concentrated bending moment at the right end where the vertical lever arm attaches to the horizontal part. The free-body diagram shows the vertical arm cut off and an internal axial force and moment replacing the effect of the extended arm. The concentrated moment causes the abrupt change in the value of the bending moment at the right end of the arm as shown in the bending moment diagram. Example Problem 3–14 gives the details of the analysis.

Figure 3–20 shows an isometric view of a crankshaft that is actuated by the vertical force acting at the end of the crank. One result is an applied torque that tends to rotate the shaft ABC clockwise about its x-axis. The reaction torque is shown acting at the forward end of the crank. A second result is that the vertical force acting

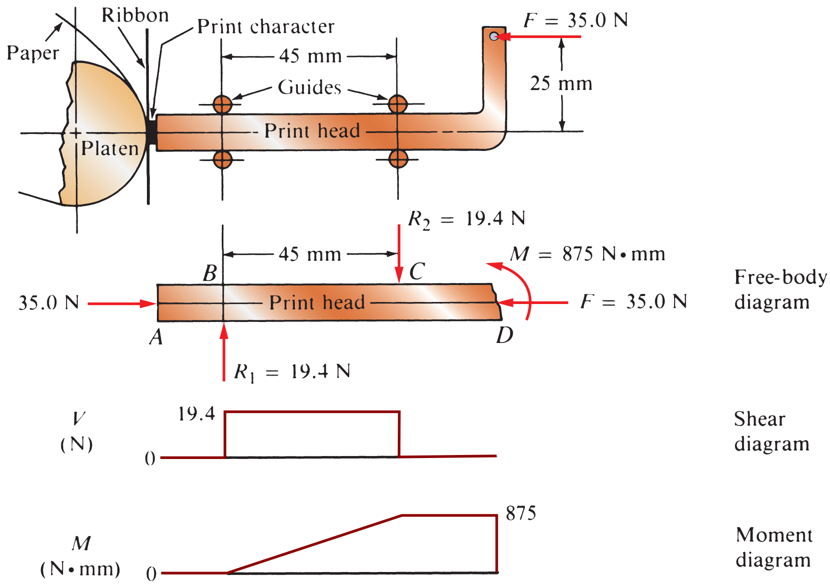


FIGURE 3–19 Bending moment on a print head

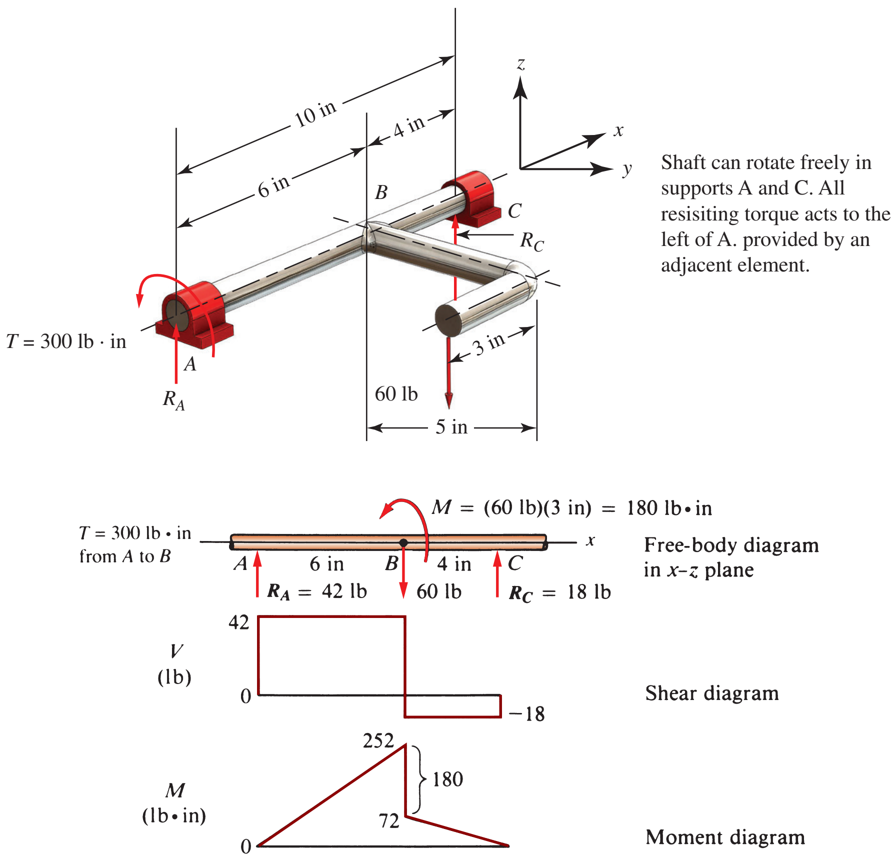


FIGURE 3–20 Bending moment on a shaft carrying a crank

at the end of the crank creates a twisting moment in the rod attached at B and thus tends to bend the shaft ABC in the x–z plane. The twisting moment is treated as a concentrated moment acting at B with the resulting abrupt change in the bending moment at that location as can be seen in the bending moment diagram. Example Problem 3–15 gives the details of the analysis.

When drawing the bending moment diagram for a member to which a concentrated moment is applied, the following sign convention will be used.

When a concentrated bending moment acts on a beam in a counterclockwise direction, the moment diagram drops; when a clockwise concentrated moment acts, the moment diagram rises.

Example Problem 3–13

The bell crank shown in Figure 3–18 is part of a linkage in which the 80-lb horizontal force is transferred to F_2 acting vertically. The crank can pivot about the pin at O. Draw a free-body diagram of the horizontal part of the crank from O to A. Then draw the shearing force and bending moment diagrams that are necessary to complete the design of the horizontal arm of the crank.

Solution

Objective: Draw the free-body diagram of the horizontal part of the crank in Figure 3–18. Draw the shearing force and bending moment diagrams for that part.

Given: The layout from Figure 3–18.

Analysis: Use the entire crank first as a free body to determine the downward force F_2 that reacts to the applied horizontal force F_1 of 80 lb by summing moments about the pin at O.

Then create the free-body diagram for the horizontal part by breaking it through the pin and replacing the removed part with the internal force and moment acting at the break.

Results: We can first find the value of F_2 by summing moments about the pin at O using the entire crank:

$$F_1 \cdot a = F_2 \cdot b$$
$$F_2 = F_1(a/b) = 80\text{ lb}(1.50/2.00) = 60\text{ lb}$$

Below the drawing of the complete crank, we have drawn a sketch of the horizontal part, isolating it from the vertical part. The internal force and moment at the cut section are shown. The externally applied downward force F_2 is reacted by the upward reaction at the pin. Also, because F_2 causes a moment with respect to the section at the pin, an internal reaction moment exists, where

$$M = F_2 \cdot b = (60\text{ lb})(2.00\text{ in}) = 120\text{ lb}\cdot\text{in}$$

The shear and moment diagrams can then be shown in the conventional manner. The result looks much like a cantilever that is built into a rigid support. The difference here is that the reaction moment at the section through the pin is developed in the vertical arm of the crank.

Comments: Note that the shape of the moment diagram for the horizontal part shows that the maximum moment occurs at the section through the pin and that the moment decreases linearly as we move out toward point A. As a result, the shape of the crank is optimized, having its largest cross section (and section modulus) at the section of highest bending moment. You could complete the design of the crank using the techniques reviewed in Section 3–15.

Example Problem 3–14

Figure 3–19 represents a print head for a computer printer. The force F moves the print head toward the left against the ribbon, imprinting the character on the paper that is backed up by the platen. Draw the free-body diagram for the horizontal portion of the print head, along with the shearing force and bending moment diagrams.

Solution

Objective: Draw the free-body diagram of the horizontal part of the print head in Figure 3–19. Draw the shearing force and bending moment diagrams for that part.

Given: The layout from Figure 3–19.

Analysis: The horizontal force of 35 N acting to the left is reacted by an equal 35 N horizontal force produced by the platen pushing back to the right on the print head. The guides provide simple supports in the vertical direction. The applied force also produces a moment at the base of the vertical arm where it joins the horizontal part of the print head.

We create the free-body diagram for the horizontal part by breaking it at its right end and replacing the removed part with the internal force and moment acting at the break. The shearing force and bending moment diagrams can then be drawn.

Results: The free-body diagram for the horizontal portion is shown below the complete sketch. Note that at the right end (section D) of the print head, the vertical arm has been removed and replaced with the internal horizontal force of 35.0 N and a moment of 875 N·mm caused by the 35.0 N force acting 25 mm above it. Also note that the 25 mm-moment arm for the force is taken from the line of action of the force *to the neutral axis of the horizontal part*. The 35.0 N reaction of the platen on the print head tends to place the head in compression over the entire length. The rotational tendency of the moment is reacted by the couple created by R_1 and R_2 acting 45 mm apart at B and C.

Below the free-body diagram is the vertical shearing force diagram in which a constant shear of 19.4 N occurs only between the two supports.

The bending moment diagram can be derived from either the left end or the right end. If we choose to start at the left end at *A*, there is no shearing force from *A* to *B*, and therefore there is no change in bending moment. From *B* to *C*, the positive shear causes an increase in bending moment from 0 to 875 N·mm. Because there is no shear from *C* to *D*, there is no change in bending moment, and the value remains at 875 N·mm. The counterclockwise-directed concentrated moment at *D* causes the moment diagram to drop abruptly, closing the diagram.

Example Problem 3–15

Figure 3–20 shows a crank in which it is necessary to visualize the three-dimensional arrangement. The 60-lb downward force tends to rotate the shaft *ABC* around the *x*-axis. The reaction torque acts only at the end of the shaft outboard of the bearing support at *A*. Bearings *A* and *C* provide simple supports. Draw the complete free-body diagram for the shaft *ABC*, along with the shearing force and bending moment diagrams.

Solution

Objective: Draw the free-body diagram of the shaft *ABC* in Figure 3–20. Draw the shearing force and bending moment diagrams for that part.

Given: The layout from Figure 3–20.

Analysis: The analysis will take the following steps:

1. Determine the magnitude of the torque in the shaft between the left end and point *B* where the crank arm is attached.
2. Analyze the connection of the crank at point *B* to determine the force and moment transferred to the shaft *ABC* by the crank.
3. Compute the vertical reactions at supports *A* and *C*.
4. Draw the shearing force and bending moment diagrams considering the concentrated moment applied at point *B*, along with the familiar relationships between shearing force and bending moments.

Results: The free-body diagram is shown as viewed looking at the *x–z* plane. Note that the free body must be in equilibrium in all force and moment directions. Considering first the torque (rotating moment) about the *x*-axis, note that the crank force of 60 lb acts 5.0 in from the axis. The torque, then, is

$$T = (60\ \text{lb})(5.0\ \text{in}) = 300\ \text{lb}\cdot\text{in}$$

This level of torque acts from the left end of the shaft to section *B*, where the crank is attached to the shaft.

Now the loading at *B* should be described. One way to do so is to visualize that the crank itself is separated from the shaft and is replaced with a force and moment caused by the crank. First, the downward force of 60 lb pulls down at *B*. Also, because the 60-lb applied force acts 3.0 in to the left of *B*, it causes a concentrated moment in the *x–z plane* of 180 lb·in to be applied at *B*.

Both the downward force and the moment at *B* affect the magnitude and direction of the reaction forces at *A* and *C*. First, summing moments about *A*,

$$(60\ \text{lb})(6.0\ \text{in}) - 180\ \text{lb}\cdot\text{in} - R_C(10.0\ \text{in}) = 0$$
$$R_C = [(360 - 180)\ \text{lb}\cdot\text{in}]/(10.0\ \text{in}) = 18.0\ \text{lb upward}$$

Now, summing moments about C,

$$(60\ \text{lb})(4.0\ \text{in}) + 180\ \text{lb}\cdot\text{in} - R_A(10.0\ \text{in}) = 0$$
$$R_A = [(240 + 180)\ \text{lb}\cdot\text{in}]/(10.0\ \text{in}) = 42.0\ \text{lb upward}$$

Now the shear and bending moment diagrams can be completed. The moment starts at zero at the simple support at *A*, rises to 252 lb·in at *B* under the influence of the 42-lb shear force, then drops by 180 lb·in due to the counterclockwise concentrated moment at *B*, and finally returns to zero at the simple support at *C*.

Comments: In summary, shaft *ABC* carries a torque of 300 lb·in from point *B* to its left end. The maximum bending moment of 252 lb·in occurs at point *B* where the crank is attached. The bending moment then suddenly drops to 72 lb·in under the influence of the concentrated moment of 180 lb·in applied by the crank.

3–17 FLEXURAL CENTER FOR BEAM BENDING

A beam section must be loaded in a way that ensures symmetrical bending; that is, there must be no tendency for the section to twist under the load. Figure 3–21 shows several shapes that are typically used for beams having a vertical axis of symmetry. If the line of action of the loads on such sections passes through the axis of symmetry, then there is no tendency for the section to twist, and the flexure formula applies.

When there is no vertical axis of symmetry, as with the sections shown in Figure 3–22, care must be exercised in placement of the loads. If the line of action of the loads were shown as F_1 in the figure, the beam would twist and bend, so the flexure formula would not give accurate results for the stress in the section. For such sections, the load must be placed in line with the *flexural center,* sometimes called the *shear center.* Figure 3–22 shows the approximate location of the flexural center for these shapes (indicated by the symbol Q). Applying the load in line with Q, as shown with the forces labeled F_2, would result in pure bending. A table of formulas for the location of the flexural center is available (see Reference 7).

3–18 BEAM DEFLECTIONS

The bending loads applied to a beam cause it to deflect in a direction perpendicular to its axis. A beam that was originally straight will deform to a slightly curved shape. In most cases, the critical factor is either the maximum deflection of the beam or its deflection at specific locations.

Consider the double-reduction speed reducer shown in Figure 3–23. The four gears (A, B, C, and D) are mounted on three shafts, each of which is supported by two bearings. The action of the gears in transmitting power creates a set of forces that in turn act on the shafts to cause bending. One component of the total force on the gear teeth acts in a direction that tends to separate the two gears. Thus, gear A is forced upward, while gear B is forced downward. For good gear performance, the net deflection of one gear relative to the other should not exceed 0.005 in (0.13 mm) for medium-sized industrial gearing.

To evaluate the design, there are many methods of computing shaft deflections. We will review briefly those methods using deflection formulas, superposition, and a general analytical approach.

A set of formulas for computing the deflection of beams at any point or at selected points is useful in many practical problems. Appendix 14 includes several cases.

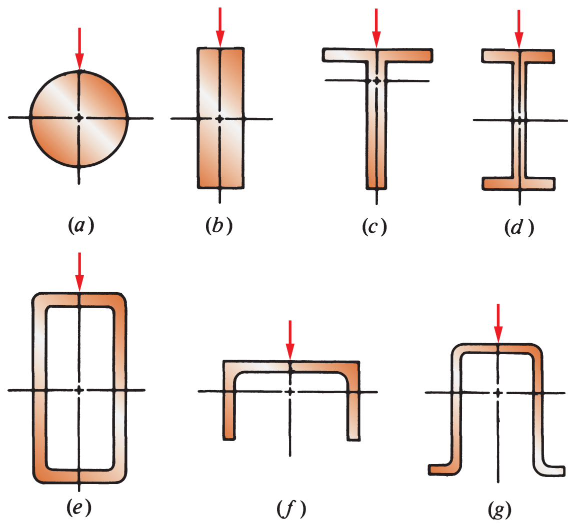


FIGURE 3–21 Symmetrical sections. A load applied through the axis of symmetry results in pure bending in the beam.

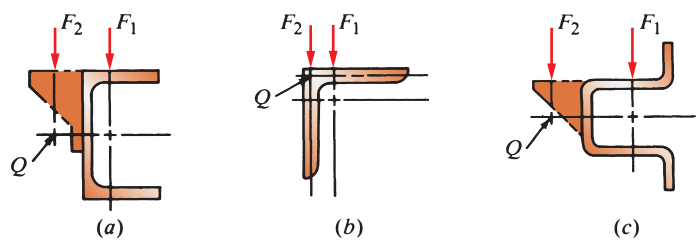


FIGURE 3–22 Nonsymmetrical sections. A load applied as at F_1 would cause twisting; loads applied as at F_2 through the flexural center Q would cause pure bending.

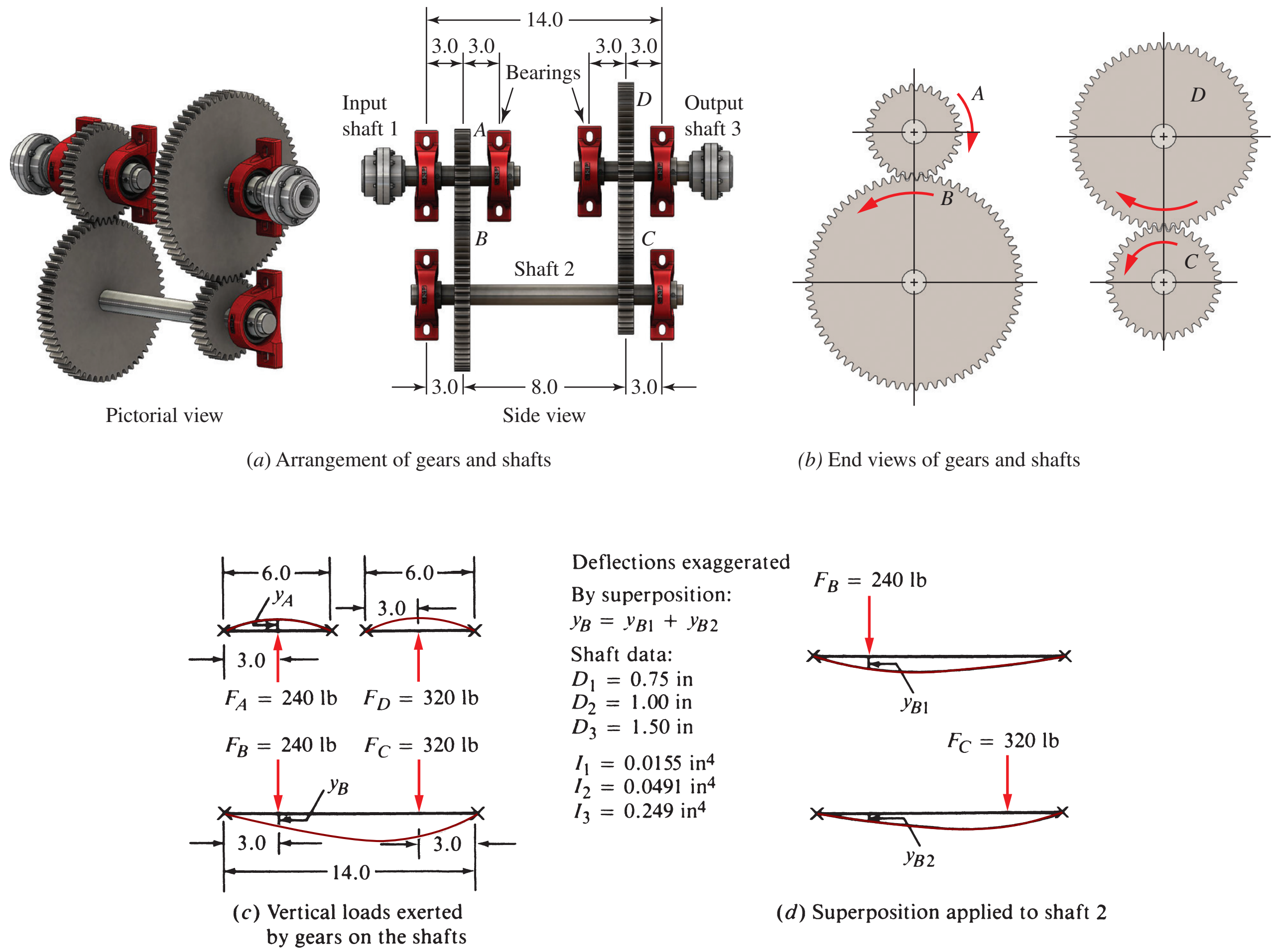


FIGURE 3–23 Shaft deflection analysis for a double-reduction speed reducer

For many additional cases, superposition is useful if the actual loading can be divided into parts that can be computed by available formulas. The deflection for each loading is computed separately, and then the individual deflections are summed at the points of interest.

Many commercially available computer software programs allow the modeling of beams having rather complex loading patterns and varying geometry. The results include reaction forces, shearing force and bending moment diagrams, and deflections at any point. It is important that you understand the principles of beam deflection, studied in strength of materials and reviewed here, so that you can apply such programs accurately and interpret the results carefully.

Example Problem 3–16

For the two gears, *A* and *B*, in Figure 3–23, compute the relative deflection between them in the plane of the paper that is due to the forces shown in Figure 3–23 (c). These *separating forces*, or *normal forces*, are discussed in Chapters 9 and 10. It is customary to consider the loads at the gears and the reactions at the bearings to be concentrated. The shafts carrying the gears are steel and have uniform diameters as listed in the figure.

Solution

Objective: Compute the relative deflection between gears *A* and *B* in Figure 3–23.

Given: The layout and loading pattern are shown in Figure 3–23. The separating force between gears *A* and *B* is 240 lb. Gear *A* pushes downward on gear *B*, and the reaction force of gear *B* pushes upward on gear *A*. Shaft 1 has a diameter of 0.75 in and a moment of inertia of 0.0155 in^4. Shaft 2 has a diameter of 1.00 in and a moment of inertia of 0.0491 in^4. Both shafts are steel. Use $E = 30 \times 10^6$ psi.

Analysis: Use the deflection formulas from Appendix 14 to compute the upward deflection of shaft 1 at gear *A* and the downward deflection of shaft 2 at gear *B*. The sum of the two deflections is the total deflection of gear *A* with respect to gear *B*.

Case (a) from Table A14–1 applies to Shaft 1 because there is a single concentrated force acting at the midpoint of the shaft between the supporting bearings. We will call that deflection y_A.

Shaft 2 is a simply supported beam carrying two nonsymmetrical loads. No single formula from Appendix 14 matches that loading pattern. But we can use superposition to compute the deflection of the shaft at gear B by considering the two forces separately as shown in Part (d) of Figure 3–23. Case (b) from Table A14–1 is used for each load.

We first compute the deflection at B due only to the 240-lb force, calling it y_{B1}. Then we compute the deflection at B due to the 320-lb force, calling it y_{B2}. The total deflection yat B is $y_B = y_{B1} + y_{B2}$.

Results The deflection of shaft 1 at gear A is

$$y_A = \frac{F_A L_1^3}{48\, EI} = \frac{(240)(6.0)^3}{48(30 \times 10^6)(0.0155)} = 0.0023 \text{ in}$$

The deflection of shaft 2 at B due only to the 240-lb force is

$$y_{B1} = -\frac{F_B a^2 b^2}{3\, EI_2\, L_2} = -\frac{(240)(3.0)^2(11.0)^2}{3(30 \times 10^6)(0.0491)(14)} = -0.0042 \text{ in}$$

The deflection of shaft 2 at B due only to the 320-lb force at C is

$$y_{B2} = -\frac{F_c\, bx}{6\, EI_2\, L_2}(L_2^2 - b^2 - x^2)$$

$$y_{B2} = -\frac{(320)(3.0)(3.0)}{6(30 \times 10^6)(0.0491)(14)}[(14)^2 - (3.0)^2 - (3.0)^2]$$

$$y_{B2} = -0.0041 \text{ in}$$

Then the total deflection at gear B is

$$y_B = y_{B1} + y_{B2} = -0.0042 - 0.0041 = -0.0083 \text{ in}$$

Because shaft 1 deflects upward and shaft 2 deflects downward, the total relative deflection is the sum of y_A and y_B:

$$y_{\text{total}} = y_A + y_B = 0.0023 + 0.0083 = 0.0106 \text{ in}$$

Comment This deflection is very large for this application. How could the deflection be reduced?

3–19 EQUATIONS FOR DEFLECTED BEAM SHAPE

The general principles relating the deflection of a beam to the loading on the beam and its manner of support are presented here. The result will be a set of relationships among the load, the vertical shearing force, the bending moment, the slope of the deflected beam shape, and the actual deflection curve for the beam. Figure 3–17 shows diagrams for these five factors, with θ as the slope and y indicating deflection of the beam from its initial straight position. The product of modulus of elasticity and the moment of inertia, EI, for the beam is a measure of its stiffness or resistance to bending deflection. It is convenient to combine EI with the slope and deflection values to maintain a proper relationship, as discussed next.

One fundamental concept for beams in bending is

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

where M = bending moment

x = position on the beam measured along its length

y = deflection

Thus, if it is desired to create an equation of the form $y = f(x)$ (i.e., y as a function of x), it would be related to the other factors as follows:

$$y = f(x)$$

$$\theta = \frac{dy}{dx}$$

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$\frac{V}{EI} = \frac{d^3y}{dx^3}$$

$$\frac{w}{EI} = \frac{d^4y}{dx^4}$$

where w = general term for the load distribution on the beam

The last two equations follow from the observation that there is a derivative (slope) relationship between shear and bending moment and between load and shear.

In practice, the fundamental equations just given are used in reverse. That is, the load distribution as a function of x is known, and the equations for the other factors are derived by successive integrations. The results are

$$w = f(x)$$
$$V = \int w\,dx + V_0$$
$$M = \int V\,dx + M_0$$

where V_0 and M_0 = constants of integration evaluated from the boundary conditions.

In many cases, the load, shear, and bending moment diagrams can be drawn in the conventional manner, and the equations for shear or bending moment can be created directly by the principles of analytic geometry. With M as a function of x, the slope and deflection relations can be found:

$$\theta EI = \int M\,dx + C_1$$
$$yEI = \int \theta EI\,dx + C_2$$

The constants of integration must be evaluated from boundary conditions. Texts on strength of materials show the details. (See Reference 4.)

3–20 CURVED BEAMS

The analysis of stress due to bending in beams summarized earlier in this chapter is limited to beams that are straight or very nearly so. It was noted that when a beam, that is originally straight, is bent:

1. The radius of curvature of the neutral axis of the beam is very large.
2. Plane sections perpendicular to the axis of the beam remain plane as the beam bends and they rotate in opposite directions under the influence of the bending moment placing part of the section in compression and part in tension.
3. The location of the neutral axis of the cross section of the beam is:
   a. Where the bending stress is zero.
   b. Coincident with the centroidal axis of the cross section of the beam.
4. The maximum tensile and compressive stresses occur at the outermost fibers of the beam.
5. There is a linear variation of the bending stress from the place of maximum compression to the place of maximum tension.
6. Under these conditions, the maximum bending stress is computed from the familiar flexure formula:
   a. $\sigma = Mc/I$.
   b. where M is the bending moment at the section; c is the distance from the neutral axis to the outermost fiber; and I is the moment of inertia of the cross section with respect to its centroidal axis.

The flexure formula applies reasonably well when the ratio of the radius of curvature, R, to the depth of the cross section is larger than 10. That is $R/h > 10$ (Reference 6).

When a component has a smaller radius of curvature, the maximum stress due to bending is significantly higher than that which would be predicted from the flexure formula and different analysis procedures apply. There are many practical cases where smaller radii of curvature for beams occur. Examples include:

- Crane hooks used for lifting loads.
- Clamps such as the C-clamp.
- Open, C-section frames for machinery such as punch presses.
- Building beams that are curved for architectural and aesthetic reasons.
- Parts of hand tools such as saws, pliers, and wire cutters.
- Structural or functional members of machinery where the curvature of the member allows clearance of obstructions.
- Parts of some types of furniture such as tables and chairs.
- Automotive frames, suspension elements, and actuation arms for window lifts and seats.

Major features and the general nature of the results of the analysis for curved beams include:

1. When the applied bending moment tends to *straighten the beam* by increasing the radius of curvature, it is considered to be *positive*.
2. When the applied bending moment tends to decrease the radius of curvature, it is considered to be *negative*.
3. The stress distribution within the cross section of the member is *not* linear; rather it is hyperbolic. Figure 3–24 shows the general pattern of the stress distribution for the case of a curved beam with a rectangular cross section when a positive bending moment is applied. Note the following:
   a. The inside surface is placed in tension and the maximum tensile stress occurs on that surface.
   b. The outside surface is placed in compression and the maximum compressive stress occurs at that surface.
   c. The place within the cross section where the stress becomes zero is called the *neutral axis* as it is for straight beams. However, the neutral axis *is not* coincident with the centroidal axis of the shape of the cross section. For the case shown in Figure 3–24, it is displaced toward the center of curvature by an amount called e.

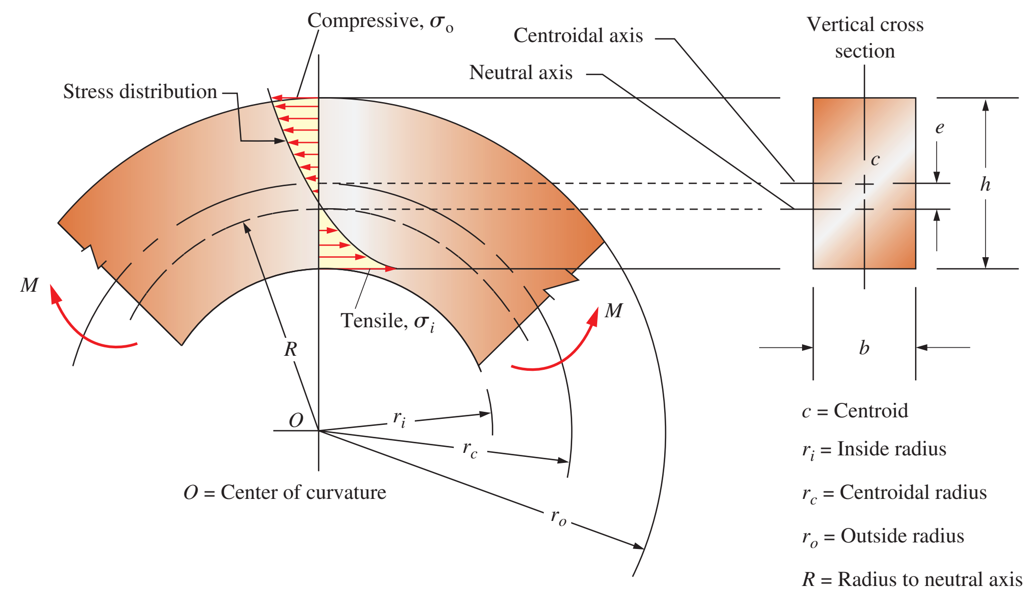


FIGURE 3–24 Segment of a curved beam carrying a positive bending moment

4. The *curved beam formula* given below for the stress distribution has been derived in References 1, 2, and 6.

$$\sigma = \frac{M(R - r)}{Ar(r_c - R)} \quad (3\text{–}26)$$

where:

A = cross-sectional area of the shape

M = bending moment applied

The three radii are measured from the center of curvature of the beam:

r to the place where the stress is to be computed

r_c to the centroid of the area

R to the *neutral axis* of the cross section

R depends on the shape of the cross section using the *area shape factor, ASF,* found from the relations shown in Figure 3–25. Then compute:

$$R = A/ASF \quad (3\text{–}27)$$

The quantity $(r_c - R)$ in the denominator of Equation (3–26) is the radial distance from the centroidal axis of the cross section to the neutral axis. This value, called e, is typically a very small number, requiring that it be computed with high accuracy, at least three significant figures.

General Procedure for Analyzing Curved Beams Carrying a Pure Bending Moment

This procedure is used to compute the maximum tensile and compressive stresses for a curved beam at the inside (at r_i) and outside (at r_o) surfaces. The results are then compared to determine which value is the true maximum stress.

1. Determine the value of the applied bending moment, M, including its sign.
2. For the cross-sectional area of the beam:
   a. Compute the total area, A.
   b. Compute the location of the centroid of the area.
   c. Determine the value of four radii from the center of curvature of the beam; r_i, r_o, r_c, and R, using Equation (3–27).
   d. Compute the area shape factor, ASF. See Figure 3–25.
3. Compute the stress at the outside surface from:

$$\sigma_o = \frac{M(R - r_o)}{Ar_o(r_c - R)} \quad (3\text{–}28)$$

4. Compute the stress at the inside surface from:

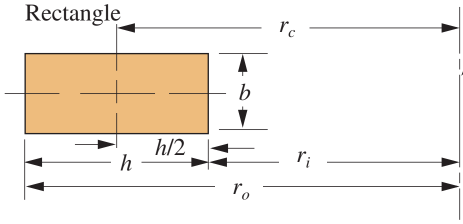
$$\sigma_i = \frac{M(R - r_i)}{Ar_i(r_c - R)} \quad (3\text{–}29)$$

5. Compare σ_o and σ_i to determine the maximum value.

Cross Sections Comprised Two or More Shapes. Designers often work toward optimizing the shape of the cross section of curved beams to more nearly balance the maximum tensile and compressive stresses when the material is isotropic. Similarly, for materials like cast irons having different strengths in compression and tension, the goal is to achieve a nearly equal design factor for both the tensile and compressive parts of the section. The cross-sectional shape of the curved beam may also be modified to optimize the casting or machining processes or to make the shape more pleasing. Adjusting the shape and dimensions of the cross section of the curved beam can accomplish these goals. Reaching these goals typically results in a cross section that is a composite of two or more of the standard shapes shown in Figure 3–25. Examples of the composite shapes are shown in Figure 3–27.

Shape of cross section	Area, A	Area shape factor, ASF

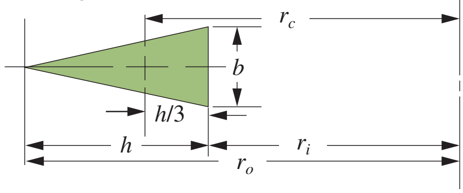
Rectangle



Center of curvature

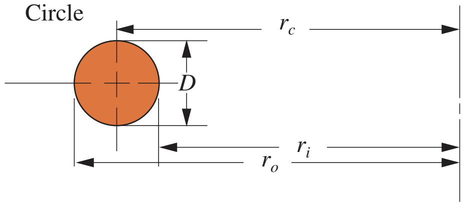
$bh = b(r_o - r_i)$ $\quad$ $b \ln(r_o / r_i)$

Triangle



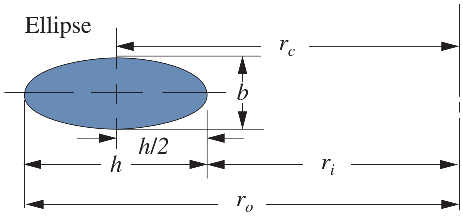
$\frac{1}{2}\, b(r_o - r_i)$ $\quad$ $\dfrac{b\, r_o}{(r_o - r_i)}[\ln(r_o/r_i)] - b$

Circle



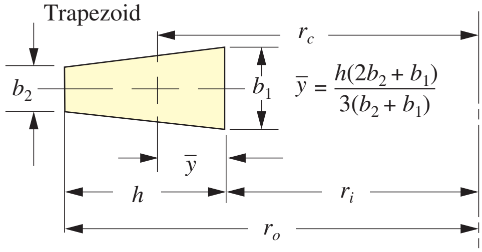
$\pi D^2/4$ $\quad$ $2\pi\, [r_c - \sqrt{r_c^2 - D^2/4}]$

Ellipse



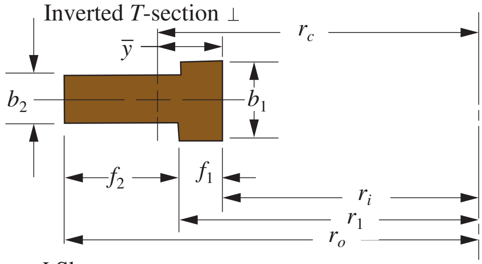
$\pi\, h\, b/4$ $\quad$ $(2\pi b/h)\, [r_c - \sqrt{r_c^2 - h^2/4}]$

Trapezoid



$\frac{1}{2}\,(r_o - r_i)(b_1 + b_2)$ $\quad$ $\dfrac{b_1 r_o - b_2 r_i}{(r_o - r_i)} \ln(r_o/r_i) - (b_1 - b_2)$

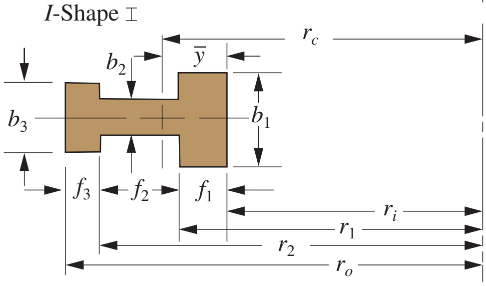
Inverted T-section ⊥



$b_1 f_1 + b_2 f_2$ $\quad$ $b_1 \ln(r_1/r_i) + b_2 \ln(r_o/r_1)$

$$\bar{y} = \frac{1}{A}\left[(b_1 f_1)(f_1/2) + (b_2 f_2)(f_1 + f_2/2)\right]$$

I-Shape ⌶



$b_1 f_1 + b_2 f_2 + b_3 f_3$ $\quad$ $b_1 \ln(r_1/r_i) + b_2 \ln(r_2/r_1) + b_3 \ln(r_o/r_2)$

$$\bar{y} = \frac{1}{A}\left[(b_1 f_1)(f_1/2) + (b_2 f_2)(f_1 + f_2/2) + (b_3 f_3)(f_1 + f_2 + f_3/2)\right]$$

FIGURE 3–25 Area shape factors for selected cross sections of curved bars

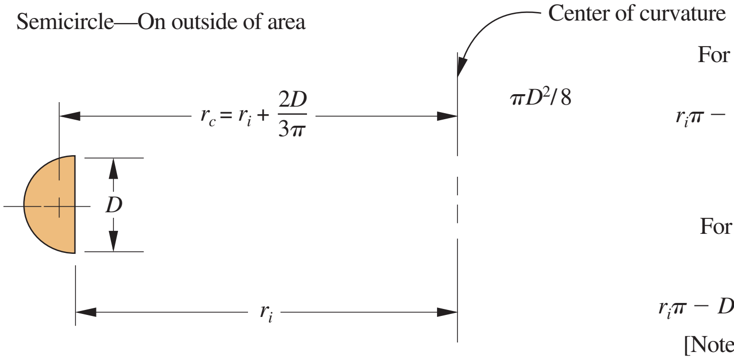
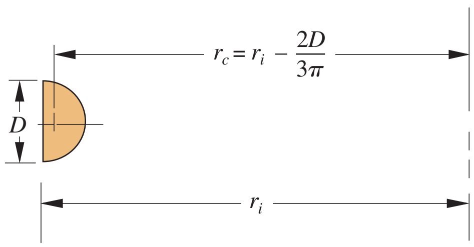
Shape of cross section	Area, A	Area shape factor, ASF
Semicircle—On outside of area  	$\pi D^2/8$	For $D/2 > r_i$ $r_i\pi - D + 2\sqrt{D^2/4 - r_i^2}\ \ln\left[\frac{D/2+\sqrt{D^2/4-r_i^2}}{r_i}\right]$ For $r_i > D/2$ $r_i\pi - D - \pi\sqrt{r_i^2 - D^2/4} + 2\sqrt{r_i^2 - D^2/4}\ \sin^{-1}(D/2r_i)$ [Note: Argument of inverse sine is in radians]
Semicircle—On inside of area  	$\pi D^2/8$	$r_i\pi + D - \pi\sqrt{r_i^2 - D^2/4} - 2\sqrt{r_i^2 - D^2/4}\ \sin^{-1}(D/r_i)$ [Note: Argument of inverse sine is in radians]

FIGURE 3–25 Area shape factors for selected cross sections of curved bars (*Continued*)

Example Problem 3–17 A curved bar has a rectangular cross section 15.0 mm thick by 25.0 mm deep as shown in Figure 3–26. The bar is bent into a circular arc producing an inside radius of 25.0 mm. For an applied bending moment of +400 N·m, compute the maximum tensile and compressive stresses in the bar.

Solution

Objective: Compute the maximum tensile and compressive stresses.

Given: $M = +400$ N·m that tends to straighten the bar. $r_i = 25.0$ mm.

Analysis: Apply Equations (3–28) and (3–29).

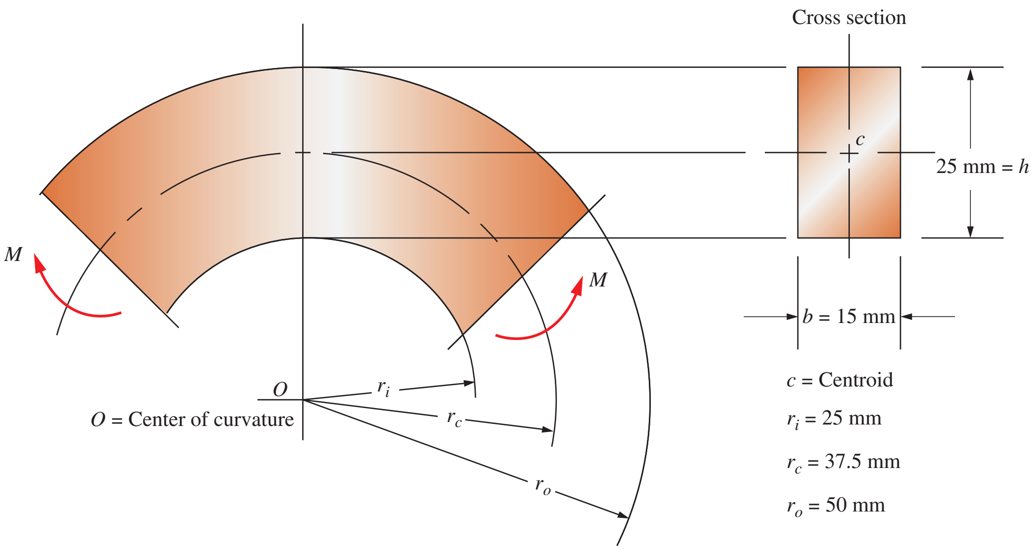


FIGURE 3–26 Curved bar with a rectangular cross section for Example Problem 3–17

Results First compute the cross-sectional area:

$$A = bh = (15.0\text{ mm})(25.0\text{ mm}) = 375.0\text{ mm}^2$$

Now compute the quantities involving radii. See Figure 3–26 for related dimensions:

$$r_o = r_i + h = 25.0\text{ mm} + 25.0\text{ mm} = 50.0\text{ mm}$$
$$r_c = r_i + h/2 = 25.0\text{ mm} + (25.0/2)\text{ mm} = 37.5\text{ mm}$$
$$R = A/ASF$$

From Figure 3–25, for a rectangular cross section:

$$ASF = b \cdot \ln(r_o/r_i) = (15\text{ mm})[\ln(50.0/25.0)] = 10.3972\text{ mm}$$

Then, $R = A/ASF = (375\text{ mm}^2)/10.3972\text{ mm} = 36.0674\text{ mm}$
Quantities needed in the stress equations include:

$$r_c - R = 37.5\text{ mm} - 36.0674\text{ mm} = 1.4326\text{ mm}$$

This is the distance e as shown in Figure 3–24.

$$R - r_o = 36.0674\text{ mm} - 50.0\text{ mm} = -13.9326\text{ mm}$$
$$R - r_i = 36.0674\text{ mm} - 25.0\text{ mm} = 11.0674\text{ mm}$$

This is the distance from the inside surface to the neutral axis.
Stress at outer surface, using Equation (3–28):

$$\sigma_o = \frac{M(R - r_o)}{Ar_o(r_c - R)} = \frac{(400\text{ N}\cdot\text{m})(-13.9326\text{ mm})[1000\text{ mm/m}]}{(375\text{ mm}^2)(50.0\text{ mm})(1.4326\text{ mm})}$$

$$\sigma_o = -207.5\text{ N/mm}^2 = -207.5\text{ MPa}$$

This is the maximum compressive stress in the bar.
Stress at inner surface, using Equation (3–29):

$$\sigma_i = \frac{M(R - r_i)}{Ar_i(r_c - R)} = \frac{(400\text{ N}\cdot\text{m})(11.0674\text{ mm})[1000\text{ mm/m}]}{(375\text{ mm}^2)(25.0\text{ mm})(1.4326\text{ mm})}$$

$$\sigma_i = 329.6\text{ N/mm}^2 = 329.6\text{ MPa}$$

This is the maximum tensile stress in the bar.

Comment The stress distribution between the outside and the inside is similar to that shown in Figure 3–24.

The analysis process introduced earlier can be modified to consider a composite shape for the cross section of the curved beam.

Procedure for Analyzing Curved Beams with Composite Cross-sectional Shapes Carrying a Pure Bending Moment

This procedure is used to compute the maximum tensile and compressive stresses for a curved beam at the inside (at r_i) and outside (at r_o) surfaces. The results are then compared to determine which value is the true maximum stress.

1. Determine the value of the applied bending moment, M, including its sign.
2. For the composite cross-sectional area of the beam:
   a. Determine the inside radius, r_i, and the outside radius, r_o.
   b. Separate the composite area into two or more parts that are shapes from Figure 3–25.
   c. Compute the area of each component part, A_i, and the total area, A.
   d. Locate of the centroid of each component area.
   e. Compute the radius of the centroid of the composite area, r_c.
   f. Compute the value of the area shape factor, ASF_i, for each component area using the equations in Figure 3–25.
   g. Compute the radius, R, from the center of curvature to the neutral axis from:

$$R = A/\Sigma(ASF_i)$$

3. Compute the stress at the outside surface from:

$$\sigma_o = \frac{M(R - r_o)}{Ar_o(r_c - R)} \tag{3–30}$$

4. Compute the stress at the inside surface from:

$$\sigma_i = \frac{M(R - r_i)}{Ar_i(r_c - R)} \tag{3–31}$$

5. Compare σ_o and σ_i to determine the maximum value.

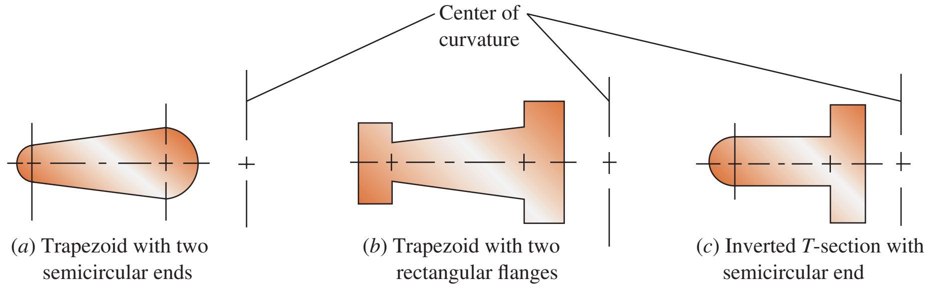


FIGURE 3–27 Examples of composite shapes for cross sections of curved beams

Example Problem 3–18 A curved beam has the shape shown in Figure 3–28 and is subjected to a pure bending moment of −640 lb·in. The inside radius is 2.90 in. Compute the maximum tensile and compressive stresses in the beam.

Solution

Objective: Compute the maximum tensile and compressive stresses.

Given: $M = -640$ lb·in that tends to decrease the radius of curvature.
$r_i = 2.90$ in (see Figure 3–28 for related dimensions).

Analysis: Apply Equations (3–28) and (3–29)

Results: Separate the shape into a composite of an inverted T-section 1 and a semicircular area 2. Compute cross-sectional area for each part:

$$A_1 = b_1 f_1 + b_2 f_2 = (0.80\text{ in})(0.20\text{ in}) + (1.0\text{ in})(0.40\text{ in})$$
$$A_1 = 0.560\text{ in}^2$$
$$A_2 = \pi D^2/8 = \pi(0.40\text{ in})^2/8 = 0.06283\text{ in}^2$$

Total area $= A = A_1 + A_2 = 0.560 + 0.06283 = 0.62283\text{ in}^2$
Now locate the centroid of each area with respect to the inside surface; from Figure 3–25:

$$\bar{y}_1 = [1/A_1][(b_1 f_1)(f_1/2) + (b_2 f_2)(f_1 + f_2/2)]$$
$$\bar{y}_1 = [1/0.56\text{ in}^2][(0.80)(0.20)(0.10) + (1.0)(0.40)(0.70)]\text{ in}^3$$
$$\bar{y}_1 = 0.5286\text{ in}$$
$$\bar{y}_2 = 1.20\text{ in} + 2D/3\pi = 1.20\text{ in} + (2)(0.40\text{ in})/(3)(\pi)$$
$$\bar{y}_2 = 1.2849\ in$$

Now locate the centroid of the composite area:

$$\bar{y}_c = [1/A][A_1\bar{y}_1 + A_2\bar{y}_2]$$
$$\bar{y}_c = [1/0.62283\text{ in}^2][(0.560)(0.5286) + (0.06283)(1.2849)]\text{ in}^3$$
$$\bar{y}_c = 0.60489\text{ in}$$

Now define the pertinent radii from the center of curvature:

$$r_o = r_i + 1.40\text{ in} = 2.90\text{ in} + 1.40\text{ in} = 4.30\text{ in}$$
$$r_c = r_i + \bar{y}_c = 1.40\text{ in} + 0.60489\text{ in} = 3.5049\text{ in}$$

Compute the value of the area shape factor, ASF_i, for each part:
For the T-shape:

$$ASF_1 = b_1 \ln(r_1/r_i) + b_2 \ln(r_{o1}/r_1)$$
$$ASF_1 = (0.80\text{ in}) \ln(3.1/2.9) + (0.40\text{ in}) \ln(4.1/3.1)$$
$$ASF_1 = 0.16519\text{ in}$$

For the semicircular area, we must determine the relationship between ri_2 and $D/2$:

$$r_{i2} = 4.10\text{ in at the base of the semicircle}$$
$$D/2 = (0.40\text{ in})/2 = 0.20\text{ in}$$

Because $r_{i2} > D/2$, we use the equation,

$$ASF_2 = r_i\pi - D - \pi\sqrt{r_{i2}^2 - D^2/4} + 2\sqrt{r_{i2}^2 - D^2/4}[\sin^{-1}(D/2r_i)]$$

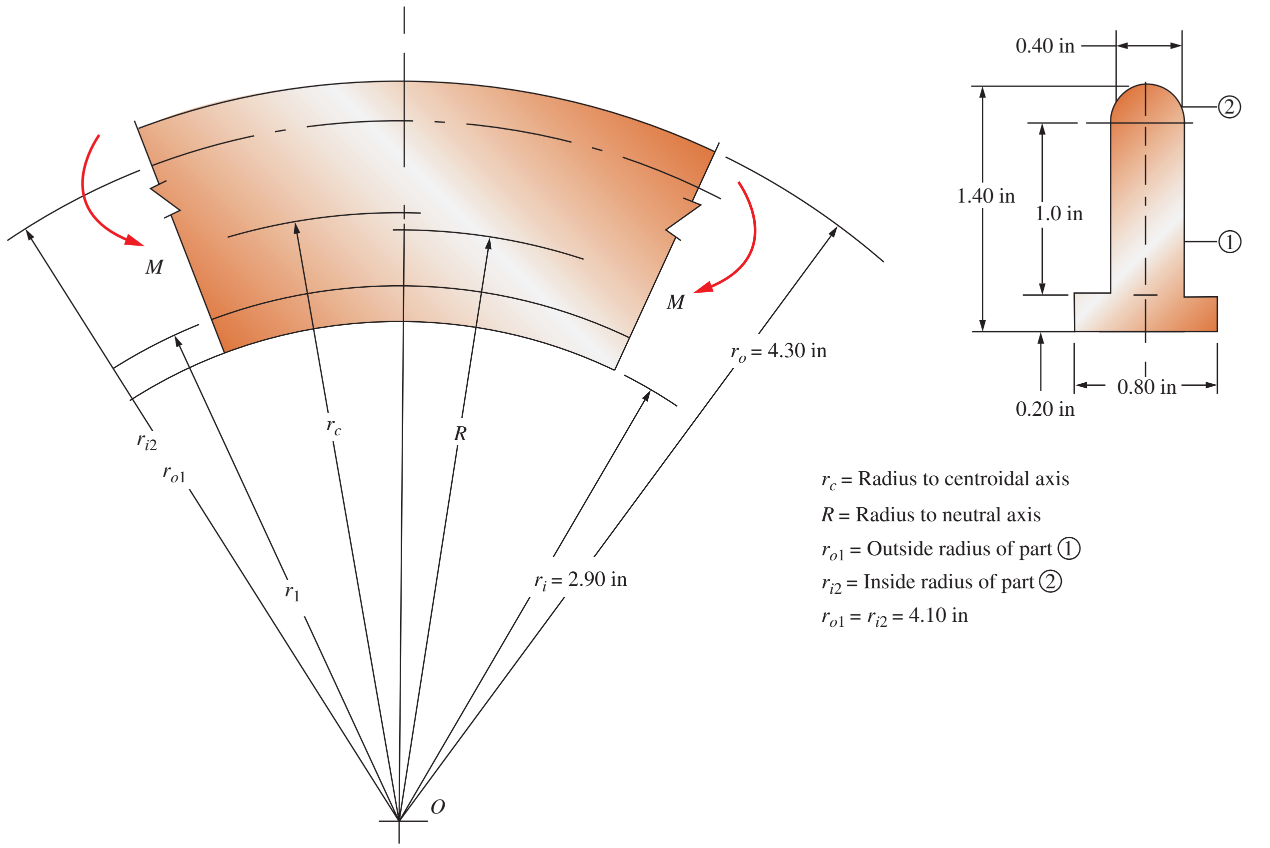


FIGURE 3–28 Curved beam with a composite shape carrying a negative bending moment for Example Problem 3–18

[Note: Argument of inverse sine is in radians]

$$ASF_2 = (4.10)\pi - 0.40 - \pi\sqrt{(4.1)^2 - (0.40)^2/4}$$
$$+ 2\sqrt{(4.1)^2 - (0.40)^2/4}\,[\sin^{-1}(0.40)/(2)(4.1)] = 0.015016 \text{ in}$$

Compute the radius, R, from the center of curvature to the neutral axis from:

$$R = A/\Sigma(ASF_i) = 0.62283 \text{ in}^2/(0.16519 + 0.015016) \text{ in}$$
$$R = 3.4563 \text{ in}$$

Now compute:

$$r_c - R = 3.5049 \text{ in} - 3.4563 \text{ in} = 0.0486 \text{ in} = e$$
$$R - r_o = 3.4563 \text{ in} - 4.30 \text{ in} = -0.8437 \text{ in}$$
$$R - r_i = 3.4563 \text{ in} - 2.90 \text{ in} = 0.5563 \text{ in}$$

This locates the neutral axis from the inner surface.
Now compute the stress at outer surface, using Equation (3–28):

$$\sigma_o = \frac{M(R - r_o)}{Ar_o(r_c - R)} = \frac{(-640 \text{ lb}\cdot\text{in})(-0.8437 \text{ in})}{(0.62283 \text{ in}^2)(4.30 \text{ in})(0.0486 \text{ in})}$$
$$\sigma_o = 4149 \text{ lb/in}^2 = 4149 \text{ psi}$$

This is the maximum tensile stress in the bar.
Stress at inner surface, using Equation (3–29):

$$\sigma_i = \frac{M(R - r_i)}{Ar_i(r_c - R)} = \frac{(-640 \text{ lb}\cdot\text{in})(0.5563 \text{ in})}{(0.62283 \text{ in}^2)(2.90 \text{ in})(0.0486 \text{ in})}$$
$$\sigma_i = -4056 \text{ lb/in}^2 = -4056 \text{ psi}$$

This is the maximum compressive stress in the bar.

Comment This problem demonstrated the process for analyzing a composite section. Note that the maximum tensile and compressive stresses are very nearly equal for this design, a desirable condition for homogeneous, isotropic materials.

Curved Beams with Combined Bending Moment and Normal Load. Each of the example curved beams considered thus far carried only a moment that tended to either increase or decrease the radius of curvature for the beam. Only the bending stress was computed. However, many curved beams carry a combination of a moment and a normal load acting perpendicular to the cross section of the beam. Examples are a crane hook and a C-clamp.

The complete stress analysis requires that the normal stress, $\sigma = P/A$, be added to the bending stress computed from Equations (3–28) and (3–29). This process is discussed in Section 3–21, "Superposition Principle." Apply Equation (3–30) as illustrated in Example Problem 3–19.

Alternate Approaches to Analyzing Stresses in Curved Beams. An alternate approach to analyzing curved beams is reported in Reference 5. Using the concept of stress concentration factors, data are reported for an equivalent K_t value for curved beams having five different shapes: Circular, elliptical, hollow circular, rectangular, and I-shapes for one specific set of proportions. The controlling parameter is r_c/c, where r_c is the radius to the centroidal axis of the shape and c is one-half of the total height of each section. Then K_t is used in the same manner that is discussed in Section 3–22. That is,

$$K_t = \frac{\sigma_{\max}}{\sigma_{\text{nom}}} \quad \text{or} \quad \sigma_{\max} = K_t\sigma_{\text{nom}} = K_t(Mc/I)$$

Note that this is *not* strictly a stress concentration in the sense that there is no geometric discontinuity or abrupt change in shape or size of the cross section. However, the approach gives a reasonable approximation of the maximum stress in the curved beam.

Reference 7 presents a similar approach to the analysis of curved beams but data are presented in tabular form. Again, the primary parameter is r_c/c which is used to compute the eccentricity, e, the radial distance from the centroidal axis to the true neutral axis. Then two factors are determined from the table: k_o for the stress at the outside surface and k_i for the stress at the inside surface. Then,

$$\sigma_{o-\max} = k_o\sigma_{\text{nom}} = k_o(Mc/I) \quad \text{and}$$
$$\sigma_{i-\max} = k_i\sigma_{\text{nom}} = k_i(Mc/I)$$

3–21 SUPERPOSITION PRINCIPLE

When the same cross section of a load-carrying member is subjected to both a direct tensile or compressive stress and a stress due to bending, the resulting normal stress can be computed by the method of superposition. The formula is

$$\sigma = \pm Mc/I \pm F/A \tag{3–32}$$

where *tensile stresses are positive and compressive stresses are negative*.

An example of a load-carrying member subjected to combined bending and axial tension is shown in Figure 3–29. It shows a beam subjected to a load applied downward and to the right through a bracket below the beam. Resolving the load into horizontal and vertical components shows that its effect can be broken into three parts:

1. The vertical component tends to place the beam in bending with tension on the top and compression on the bottom.
2. The horizontal component, because it acts away from the neutral axis of the beam, causes bending with tension on the bottom and compression on the top.
3. The horizontal component causes direct tensile stress across the entire cross section.

We can proceed with the stress analysis by using the techniques from the previous section to prepare the shearing force and bending moment diagrams and then using Equation (3–30) to combine the effects of the bending stress and the direct tensile stress at any point. The details are shown within Example Problem 3–19.

Example Problem 3–19 The cantilever beam in Figure 3–29 is a steel American Standard beam, S6×12.5. The force F is 10 000 lb, and it acts at an angle of 30° below the horizontal, as shown. Use a = 24 in and e = 6.0 in. Draw the free-body diagram and the shearing force and bending moment diagrams for the beam. Then compute the maximum tensile and maximum compressive stresses in the beam and show where they occur.

Solution Objective Determine the maximum tensile and compressive stresses in the beam.

Given The layout from Figure 3–29(a). Force = F = 10 000 lb; angle θ = 30°.
The beam shape: S6×12.5; length = a = 24 in.
Section modulus = S = 7.37 in^3; area = A = 3.67 in^2 (Appendix 15–10).

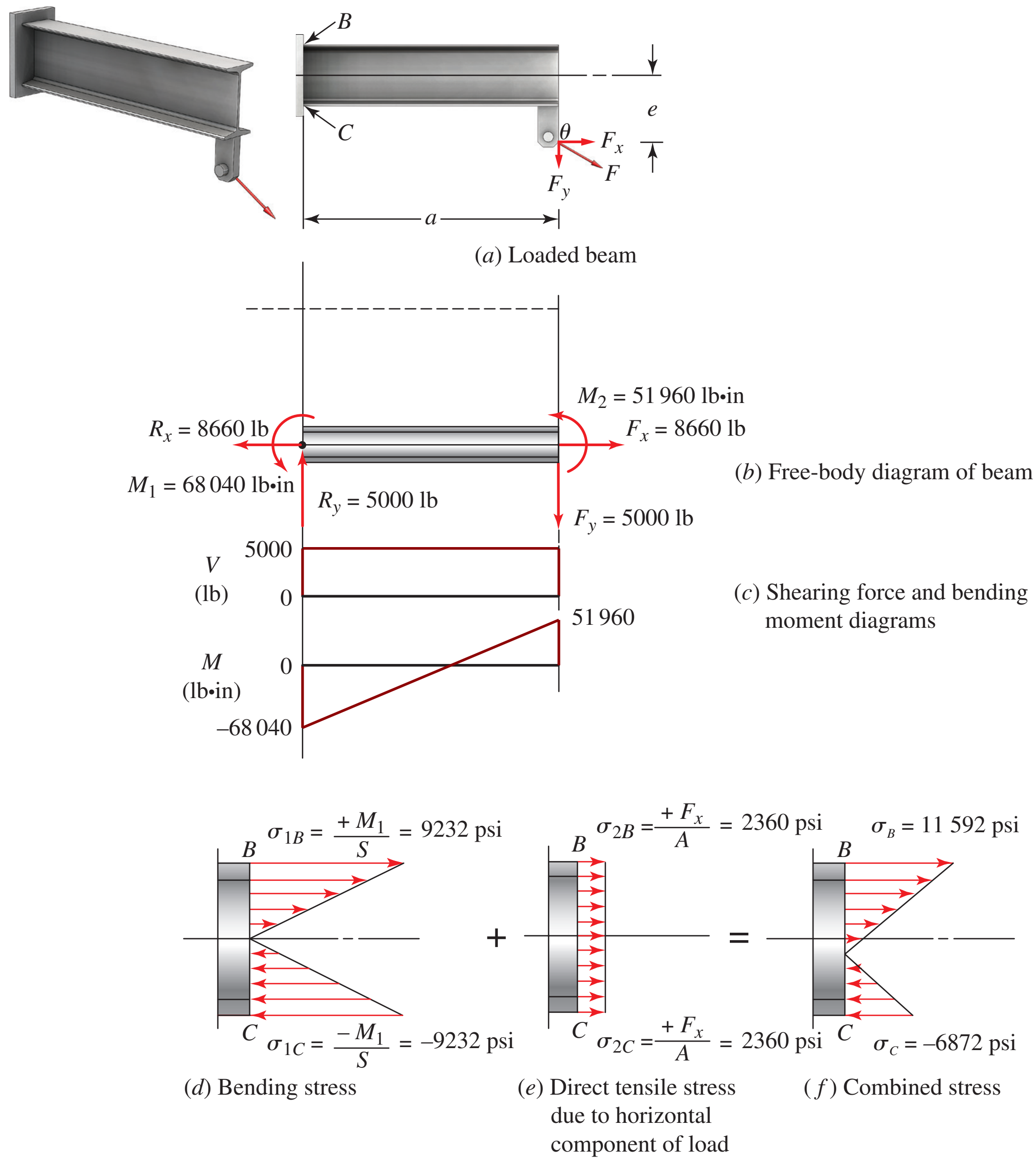


FIGURE 3–29 Beam subjected to combined stresses

Eccentricity of the load = e = 6.0 in from the neutral axis of the beam to the line of action of the horizontal component of the applied load.

Analysis The analysis takes the following steps:

1. Resolve the applied force into its vertical and horizontal components.
2. Transfer the horizontal component to an equivalent loading at the neutral axis having a direct tensile force and a moment due to the eccentric placement of the force.
3. Prepare the free-body diagram using the techniques from Section 3–16.
4. Draw the shearing force and bending moment diagrams and determine where the maximum bending moment occurs.
5. Complete the stress analysis at that section, computing both the maximum tensile and maximum compressive stresses.

Results The components of the applied force are:

$$F_x = F\cos(30°) = (10\,000 \text{ lb})[\cos(30°)] = 8660 \text{ lb acting to the right}$$

$$F_y = F\sin(30°) = (10\,000 \text{ lb})[\sin(30°)] = 5000 \text{ lb acting downward}$$

The horizontal force produces a counterclockwise concentrated moment at the right end of the beam with a magnitude of:

$$M_2 = F_x(6.0 \text{ in}) = (8660 \text{ lb})(6.0 \text{ in}) = 51\,960 \text{ lb} \cdot \text{in}$$

The free-body diagram of the beam is shown in Figure 3–29(*b*).

Figure 3–29(*c*) shows the shearing force and bending moment diagrams.

The maximum bending moment, 68 040 lb in, occurs at the left end of the beam where it is attached firmly to a column.

The bending moment, taken alone, produces a tensile stress (+) on the top surface at point *B* and a compressive stress (−) on the bottom surface at *C*. The magnitudes of these stresses are:

$$\sigma_1 = \pm M_1/S = \pm(68\,040 \text{ lb in})/(7.37 \text{ in}^3) = \pm 9232 \text{ psi}$$

Figure 3–29(*d*) shows the stress distribution due only to the bending stress.

Now we compute the tensile stress due to the axial force of 8660 lb.

$$\sigma_2 = F_x/A = (8660 \text{ lb})/(3.67 \text{ in}^2) = 2360 \text{ psi}$$

Figure 3–29(*e*) shows this stress distribution, uniform across the entire section.

Next, let's compute the combined stress at *B* on the top of the beam.

$$\sigma_B = +\sigma_1 + \sigma_2 = 9232 \text{ psi} + 2360 \text{ psi} = 11\,592 \text{ psi—Tensile}$$

At *C* on the bottom of the beam, the stress is:

$$\sigma_C = -\sigma_1 + \sigma_2 = -9232 \text{ psi} + 2360 \text{ psi} = -6872 \text{ psi—Compressive}$$

Figure 3–29(*f*) shows the combined stress condition that exists on the cross section of the beam at its left end at the support. It is a superposition of the component stresses shown in Figure 3–29(*d*) and (*e*).

3–22 STRESS CONCENTRATIONS

The formulas reviewed earlier for computing simple stresses due to direct tensile and compressive forces, bending moments, and torsional moments are applicable under certain conditions. One condition is that the geometry of the member is uniform throughout the section of interest.

In many typical machine design situations, inherent geometric discontinuities are necessary for the parts to perform their desired functions. For example, as shown in Figure 12–2 shafts carrying gears, chain sprockets, or belt sheaves usually have several diameters that create a series of shoulders that seat the power transmission members and support bearings. Grooves in the shaft allow the installation of retaining rings. Keyseats milled into the shaft enable keys to drive the elements. Similarly, tension members in linkages may be designed with retaining ring grooves, radial holes for pins, screw threads, or reduced sections.

Any of these geometric discontinuities will cause the actual maximum stress in the part to be higher than the simple formulas predict. Defining *stress concentration factors* as the factors by which the actual maximum stress exceeds the nominal stress, σ_{nom} or τ_{nom}, predicted from the simple equations allows the designer to analyze these situations. The symbol for these factors is K_t. In general, the K_t factors are used as follows:

$$\sigma_{max} = K_t\sigma_{nom} \quad or \quad \tau_{max} = K_t\tau_{nom} \qquad (3\text{–}33)$$

depending on the kind of stress produced for the particular loading. The value of K_t depends on the shape of the discontinuity, the specific geometry, and the type of stress.

Figure 3–30 shows two examples of how geometric discontinuities affect the stress levels in mechanical components, creating stress concentrations. Figure 3–30 (a) shows a drawing of a model of a beam made from a flat plate using a photoelastic material that reacts to stress levels by producing light and dark areas proportional to variations in stresses when viewed through a polarizing filter. Each dark line, called a *fringe,* indicates a change in stress of a certain amount.

The beam is simply supported at its ends and carries a concentrated load at its middle, where the breadth of the section is made larger. To the right side of the middle, there is a reduction in the breadth with a circular fillet. To the left side of the middle, the breadth of the section is much smaller and the circular fillet is also used. You should visualize that for this simply supported beam, the maximum bending moment occurs at the middle and it decreases linearly to zero at the supports. The predicted maximum bending stress would be in the left portion where the cross section is smaller and toward the right end of that portion where the moment is higher. The fringe lines verify this observation because the number of fringes increases as you look toward the right end of the smaller section. However, note that near where the fillet is located, there are many more, closely spaced fringe lines. This indicates that the local stress around this geometric discontinuity is much higher than would be predicted by simple bending stress calculations. This illustrates the concept of *stress concentrations*. A similar pattern exists in the right portion of the beam, but the overall stress levels are lower because the section size is larger.

Another detail that can be observed from the beam in Figure 3–30(a) is in the area under where the force is applied. The numerous, closely spaced fringes there illustrate the concept of *contact stress* that can be very high

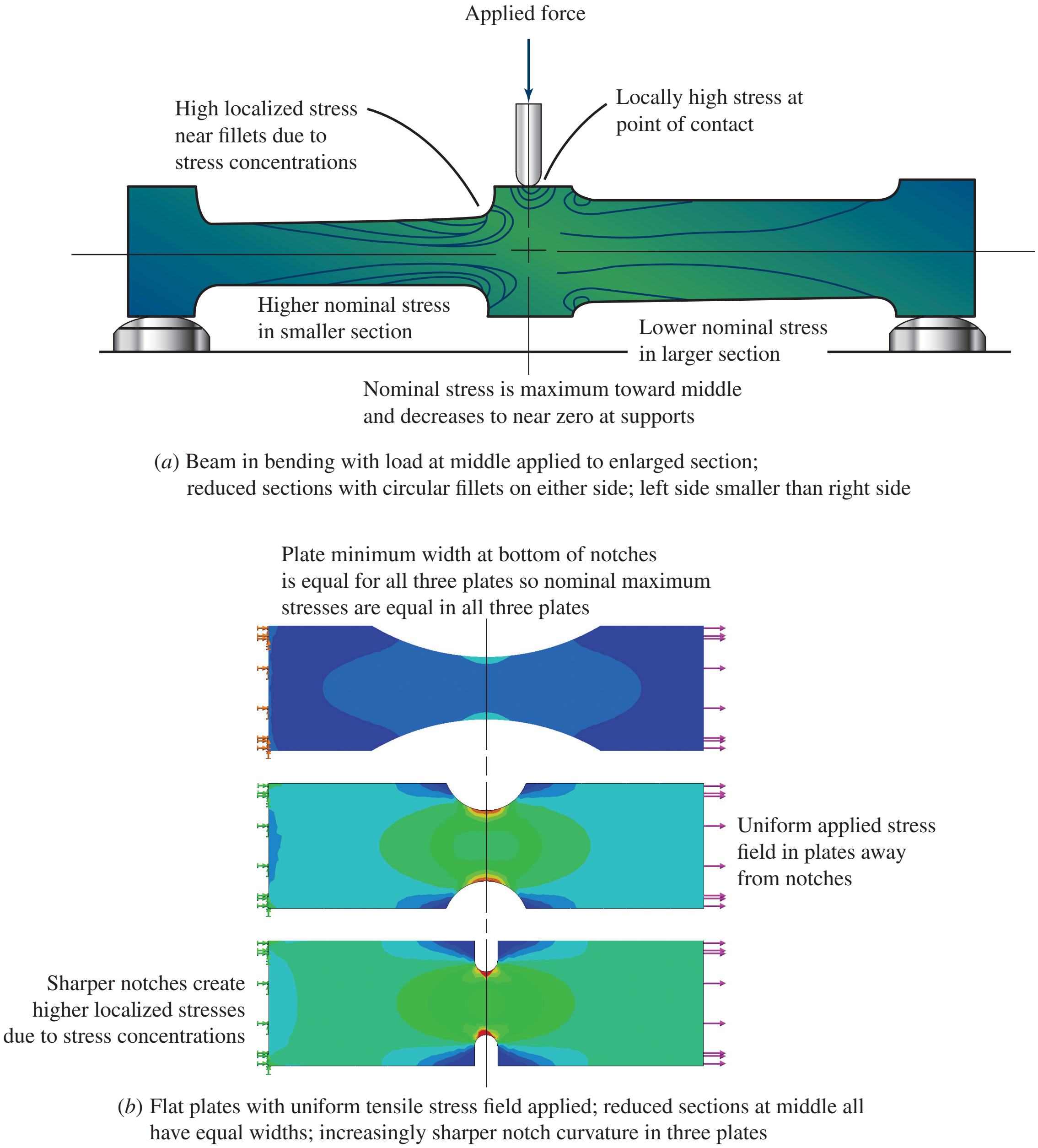


FIGURE 3–30 Sketches of stress distributions near stress concentrations at circular fillets in a beam in bending

locally. Special care should be applied in the design of that part of the beam to withstand the high contact stress. This concept is also pertinent to stresses in gear teeth (Chapters 9 and 10) and rolling contact bearings (Chapter 14).

Figure 3–30(b) shows a different loading and stress concentration phenomenon. Three flat plates are each subjected to identical levels of uniformly distributed tensile stresses in the larger part of each plate toward the ends. Each plate has an equal-width, reduced-breadth section at the middle but with quite different geometry for each. The top plate's width is reduced very gradually and the stress in the material increases essentially only due to the reduction in cross-sectional area. In the plate in the middle, the reduction is made by machining a circular notch with a relatively large radius. The stress at the notch again increases because of the reduction in cross-sectional area, but there is also an increased stress level near the two notches because of the sudden change in geometry. The larger number of fringe lines is the indication of a *stress concentration* near the notches. The lower plate also contains a notch, but it is much narrower and the radius at its bottom is much sharper than that for the middle plate. Here, there are even more fringe lines, more closely spaced, indicating a much higher local stress near the small notches.

Stress concentrations are generally highly local phenomena, affecting only the region close to the geometric discontinuity. The entire component must be examined to determine where maximum stresses may occur and often two or more areas must be analyzed to ensure that no failure occurs. More is said about this in Chapter 5.

As a designer, you are responsible for applying stress concentration factors correctly and careful judgment is required. Numerous geometries and loading cases have been evaluated analytically, empirically, and by the use of finite element stress analysis methods to produce data from which values for K_t can be found. This section summarizes the basic principles and shows examples of the application of stress concentration factors. General guidelines are listed next.

General Guidelines for the Use of Stress Concentration Factors

1. Stress concentration factors must always be applied for brittle materials because locally high stresses typically lead to fracture.
2. For ductile materials under static tensile loading, it is typical to ignore stress concentrations in small local areas because the ductility permits yielding that redistributes the stress over a larger area and ultimate failure does not occur. However, some designers prefer to ensure that no yielding occurs anywhere in the component.
3. Areas of ductile members under cyclical compressive stress are unlikely to produce failure unless the local maximum stress exceeds the ultimate compressive stress. At lower stresses, a small amount of material near the highest stress area may yield and permit the load to be redistributed over a larger area.
4. Flexing a member in tension tends to cause excessively high local stresses where there are geometric discontinuities, where small internal voids occur, or where surface imperfections occur such as cracks, tool marks, nicks, roughness, corrosion, plating, or scratches. Small cracks appear that will grow to cover larger areas over time, leading to ultimate failure. This type of failure, called *fatigue*, is discussed further in Chapter 5. ***Therefore, always apply stress concentration factors in areas of cyclical tensile stress where fatigue failures initiate.***
5. Designing components to avoid the occurrence of factors listed in item 4 is recommended. Maintaining smooth surfaces and homogeneous internal structure of the material and avoiding damage in manufacturing and handling are critical.
6. Stress concentration factors are highest in regions of acutely abrupt changes in geometry. Therefore, good design will call for more gradual changes such as by providing generous fillet radii and blending mating surfaces smoothly.

Reference 5 is arguably the most comprehensive source of data for stress concentration factors. Reference 7 also contains much useful data.

Stress concentration factors are often presented in graphical form to aid designers in visualizing the effect of specific geometrical decisions on local stresses. Examples of the general forms of such graphs are shown in Figure 3–31. The horizontal axis is typically defined in terms of the ratio of the sizes for primary geometrical features such as diameters, widths, and fillet radii. *It is essential that you understand the basis for the nominal stress for use in Equation (3–33)*. For each case in Figure 3–31, the equation for the nominal stress is reported. It is typical for the geometry of the smaller section to be used to compute nominal stress because that will give the largest stress in the region around the discontinuity before accounting for the specific geometry. However, exceptions occur. Additional discussion of the four general types of graphs follows.

(a) ***Stepped flat plate in tension:*** The magnitude of the ratio of the larger width, H, to the smaller width, h, is a major factor and it is typically shown as a family of curves like the three in the figure. The horizontal axis is the ratio of r/h, where r is the fillet radius at the step. Note that the larger the ratio, the smaller the value of K_t. Small fillet radii should be avoided if possible because the stress concentration factor typically increases exponentially for very small r/t ratios. *The nominal stress is based on the applied force, F, divided by the minimum cross-sectional area in the smaller section as shown in the figure.*

(b) ***Round bar in bending:*** A family of curves is shown for the ratio of the larger diameter, D, to the smaller diameter, d, similar to the discussion for Part (a). The horizontal axis is the ratio of r/d and, again, it is obvious that small fillet radii should be avoided. *The nominal stress is based on the applied bending moment, M, divided by the section modulus of the smaller cross section as shown in the figure.*

(c) ***Flat plate in tension with a central hole:*** This case assumes that the applied force is uniformly distributed to all parts of the plate in areas away from the hole. The maximum stress rises in the near vicinity of the hole and it occurs at the top and bottom of the hole. The horizontal axis of the chart is the ratio of d/w, where d is the hole diameter and w is the width of the plate. Note that the plate thickness is called t. *The nominal stress is based on the applied force divided by the net area found at the cross section through the hole.* The figure shows that $A_{net} = (w - d)t$.

(d) ***Circular bar in torsion with a central hole:*** *The stress concentration factor in this case is based on the gross polar section modulus for the full cross section of the bar.* This is one of the few cases where the gross section is used instead of the net section. The primary reason is the complexity of the calculation for the polar section modulus around the hole. The result is that the value of K_t accounts for both the reduction in polar section modulus and the increase in the stress due to the discontinuity around the hole, resulting in rather large values for K_t. The horizontal

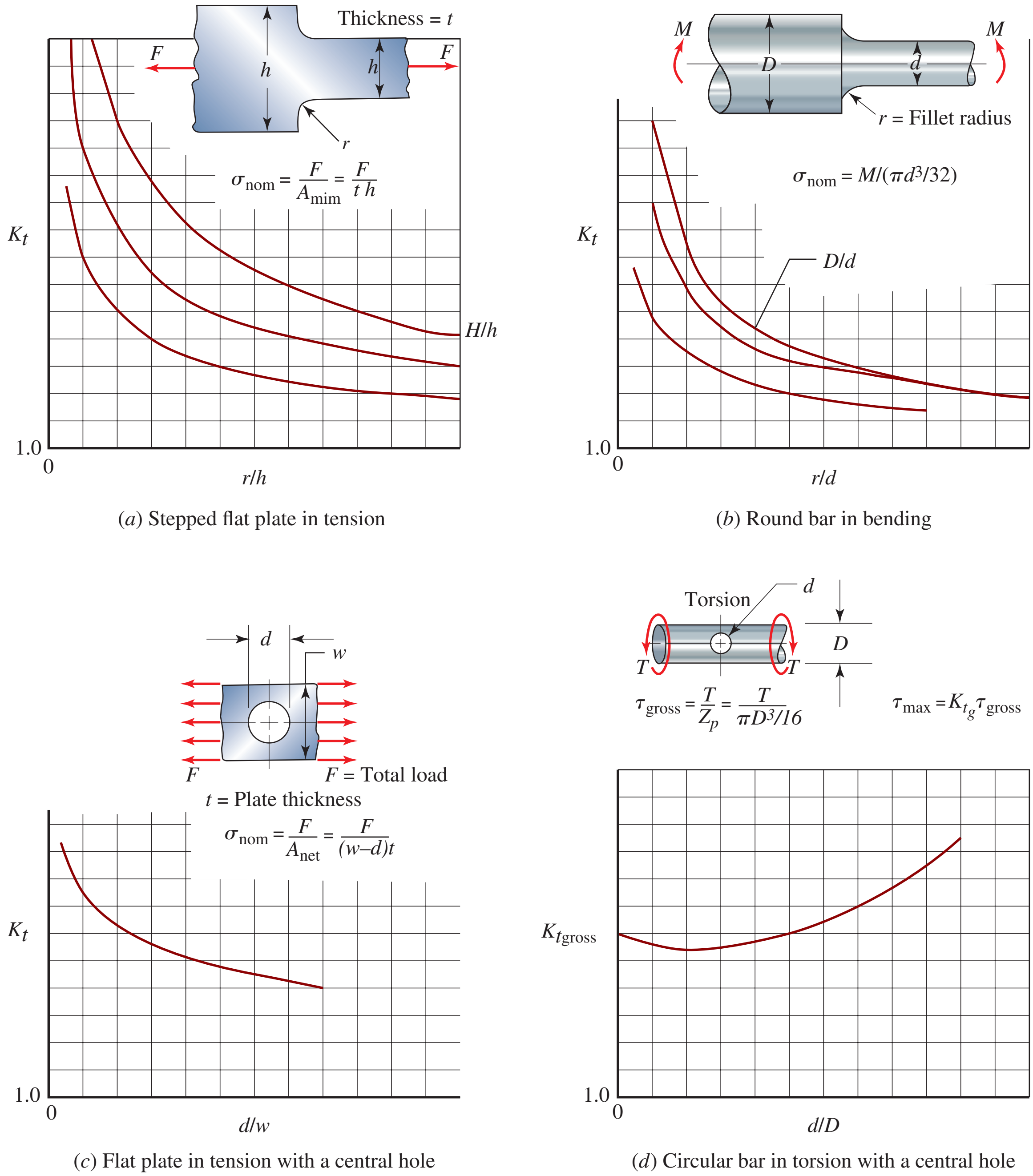


FIGURE 3–31 General Forms of Stress Concentration Factor Curves

axis is the ratio of d/D, where D is the full diameter of the bar and d is the diameter of the hole.

Appendix 18 includes 10 examples of the types of data available for stress concentration factors and should be used for problems in this book. Figures 18–1 through 18–4 are for shapes loaded in direct tension. Figures 18–5 through 18–8 are for bending loads. Figure 18–9 and 18–10 are for torsional loads.

References 5 and 7 are excellent sources of large numbers of cases for stress concentration factors.

Internet site 3 is another useful source for K_t data. Four categories of shapes, listed below, are included on the site with multiple kinds of stresses, typically axial tension, bending, and torsion.

1. Rectangular bars with several types of notches, holes, slots, and changes in thickness (17 geometry types; 31 cases).
2. Round bars and shafts with steps in diameter, grooves, keyseats, and holes (8 geometry types; 23 cases).
3. Infinite or semi-infinite plates (3 geometry types; 7 cases).
4. Special shapes (3 geometry types; 6 cases).

Example Problem 3–20 Compute the maximum stress in a stepped flat plate subjected to an axial tensile force of 9800 N. The geometry is shown in Figure 3–32.

Solution Objective Compute the maximum stress in the stepped flat plate in Figure 3–32.

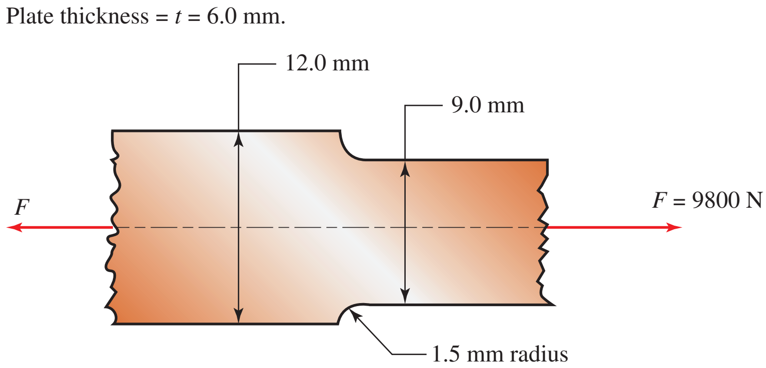


FIGURE 3–32 Stepped flat plate for Example Problem 3–20

Given The layout from Figure 3–32. Force $= F = 9800$ N
Using the notation from Figure 3–31(a): widths $H = 12.0$ mm; $h = 9.0$ mm
Plate thickness: $t = 6.0$ mm
Fillet radius at the step: $r = 1.50$ mm

Analysis The presence of the change in width at the step causes a stress concentration to occur. The geometry is of the type shown in Figure 3–31(a) and Appendix A18–2 that we will use to find the value of K_t for this problem. That value is used in Equation (3–33) to compute the maximum stress.

Results The values of the ratios H/h and r/h are required:

$$H/h = 12.0\text{ mm}/9.0\text{ mm} = 1.33$$
$$r/h = 1.5\text{ mm}/9.0\text{ mm} = 0.167$$

The value of $K_t = 1.83$ can be read from Appendix A18–2.

The nominal stress is computed for the small section having a cross section of 6.0 mm by 9.0 mm.

$$A_{net} = h \cdot t = (9.0\text{ mm})(6.0\text{ mm}) = 54.0\text{ mm}^2$$

The nominal stress is:

$$\sigma_{nom} = F/A_{net} = (9800\text{ N})/(54.0\text{ mm}^2) = 181.5\text{ N/mm}^2 = 181.5\text{ MPa}$$

The maximum stress in the area of the fillet at the step is:

$$\sigma_{max} = K_t\sigma_{nom} = (1.83)(181.5\text{ MPa}) = 332\text{ MPa}$$

Comments The maximum stress of 332 MPa occurs in the fillet area at both the top and bottom of the small section. A short distance to the right of the fillet, the local stress reduces to the nominal value of 181.5 MPa. The stress in the larger section is obviously much smaller because of the larger cross-sectional area of the plate. Note that specifying a smaller fillet radius would cause a much larger maximum stress because the curve for K_t increases sharply as the ratio of r/t decreases. Modestly smaller maximum stress would be produced for larger fillet radii.

Stress Concentration Factors for Lug Joints.

A common machine element is the familiar *clevis joint* or *lug joint,* sketched in Figure 3–33. For this discussion, the bar-shaped plate is called the *lug* that may have either a flat, square-ended design as shown or a rounded end that is sometimes used to provide clearance for rotation of the lug. The generic name for the lug in stress analysis is *flat plate with a central hole with a load applied through a pin in tension.* Figure 3–34 shows approximate data for the stress concentration factors that occur around the hole for typical conditions, described below. The plate is assumed to have a uniform distribution of the load across any cross section well away from the hole. The critical section with regard to axial tensile stress in the plate occurs in the section through the hole where the minimum net cross-sectional area occurs. The stress concentration factor accounts for the geometric discontinuity at this section.

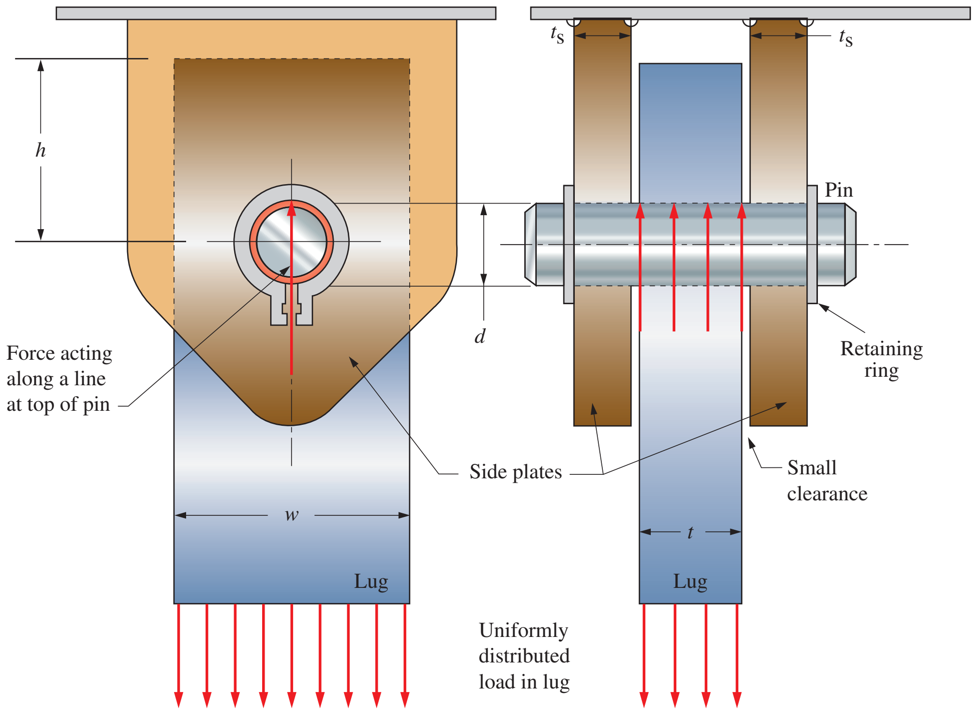


FIGURE 3–33 Clevis joint showing a pin–lug connection including a flat plate with a central hole as used in Figure 3–34.

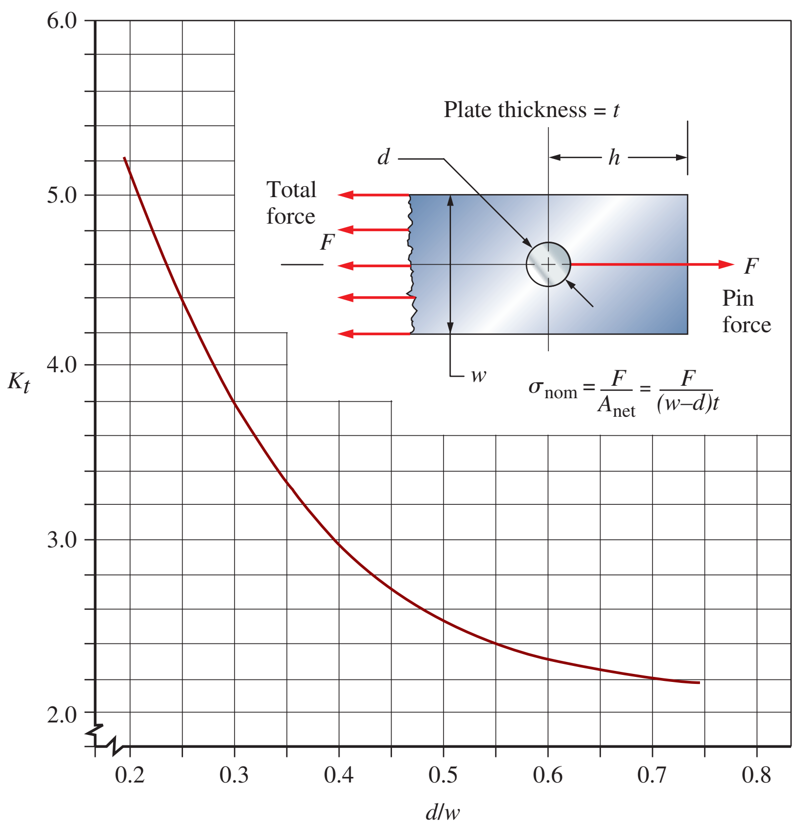


FIGURE 3–34 Stress concentration factor for flat plate with tensile load applied through a pin

Example Problem 3–21 The lug for a clevis joint of the type shown in Figure 3–33 is being designed for an applied force of 8.75 kN. The steel pin nominal diameter has been specified to be $d = 12.0$ mm. Specify the dimensions w, h, and t for the steel lug to be consistent with the parameters listed in this section and a d/w ratio of 0.40. Also specify the nominal clearance between the pin and the hole in the lug. Then determine the expected maximum stress in the lug.

Objective Specify w, h, and t and the nominal clearance.

Given $d = 12.0$ mm; $d/w = 0.40$. $F = 8.75$ kN

Analysis *Lug thickness, t*: Recommended $t/d \leq 0.50$. Using the upper limit gives:
$t = 0.50\, d = (0.50)(12.0 \text{ mm}) = 6.0 \text{ mm}$

Lug width, w: Using $d/w = 0.40$, $w = d/0.40 = 12 \text{ mm}/0.40 = 30.0 \text{ mm}$

End distance, h: Recommended ratio $h/w = 1.0$. Then,

$$h = w = 30.0 \text{ mm}$$

Close-fitting pin: $d_{hole} \cong d_{pin}(1.002) = 12.0 \text{ mm}(1.002) = 12.024 \text{ mm}$

Maximum stress: $\sigma_{max} = K_t \times \sigma_{nom}$

$$\sigma_{nom} = F/(w - d)t = [8.75 \text{ kN}/(30.0 - 12.0)(6.0)\text{mm}^2](1000 \text{ N/kN})$$
$$\sigma_{nom} = 81.8 \text{ N/mm}^2 = 81.8 \text{ MPa}$$

To find K_t: At $d/w = 0.40$ in Figure 3–33, $K_t \cong 3.0$. Then

$$\sigma_{max} = K_t \times \sigma_{nom} = (3.0)(81.8 \text{ MPa}) = 243 \text{ MPa}$$

Results The design details for the lug of the clevis joint in Figure 3–33 are:

$$d = 12.0 \text{ mm};\ w = 30.0 \text{ mm};\ t = 6.0 \text{ mm};\ h = 30.0 \text{ mm}$$

Nominal clearance between pin and hole: $\delta = 0.02$ mm

Maximum stress in lug at hole: $\sigma_{max} = 243$ mm

In general, the axial tensile stress near the hole in the lug depends on several factors:

1. The width and thickness of the lug and the size and placement of the hole relative to the end of the lug.
2. The clearances between the faces of the lug and the side plates of the clevis.
3. The diameter and length of the pin.
4. The clearance between the pin and the hole.
5. The materials of the pin and the lug.

A typical lug connection should have the following features:

- a square-ended lug.
- close-fitting pin: $d_{hole} \cong d_{pin}(1.002)$.
- $t/d \leqq 0.5$ [t = thickness of lug parallel to axis of pin; d = nominal hole diameter].
- Both pin and lug have the same modulus of elasticity; $E_{pin}/E_{lug} = 1.0$.
- $h/w = 1.0$.
  - h = distance from centerline of hole to the top of the lug.
  - w = width of the lug across the location of the hole.

Problems in this book will assume the conditions listed above and use the values for K_t from Internet site 3. The curve in Figure 3–34 gives approximate data for K_t vs. the ratio d/w.

Reference 5 includes extensive data that allow consideration of factors different from those assumed above.

Very loose fitting pins should be avoided because:

- The pin makes a virtual line contact on the top of the hole.
- The stress concentration factor for the lug is somewhat higher than shown in Figure 3–34.
- The maximum stress occurs at the line of contact.
- There is the danger that the contact stress between the pin and the hole will cause failure in bearing, a local crushing of the pin or the inside surface of the hole.

Other factors to consider in the design of a clevis or lug joint are:

1. The pin should be as short as practical to minimize the tendency for it to bend, causing nonuniform contact of the pin in the hole.
2. The diameter of the pin must be adequate to resist failure by direct shear and to keep bending deflection to an acceptable level.

3. The distance, h, from the centerline of the hole to the top of the lug should be nominally equal to the width of the plate, w, and as large as practical to minimize the bending stresses in the upper part of the lug and to prevent shearing tear-out of the lug.
4. The bearing stress between the pin surface and the inside of the hole of the lug must be acceptable, typically less than $0.9s_y$.
5. Similarly, the bearing stress between the pin surface and the holes in the side plates must be acceptable.
6. The clearance between the faces of the lug and the side plates of the clevis should be small to minimize bending of the pin.

3–23 NOTCH SENSITIVITY AND STRENGTH REDUCTION FACTOR

The amount by which a load-carrying member is weakened by the presence of a stress concentration (notch), considering both the material and the sharpness of the notch, is defined as

K_f = fatigue strength reduction factor

$$K_f = \frac{\text{endurance limit of a notch-free specimen}}{\text{endurance limit of a notched specimen}}$$

This factor could be determined by actual test. However, it is typically found by combining the stress concentration factor, K_t, defined in Section 3–22, and a material factor called the *notch sensitivity*, q. We define

$$q = (K_f - 1)/(K_t - 1) \tag{3–34}$$

When q is known, K_f can be computed from

$$K_f = 1 + q(K_t - 1) \tag{3–35}$$

Values of q range from 0 to 1.0, and therefore K_f varies from 1.0 to K_t. Under repeated bending loads, very ductile steels typically exhibit values of q from 0.5 to 0.7. High-strength steels with hardness approximately HB 400 ($s_u \cong$ 200 ksi or 1400 MPa) have values of q from 0.90 to 0.95. (See Reference 2 for further discussion of values of q.)

Because reliable values of q are difficult to obtain, the problems in this book will typically assume that $q = 1.0$ and $K_f = K_t$, the safest, most conservative value.

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6. Timoshenko, S. *Strength of Materials, Part II—Advanced Theory and Problems*. New York: D. Van Nostrand Co., 1930.
7. Young, Warren C., and Richard G. Budynas. *Roark's Formulas for Stress and Strain*. 8th ed. New York: McGraw-Hill, 2012.

INTERNET SITES RELATED TO STRESS AND DEFORMATION ANALYSIS

1. **BEAM 2D-Stress Analysis 3.1** Software for mechanical, structural, civil, and architectural designers providing detailed analysis of statically indeterminate and determinate beams. From Orand Systems.
2. **MDSolids** Educational software devoted to introductory mechanics of materials. Includes modules on basic stress and strain; beam and strut axial problems; trusses; statically indeterminate axial structures; torsion; determinate beams; section properties; general analysis of axial, torsion, and beam members; column buckling; pressure vessels; and Mohr's circle transformations.
3. **eFatigue.com** An Internet source for consultation and information related to design of machine components subjected to fatigue loading. From the home page, select "Constant Amplitude" then "Stress Concentrations." This site permits online calculation of stress concentration factors for 60 cases covering a variety of geometry types and loading conditions.

PROBLEMS

Direct Tension and Compression

1. A tensile member in a machine structure is subjected to a steady load of 4.50 kN. It has a length of 750 mm and is made from a steel tube having an outside diameter of 18 mm and an inside diameter of 12 mm. Compute the tensile stress in the tube and the axial deformation.
2. Compute the stress in a round bar having a diameter of 10.0 mm and subjected to a direct tensile force of 3500 N.
3. Compute the stress in a rectangular bar having cross-sectional dimensions of 10.0 mm by 30.0 mm when a direct tensile force of 20.0 kN is applied.
4. A link in a packaging machine mechanism has a square cross section 0.40 in on a side. It is subjected to a tensile force of 860 lb. Compute the stress in the link.
5. Two circular rods support the 3800 lb weight of a space heater in a warehouse. Each rod has a diameter of 0.375 in and carries 1/2 of the total load. Compute the stress in the rods.

6. A tensile load of 5.00 kN is applied to a square bar, 12 mm on a side and having a length of 1.65 m. Compute the stress and the axial deformation in the bar if it is made from (a) SAE 1020 hot-rolled steel, (b) SAE 8650 OQT 1000 steel, (c) ductile iron A536(60-40-18), (d) aluminum 6061-T6, (e) titanium Ti-6Al-4V, (f) rigid PVC plastic, and (g) phenolic plastic.
7. An aluminum rod is made in the form of a hollow square tube, 2.25 in outside, with a wall thickness of 0.120 in. Its length is 16.0 in. What axial compressive force would cause the tube to shorten by 0.004 in? Compute the resulting compressive stress in the aluminum.
8. Compute the stress in the middle portion of rod *AC* in Figure P3–8 if the vertical force on the boom is 2500 lb. The rod is rectangular, 1.50 in by 3.50 in.

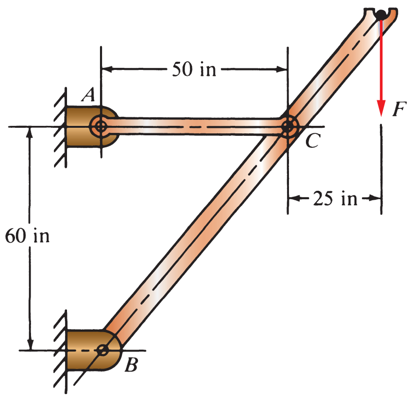


FIGURE P3–8 (Problems 8, 16, 56, and 80)

9. Compute the forces in the two angled rods in Figure P3–9 for an applied force, $F = 1500$ lb, if the angle θ is 45°.

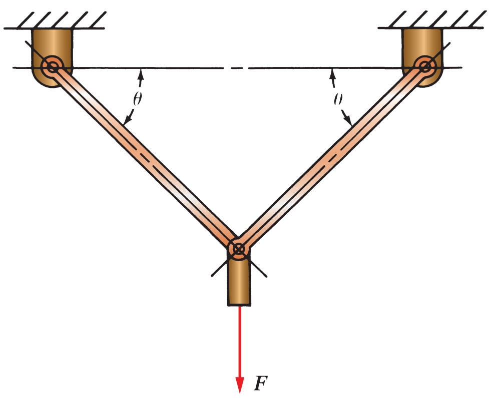
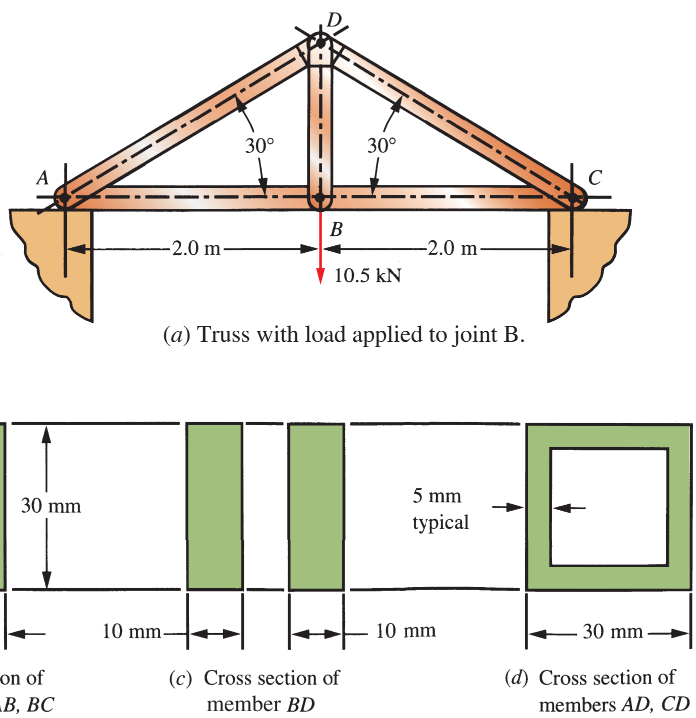


FIGURE P3–9 (Problems 9, 10, 11, 17, and 18)

10. If the rods from Problem 9 are circular, determine their required diameter if the load is static and the allowable stress is 18 000 psi.
11. Repeat Problems 9 and 10 if the angle θ is 15°.
12. Figure P3–12 shows a small truss spanning between solid supports and suspending a 10.5 kN load. The cross sections for the three main types of truss members are shown. Compute the stresses in all of the members of the truss near their midpoints away from the connections. Consider all joints to be pinned.
13. The truss shown in Figure P3–13 spans a total space of 18.0 ft and carries two concentrated loads on its top chord. The members are made from standard steel angle and channel shapes as indicated in the figure. Consider all joints to be pinned. Compute the stresses in all members near their midpoints away from the connections.



(*a*) Truss with load applied to joint B.

(*b*) Cross section of members *AB, BC*

(*c*) Cross section of member *BD*

(*d*) Cross section of members *AD, CD*

FIGURE P3–12 (Problem 12)

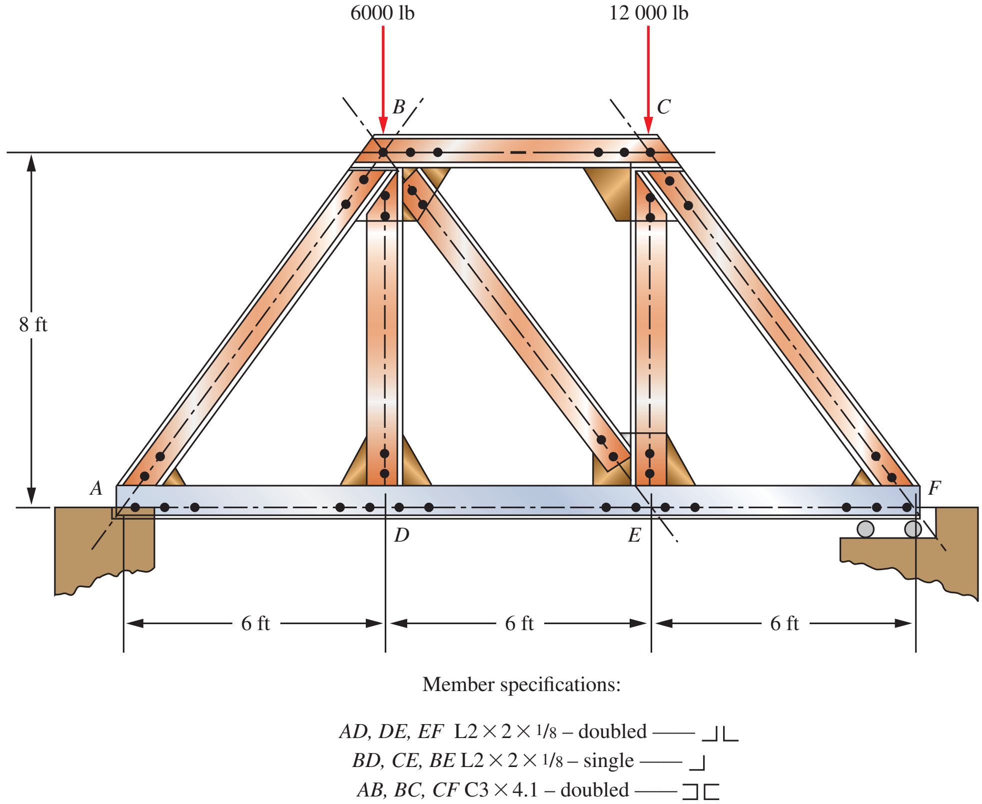


FIGURE P3–13 (Problem 13)

14. Figure P3–14 shows a short leg for a machine that carries a direct compression load. Compute the compressive stress if the cross section has the shape shown and the applied force is $F = 52\,000$ lb.
15. Consider the short compression member shown in Figure P3–15. Compute the compressive stress if the cross section has the shape shown and the applied load is 640 kN.

Direct Shear Stress

16. Refer Figure P3–8. Each of the pins at *A*, *B*, and *C* has a diameter of 0.50 in and is loaded in double shear. Compute the shear stress in each pin.
17. Compute the shear stress in the pins connecting the rods shown in Figure P3–9 when a load of $F = 1500$ lb is carried. The pins have a diameter of 0.75 in. The angle $\theta = 40°$.

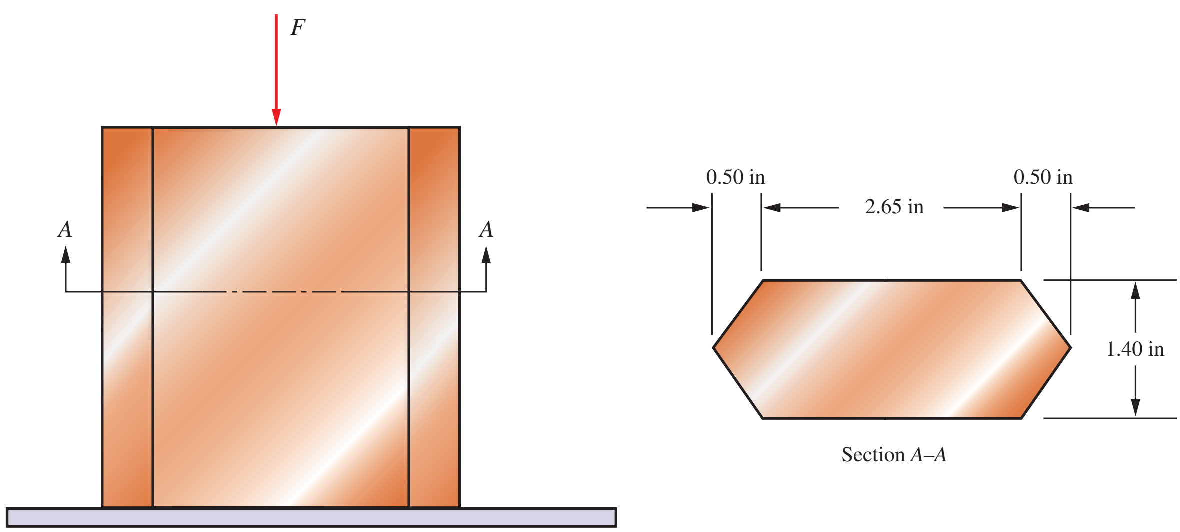


FIGURE P3–14 (Problem 14)

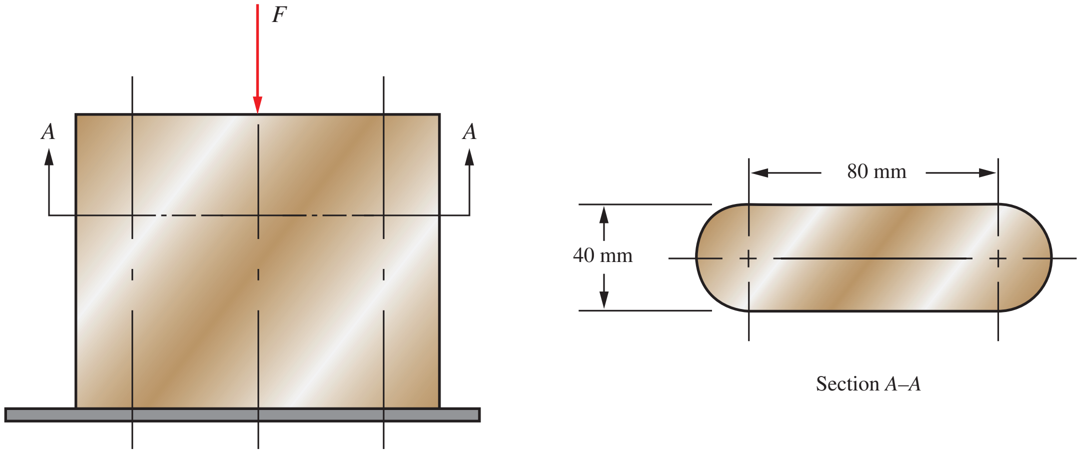


FIGURE P3–15 (Problem 15)

18. Repeat Problem 17, but change the angle to $\theta = 15°$.
19. Refer Figure 3–7. Compute the shear stress in the key if the shaft transmits a torque of 1600 N·m. The shaft diameter is 60 mm. The key is square with $b = 12$ mm, and it has a length of 45 mm.
20. A punch is attempting to cut a slug having the shape shown in Figure P3–20 from a sheet of aluminum having a thickness of 0.060 in. Compute the shearing stress in the aluminum when a force of 52 000 lb is applied by the punch.

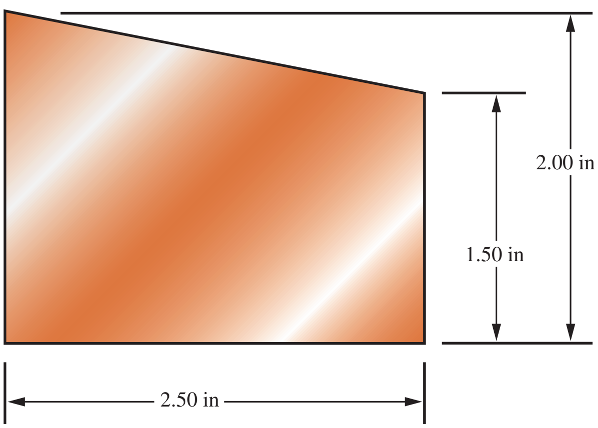


FIGURE P3–20 (Problem 20)

21. Figure P3–21 shows the shape of a slug that is to be cut from a sheet of steel having a thickness of 2.0 mm. If the punch exerts a force of 225 kN, compute the shearing stress in the steel.

Torsion

22. Compute the torsional shear stress in a circular shaft with a diameter of 50 mm that is subjected to a torque of 800 N·m.
23. If the shaft of Problem 22 is 850 mm long and is made of steel, compute the angle of twist of one end in relation to the other.
24. Compute the torsional shear stress due to a torque of 88.0 lb·in in a circular shaft having a 0.40-in diameter.

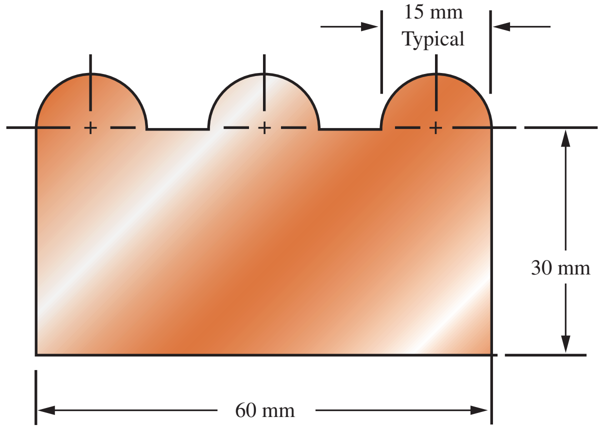


FIGURE P3–21 (Problem 21)

25. Compute the torsional shear stress in a solid circular shaft having a diameter of 1.25 in that is transmitting 110 hp at a speed of 560 rpm.
26. Compute the torsional shear stress in a hollow shaft with an outside diameter of 40 mm and an inside diameter of 30 mm when transmitting 28 kilowatts (kW) of power at a speed of 45 rad/s.
27. Compute the angle of twist for the hollow shaft of Problem 26 over a length of 400 mm. The shaft is steel.

Noncircular Members in Torsion

28. A square steel bar, 25 mm on a side and 650 mm long, is subjected to a torque of 230 N·m. Compute the shear stress and the angle of twist for the bar.
29. A 3.00 in-diameter steel bar has a flat milled on one side, as shown in Figure P3–29. If the shaft is 44.0 in long and carries a torque of 10 600 lb·in, compute the stress and the angle of twist.
30. A commercial steel supplier lists rectangular steel tubing having outside dimensions of 4.00 by 2.00 in and a wall thickness of 0.109 in. Compute the maximum torque that can be applied to such a tube if the shear stress is to be limited to 6000 psi. For this torque, compute the angle of twist of the tube over a length of 6.5 ft.

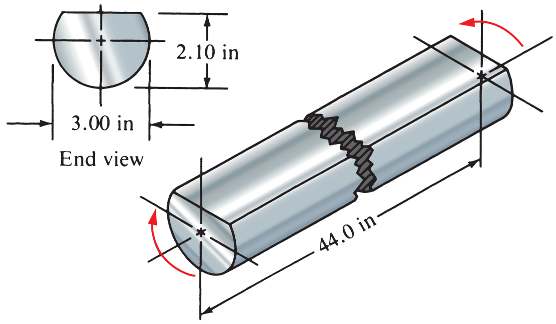


FIGURE P3–29 (Problem 29)

Beams

31. A beam is simply supported and carries the load shown in Figure P3–31. Specify suitable dimensions for the beam if it is steel and the stress is limited to 18 000 psi, for the following shapes:
(a) Square
(b) Rectangle with height three times the width
(c) Rectangle with height one-third the width
(d) Solid circular section
(e) American Standard beam section
(f) American Standard channel with the legs down
(g) Standard steel pipe

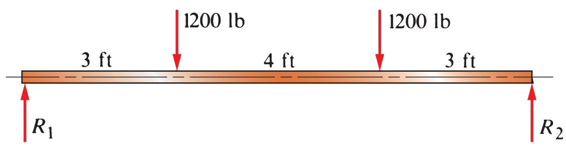


FIGURE P3–31 (Problems 31, 32, and 33)

32. For each beam of Problem 31, compute its weight if the steel weighs 0.283 lb/in^3.
33. For each beam of Problem 31, compute the maximum deflection and the deflection at the loads.
34. For the beam loading of Figure P3–34, draw the complete shearing force and bending moment diagrams, and determine the bending moments at points *A*, *B*, and *C*.
35. For the beam loading of Figure P3–34, design the beam choosing a commercially available shape in standard SI units from Appendix 15 with the smallest cross-sectional area that will limit the bending stress to 100 MPa.

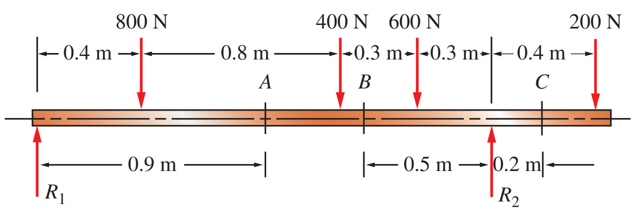


FIGURE P3–34 (Problems 34 and 35)

36. Figure P3–36 shows a beam made from 4 in schedule 40 steel pipe. Compute the deflection at points *A* and *B* for two cases: (a) the simple cantilever and (b) the supported cantilever.

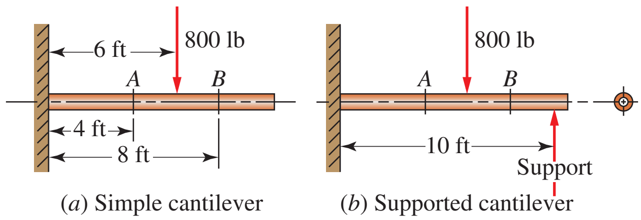


FIGURE P3–36 (Problem 36)

37. Select an aluminum I-beam shape to carry the load shown in Figure P3–37 with a maximum stress of 12 000 psi. Then compute the deflection at each load.

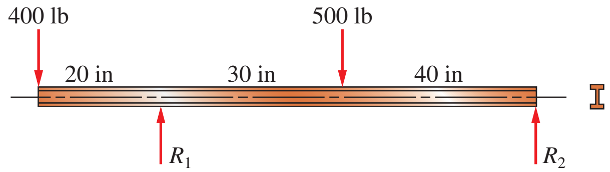


FIGURE P3–37 (Problem 37)

38. Figure P3–38 represents a wood joist for a platform, carrying a uniformly distributed load of 120 lb/ft and two concentrated loads applied by some machinery. Compute the maximum stress due to bending in the joist and the maximum vertical shear stress.

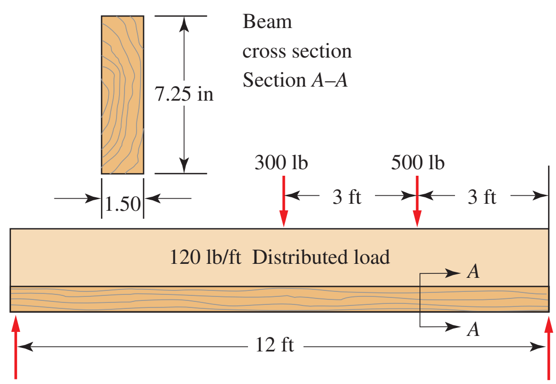


FIGURE P3–38 (Problem 38)

Beams with Concentrated Bending Moments

For Problems 39 through 50, draw the free-body diagram of only the horizontal beam portion of the given figures. Then draw the complete shear and bending moment diagrams. Where used, the symbol *X* indicates a simple support capable of exerting a reaction force in any direction but having no moment resistance. For beams having unbalanced axial loads, you may specify which support offers the reaction.

39. Use Figure P3–39.

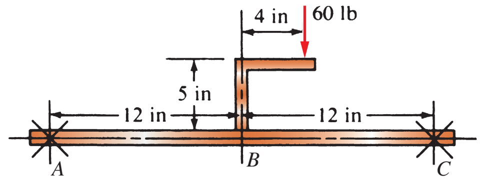


FIGURE P3–39 (Problems 39 and 57)

40. Use Figure P3–40.

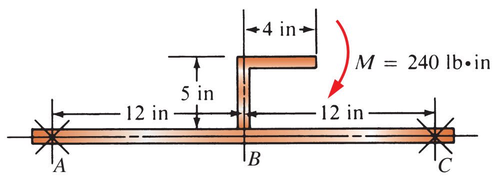


FIGURE P3–40 (Problem 40)

41. Use Figure P3–41.

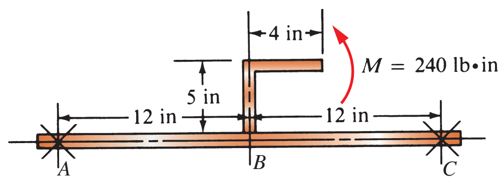


FIGURE P3–41 (Problem 41)

42. Use Figure P3–42.

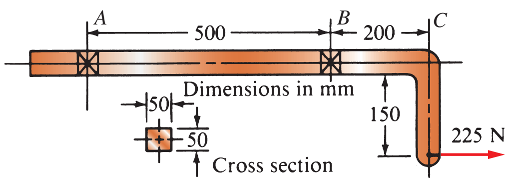


FIGURE P3–42 (Problems 42 and 58)

43. Use Figure P3–43.

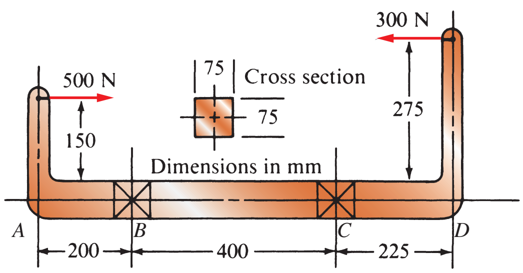


FIGURE P3–43 (Problems 43 and 59)

44. Use Figure P3–44.

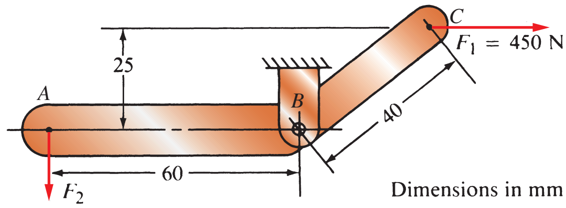


FIGURE P3–44 (Problem 44)

45. Use Figure P3–45.

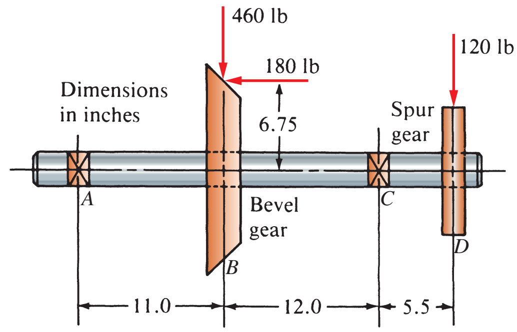


FIGURE P3–45 (Problem 45)

46. Use Figure P3–46.

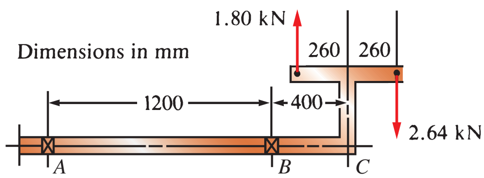


FIGURE P3–46 (Problem 46)

47. Use Figure P3–47.

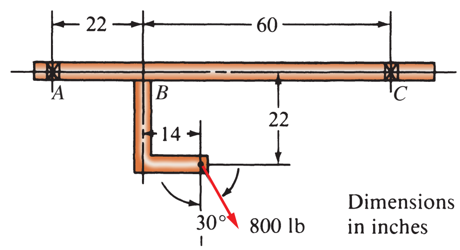


FIGURE P3–47 (Problems 47 and 60)

For Problems 48–50, draw the free-body diagram of the main shaft portion, labeled *A*, *B*, and *C*. Include any unbalanced torque on the shaft that tends to rotate it about the *z*-axis. In each case, the reaction to the unbalanced torque is taken at the right end of the shaft labeled *C*. Then draw the complete shearing force and bending moment diagrams for loading in the *y*–*z* plane. Also prepare a graph of the torque in the shaft as a function of position along the shaft from *A* to *C*.

48. Use Figure P3–48.

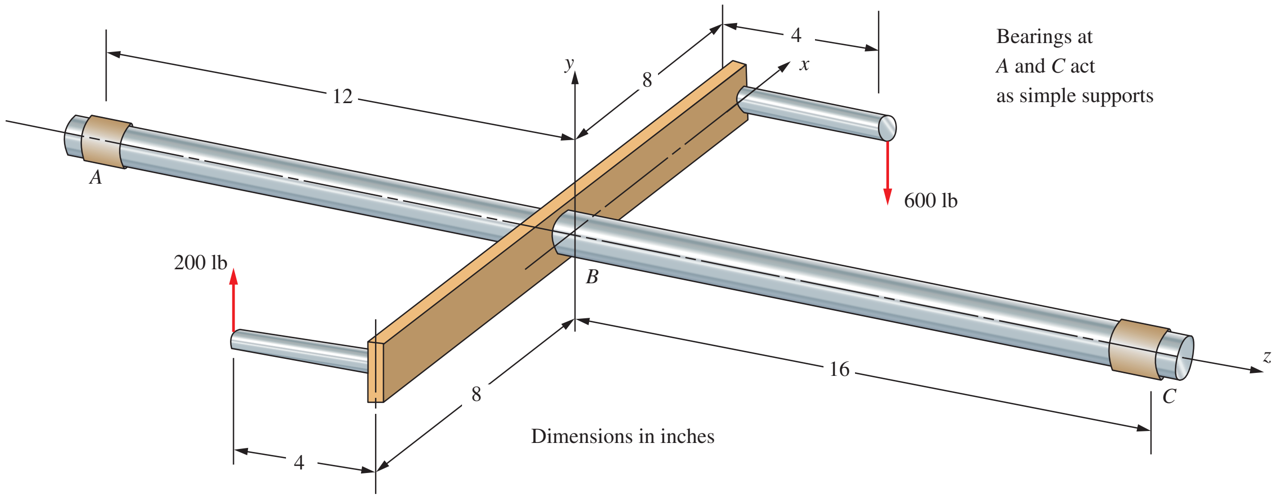


FIGURE P3–48 (Problem 48)

49. Use Figure P3–49.

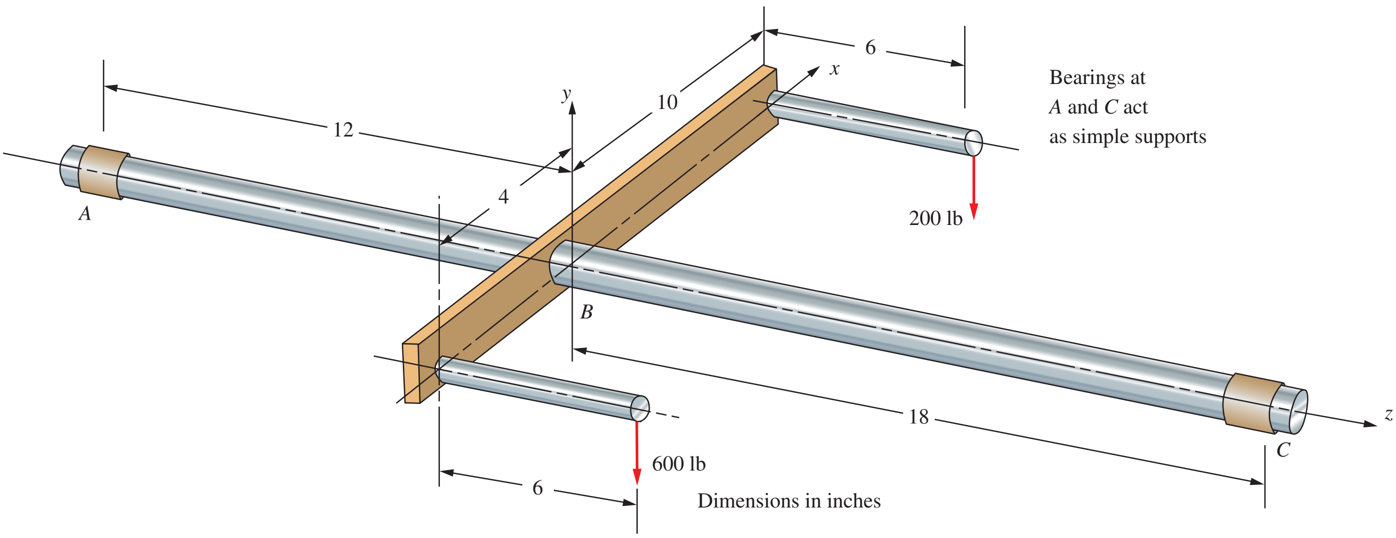


FIGURE P3–49 (Problem 49)

50. Use Figure P3–50.

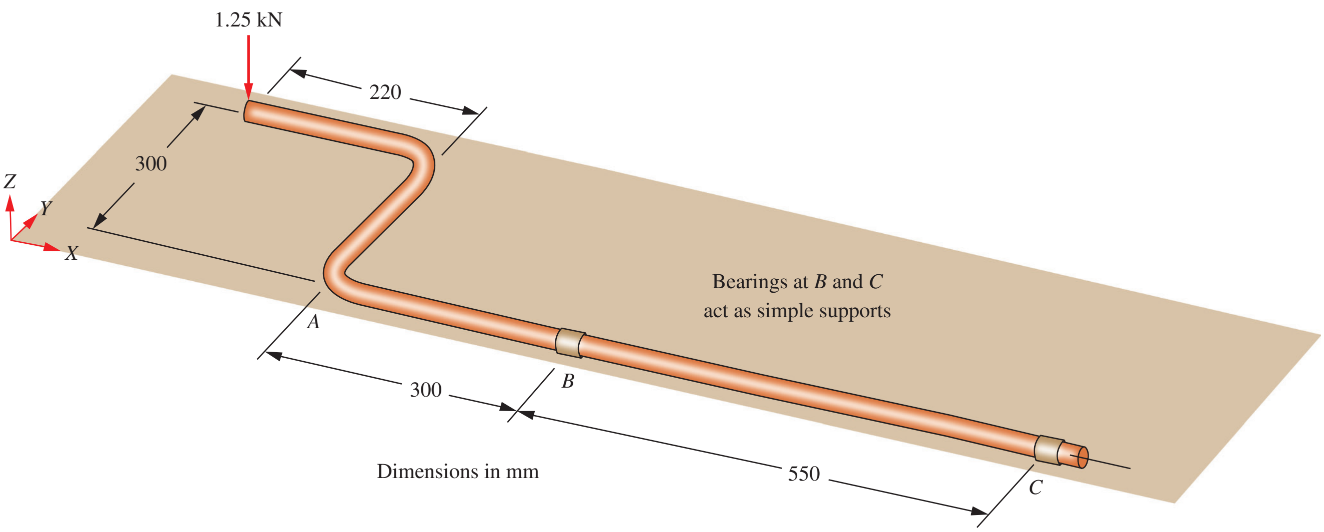


FIGURE P3–50 (Problem 50)

Combined Normal Stresses

51. Compute the maximum tensile stress in the bracket shown in Figure P3–51.

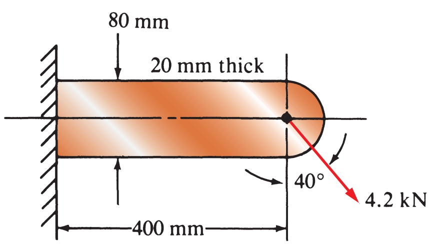


FIGURE P3–51 (Problem 51)

52. Compute the maximum tensile and compressive stresses in the horizontal beam shown in Figure P3–52.

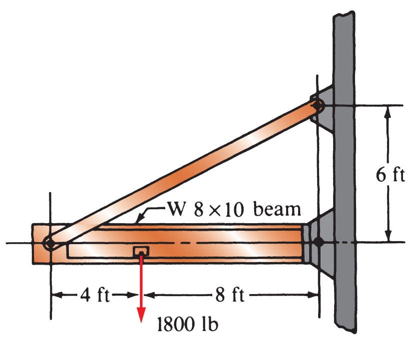


FIGURE P3–52 (Problem 52)

53. For the lever shown in Figure P3–53(a), compute the stress at section A near the fixed end. Then redesign the lever to the tapered form shown in Figure P3–5 (b) by adjusting only the height of the cross section at sections *B* and *C* so that they have no greater stress than section *A*.

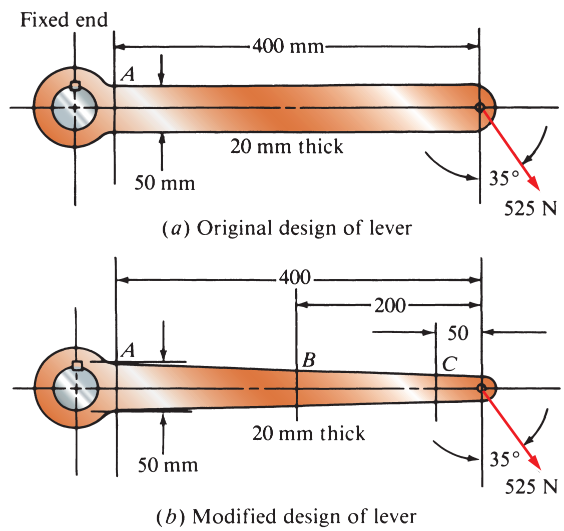


FIGURE P3–53 (Problem 53)

54. Compute the maximum tensile stress at sections *A* and *B* on the crane boom shown in Figure P3–54.
55. Refer to Figure 3–22. Compute the maximum tensile stress in the print head just to the right of the right guide. The head has a rectangular cross section, 5.0 mm high in the plane of the paper and 2.4 mm thick.
56. Refer to Figure P3–8. Compute the maximum tensile and compressive stresses in the member *B-C* if the load *F* is 1800 lb. The cross section of *B-C* is a HSS 6×4×1/4 rectangular tube.
57. Refer to P3–39. The vertical member is to be made from steel with a maximum allowable stress of 12 000 psi. Specify the required size of a standard

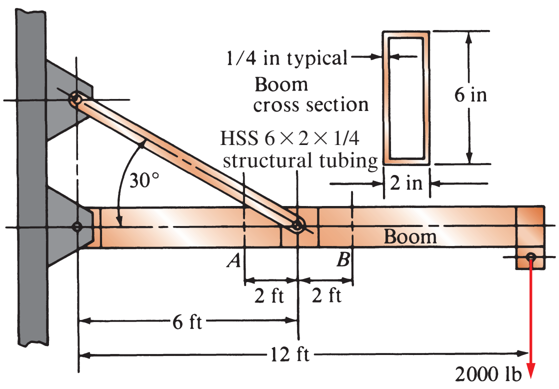


FIGURE P3–54 (Problem 54)

square cross section if sizes are available in increments of 1/16 in.

58. Refer to P3–42. Compute the maximum stress in the horizontal portion of the bar, and tell where it occurs on the cross section. The left support resists the axial force.

59. Refer to P3–43. Compute the maximum stress in the horizontal portion of the bar, and indicate where it occurs on the cross section. The right support resists the unbalanced axial force.

60. Refer to P3–47. Specify a suitable diameter for a solid circular bar to be used for the top horizontal member, which is supported in the bearings. The left bearing resists the axial load. The allowable normal stress is 25 000 psi.

Stress Concentrations

61. Figure P3–61 shows a valve stem from an engine subjected to an axial tensile load applied by the valve spring. For a force of 1.25 kN, compute the maximum stress at the fillet under the shoulder.

62. The conveyor fixture shown in Figure P3–62 carries three heavy assemblies (1200 lb each). Compute the maximum stress in the fixture, considering stress concentrations at the fillets and assuming that the load acts axially.

63. For the flat plate in tension in Figure P3–63, compute the stress at each hole, assuming that the holes are sufficiently far apart that their effects do not interact.

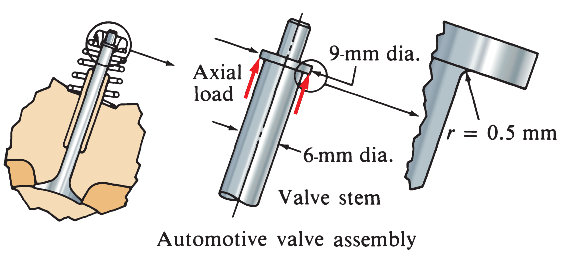


FIGURE P3–61 (Problem 61)

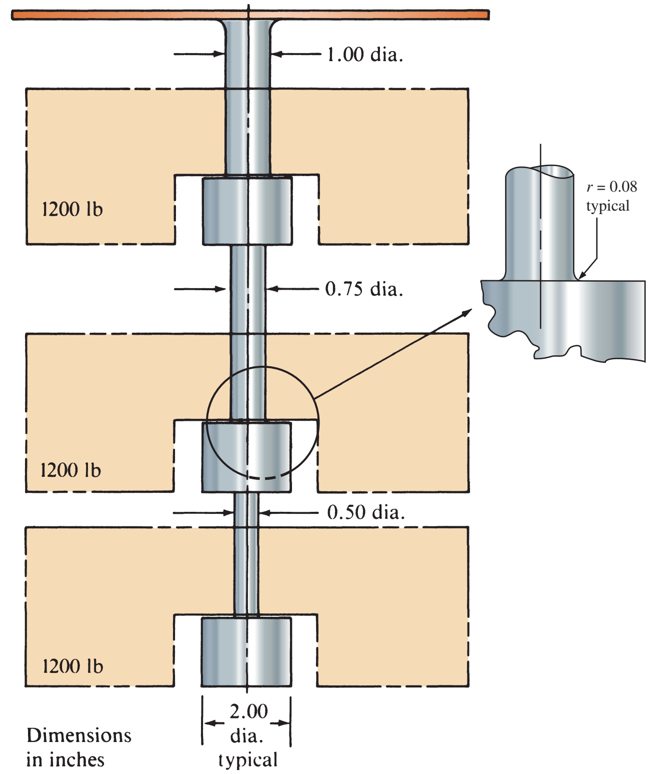


FIGURE P3–62 (Problem 62)

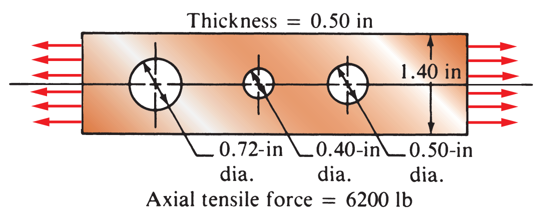


FIGURE P3–63 (Problem 63)

For Problems 64 through 68, compute the maximum stress in the member, considering stress concentrations.

64. Use Figure P3–64.

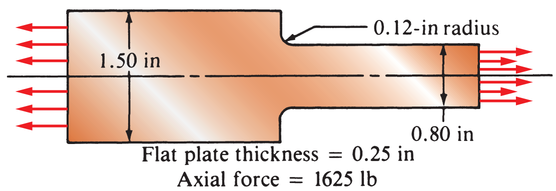


FIGURE P3–64 (Problem 64)

65. Use Figure P3–65.

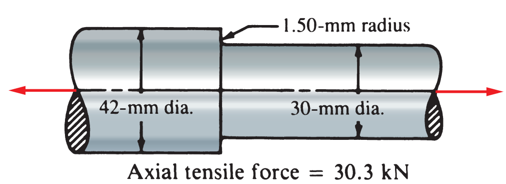


FIGURE P3–65 (Problem 65)

66. Use Figure P3–66.

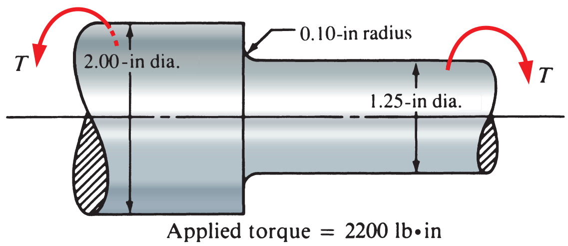


FIGURE P3–66 (Problem 66)

67. Use Figure P3–67.

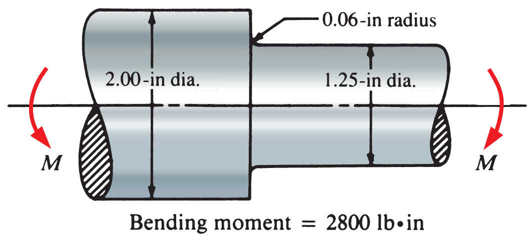


FIGURE P3–67 (Problem 67)

68. Use Figure P3–68.

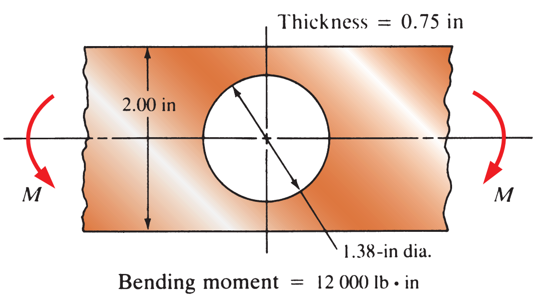


FIGURE P3–68 (Problem 68)

Problems of a General Nature

69. Figure P3–69 shows a horizontal beam supported by a vertical tension link. The cross sections of both the beam and the link are 20 mm square. All connections use 8.00 mm-diameter cylindrical pins in double shear. Compute the tensile stress in member *A-B*, the stress due to bending in *C-D*, and the shearing stress in the pins *A* and *C*.

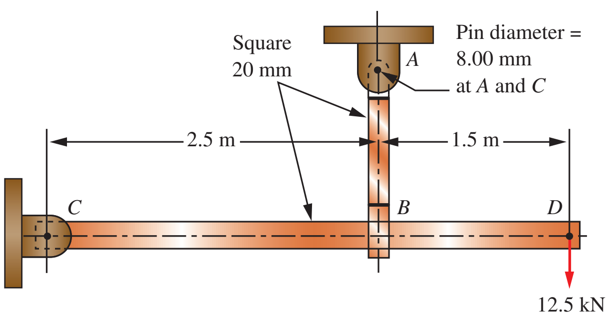


FIGURE P3–69 (Problem 69)

70. Figure P3–70 shows a tapered flat bar that has a uniform thickness of 20 mm. The depth tapers from $h_1 = 40$ mm near the load to $h_2 = 20$ mm at each support. Compute the stress due to bending in the bar at points spaced 40 mm apart from the support to the load. Let the load $P = 5.0$ kN.

71. For the flat bar shown in Figure P3–70, compute the stress in the middle of the bar if a hole of 25 mm diameter is drilled directly under the load on the horizontal centerline. The load is $P = 5.0$ kN. See data in Problem 70.

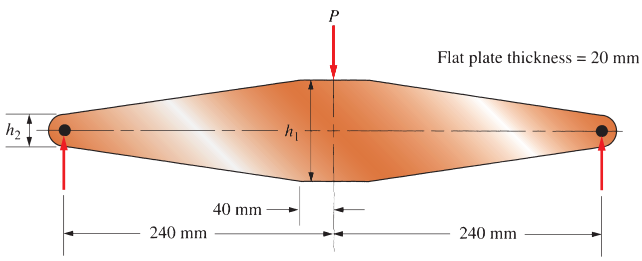


FIGURE P3–70 Tapered flat bar for Problems 70 and 71

72. The beam shown in Figure P3–72 is a stepped, flat bar having a constant thickness of 1.20 in. It carries a single concentrated load at *C* of 1500 lb. Compare the stresses at the following locations:
    (a) In the vicinity of the load
    (b) At the section through the smaller hole to the right of section *C*
    (c) At the section through the larger hole to the right of section CNear section *B* where the bar changes height

73. Figure P3–73 shows a stepped, flat bar having a constant thickness of 8.0 mm. It carries three concentrated loads as shown. Let $P = 200$ N, $L_1 = 180$ mm, $L_2 = 80$ mm, and $L_3 = 40$ mm. Compute the maximum stress due to bending, and state where it occurs. The bar is braced against lateral bending and twisting. Note that the dimensions in the figure are not drawn to scale.

74. Figure P3–74 shows a bracket carrying opposing forces of $F = 2500$ N. Compute the stress in the upper horizontal part through one of the holes as at *B*. Use $d = 15.0$ mm for the diameter of the holes.

75. Repeat Problem 74, but use a hole diameter of $d = 12.0$ mm.

76. Figure P3–76 shows a lever made from a rectangular bar of steel. Compute the stress due to bending at the fulcrum (20 in from the pivot) and at the section through the bottom hole. The diameter of each hole is 1.25 in.

77. For the lever in P3–76, determine the maximum stress if the attachment point is moved to each of the other two holes.

78. Figure P3–78 shows a shaft that is loaded only in bending. Bearings are located at points *B* and *D* to allow the shaft to rotate. Pulleys at *A*, *C*, and *E* carry cables that support loads from below while allowing

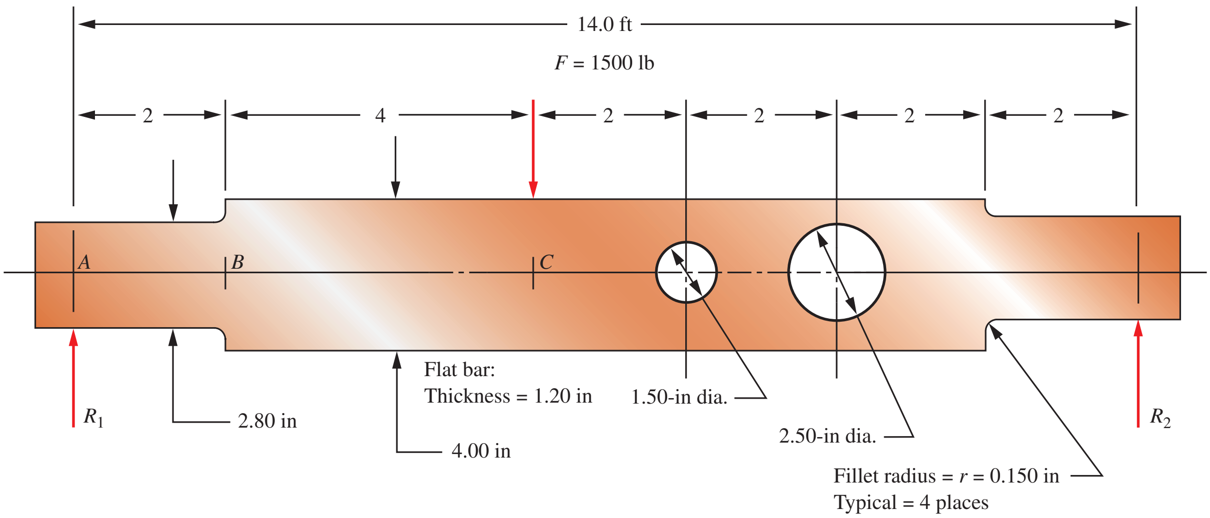


FIGURE P3–72 (Problem 72)

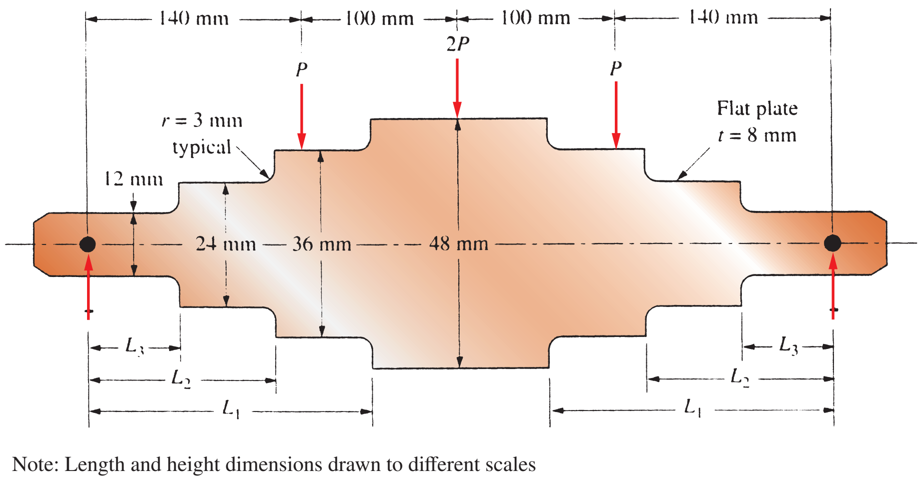


FIGURE P3–73 Stepped flat bar for Problem 73

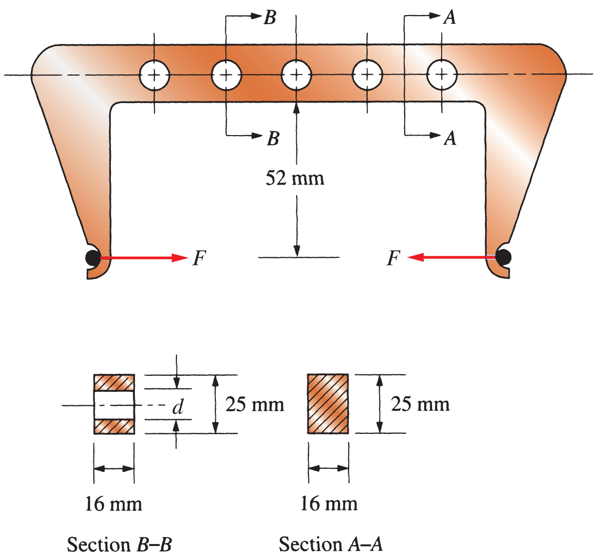


FIGURE P3–74 Bracket for Problems 74 and 75

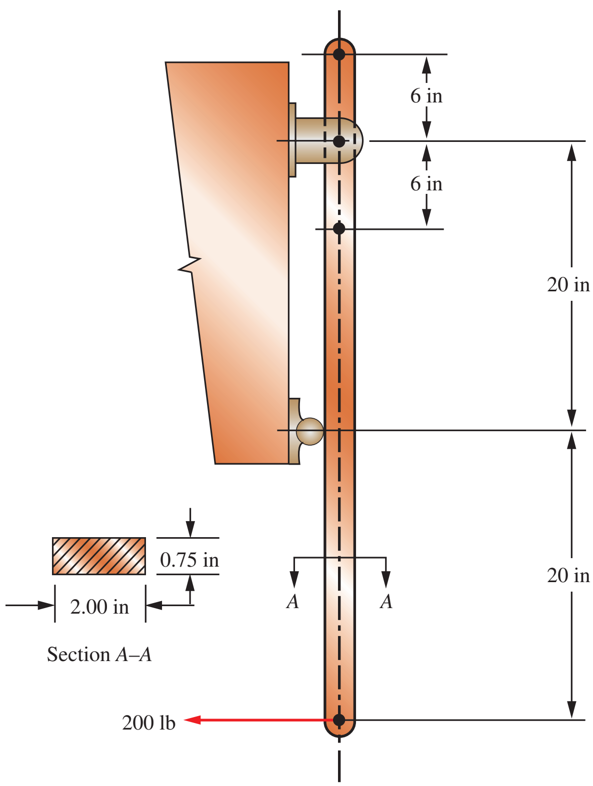


FIGURE P3–76 Lever for Problems 76 and 77

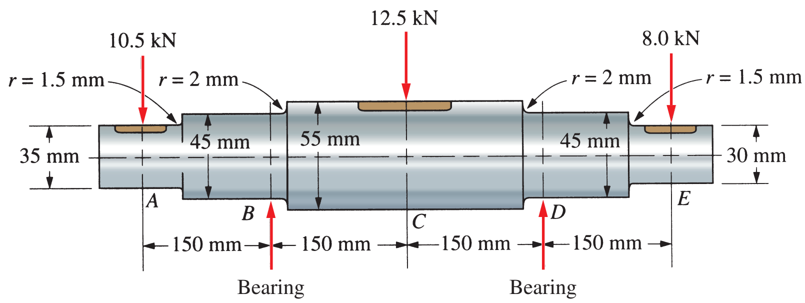


FIGURE P3–78 Data for Problem 78

the shaft to rotate. Compute the maximum stress due to bending in the shaft considering stress concentrations.

Lug Joints

79. For the vertical beam support shown in Figure P3–69, design the clevis joint at point *A* according to the recommendations described in Section 3–21 on stress concentrations. Use the given pin diameter, $d = 8.0$ mm, and lug width, $w = 20.0$ mm, using the terminology shown in Figure 3–28. The primary design decisions are the thickness of the lug, *t*, and the materials for the lug and the pin. Work toward a design factor of $N = 5$ based on ultimate strength for both tension in the lug and shearing of the pin.
80. Repeat Problem 3–79 for Joint *A* in Figure P3–8, using the data from Problem 3–8. All dimensions for the pin and the lug of the clevis are to be specified.

Curved Beams

81. A hanger is made from ASTM A36 structural steel bar with a square cross section, 10 mm on a side, as shown in Figure P3–81. The radius of curvature is 150 mm to the inside surface of the bar. Determine the load *F* that would cause yielding of the steel.

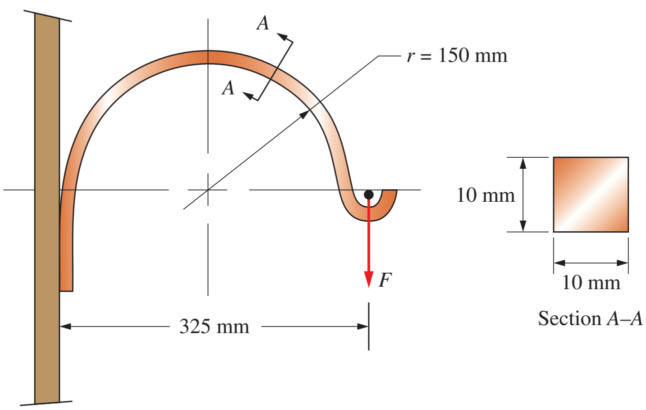


FIGURE P3–81 Hanger for Problem 3–81

82. A coping saw frame shown in Figure P3–82 is made from SAE 1020 CD steel. A screw thread in the handle draws the blade of the saw into a tension of 120 N. Determine the resulting design factor based on yield strength in the area of the corner radii of the frame.
83. Repeat Problem 3–82 for the hacksaw frame shown in Figure P3–83 when the tensile force in the blade is 480 N.

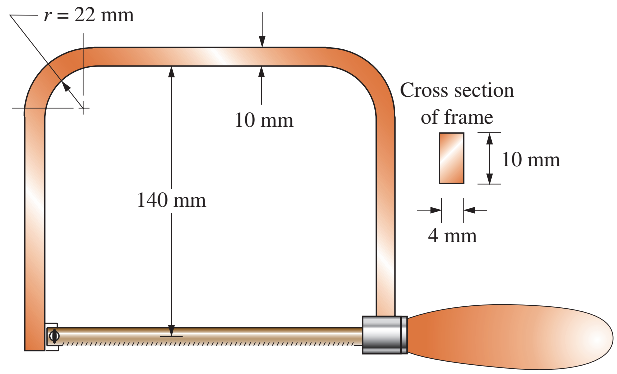


FIGURE P3–82 Coping saw frame for Problem 3–82

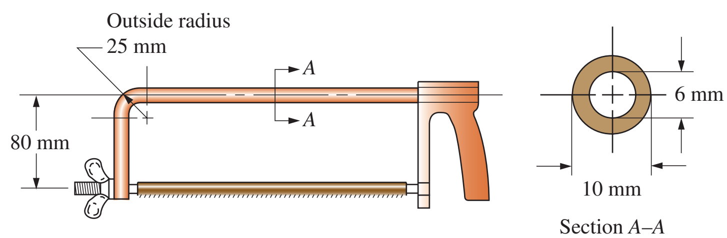


FIGURE P3–83 Hacksaw frame for Problem 3–83

84. Figure P3–84 shows a hand garden tool used to break up soil. Compute the force applied to the end of one prong that would cause yielding in the curved area. The tool is made from cast aluminum, alloy 356.0-T6.
85. Figure P3–85 shows a basketball backboard and goal attached to a steel pipe that is firmly cemented into the ground. The force, $F = 230$ lb, represents a husky player hanging from the back of the rim. Compute the design factor based on yield strength for the pipe if it is made from ASTM A53 Grade B structural steel.
86. The C-clamp in Figure P3–86 is made of cast zinc, ZA12. Determine the force that the clamp can exert for a design factor of 3 based on ultimate strength in either tension or compression.

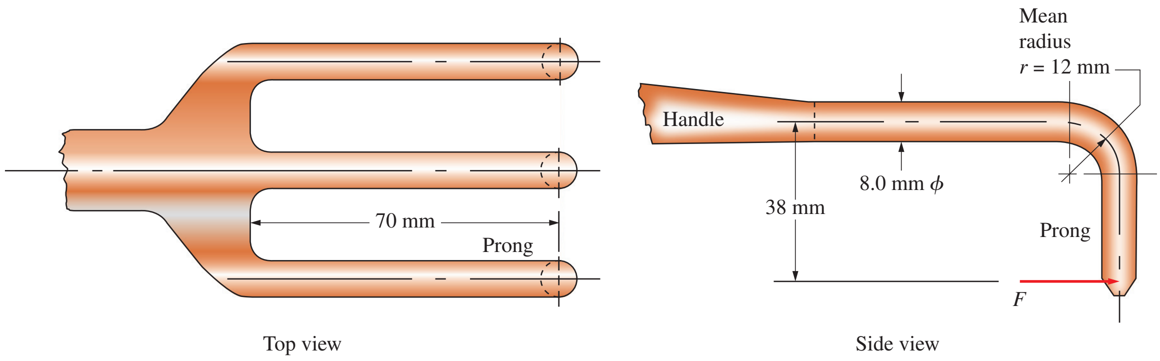


FIGURE P3–84 Garden tool for Problem 3–84

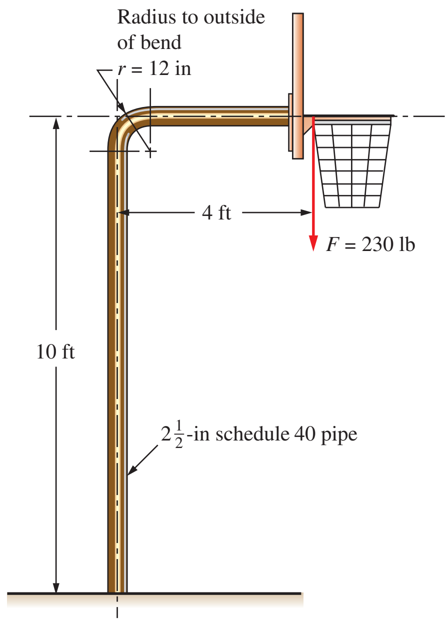


FIGURE P3–85 Basketball backboard for Problem 3–85

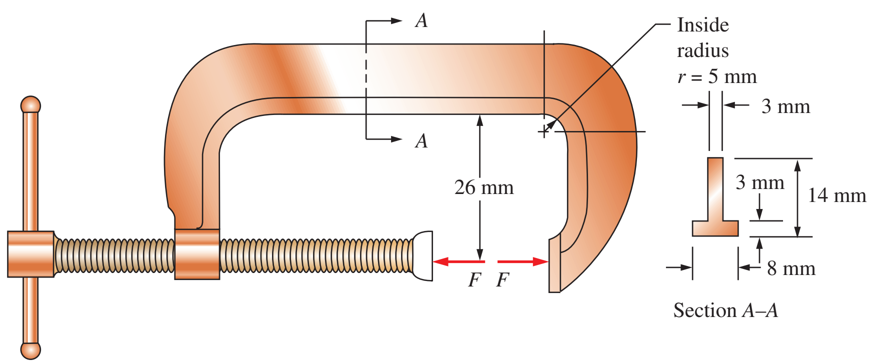


FIGURE P3–86 C-clamp for problem 3–86