The Entertainment Software Rating Board (ESRB) assigns ratings to video games to indicate the appropriate ages for players. These ratings include EC (early childhood), E (everyone), E10+ (everyone 10+), T (teen), M (mature), and AO (adults only).
In Chapter 6, you began your study of inferential statistics. There, you learned how to form a confidence interval to estimate a population parameter, such as the proportion of people in the United States who agree with a certain statement. For instance, in a nationwide poll conducted by Pew Research Center, 2001 U.S. adults were asked whether they agreed or disagreed with the statement, “People who play violent video games are more likely to be violent themselves.” Out of those surveyed, 800 adults agreed with the statement.

You have learned how to use these results to state with 95% confidence that the population proportion of U.S. adults who agree that people who play violent video games are more likely to be violent themselves is between 37.9% and 42.1%.

In this chapter, you will continue your study of inferential statistics. But now, instead of making an estimate about a population parameter, you will learn how to test a claim about a parameter.

For instance, suppose that you work for Pew Research Center and are asked to test a claim that the proportion of U.S. adults who agree that people who play violent video games are more likely to be violent themselves is \( p = 0.35 \). To test the claim, you take a random sample of \( n = 2001 \) U.S. adults and find that 800 of them think that people who play violent video games are more likely to be violent themselves. Your sample statistic is \( \hat{p} = 0.400 \).

Is your sample statistic different enough from the claim \( (p = 0.35) \) to decide that the claim is false? The answer lies in the sampling distribution of sample proportions taken from a population in which \( p = 0.35 \). The figure below shows that your sample statistic is more than 4 standard errors from the claimed value. If the claim is true, then the probability of the sample statistic being 4 standard errors or more from the claimed value is extremely small. Something is wrong! If your sample was truly random, then you can conclude that the actual proportion of the adult population is not 0.35. In other words, you tested the original claim (hypothesis), and you decided to reject it.
Introduction to Hypothesis Testing

What You Should Learn

▶ A practical introduction to hypothesis tests
▶ How to state a null hypothesis and an alternative hypothesis
▶ How to identify type I and type II errors and interpret the level of significance
▶ How to know whether to use a one-tailed or two-tailed statistical test and find a \( P \) value
▶ How to make and interpret a decision based on the results of a statistical test
▶ How to write a claim for a hypothesis test

Hypothesis Tests

A hypothesis test is a process that uses sample statistics to test a claim about the value of a population parameter. Researchers in fields such as medicine, psychology, and business rely on hypothesis testing to make informed decisions about new medicines, treatments, and marketing strategies.

For instance, consider a manufacturer that advertises its new hybrid car has a mean gas mileage of 50 miles per gallon. If you suspect that the mean mileage is not 50 miles per gallon, how could you show that the advertisement is false? Obviously, you cannot test all the vehicles, but you can still make a reasonable decision about the mean gas mileage by taking a random sample from the population of vehicles and measuring the mileage of each. If the sample mean differs enough from the advertisement’s mean, you can decide that the advertisement is wrong.

For instance, to test that the mean gas mileage of all hybrid vehicles of this type is \( \mu = 50 \) miles per gallon, you take a random sample of \( n = 30 \) vehicles and measure the mileage of each. You obtain a sample mean of \( \bar{x} = 47 \) miles per gallon with a sample standard deviation of \( s = 5.5 \) miles per gallon. Does this indicate that the manufacturer’s advertisement is false?

To decide, you do something unusual—you assume the advertisement is correct! That is, you assume that \( \mu = 50 \). Then, you examine the sampling distribution of sample means (with \( n = 30 \)) taken from a population in which \( \mu = 50 \) and \( \sigma = 5.5 \). From the Central Limit Theorem, you know this sampling distribution is normal with a mean of 50 and standard error of

\[
\frac{5.5}{\sqrt{30}} \approx 1.
\]

In the figure below, notice that the sample mean of \( \bar{x} = 47 \) miles per gallon is highly unlikely—it is about 3 standard errors (\( z = -2.99 \)) from the claimed mean! Using the techniques you studied in Chapter 5, you can determine that if the advertisement is true, then the probability of obtaining a sample mean of 47 or less is about 0.001. This is an unusual event! Your assumption that the company’s advertisement is correct has led you to an improbable result. So, either you had a very unusual sample, or the advertisement is probably false. The logical conclusion is that the advertisement is probably false.

Study Tip

As you study this chapter, do not get confused regarding concepts of certainty and importance. For instance, even if you were very certain that the mean gas mileage of a type of hybrid vehicle is not 50 miles per gallon, the actual mean mileage might be very close to this value and the difference might not be important.
Stating a Hypothesis

A statement about a population parameter is called a statistical hypothesis. To test a population parameter, you should carefully state a pair of hypotheses—one that represents the claim and the other, its complement. When one of these hypotheses is false, the other must be true. Either hypothesis—the null hypothesis or the alternative hypothesis—may represent the original claim.

1. A null hypothesis $H_0$ is a statistical hypothesis that contains a statement of equality, such as $\leq$, $=$, or $\geq$.
2. The alternative hypothesis $H_a$ is the complement of the null hypothesis. It is a statement that must be true if $H_0$ is false and it contains a statement of strict inequality, such as $>$, $\neq$, or $<$. The symbol $H_0$ is read as “H sub-zero” or “H naught” and $H_a$ is read as “H sub-a.”

To write the null and alternative hypotheses, translate the claim made about the population parameter from a verbal statement to a mathematical statement. Then, write its complement. For instance, if the claim value is $k$ and the population parameter is $m$, then some possible pairs of null and alternative hypotheses are

\[
\begin{align*}
H_0 &: m \geq k, \\
H_a &: m < k
\end{align*}
\]

Regardless of which of the three pairs of hypotheses you use, you always assume $m = k$ and examine the sampling distribution on the basis of this assumption. Within this sampling distribution, you will determine whether or not a sample statistic is unusual.

The table shows the relationship between possible verbal statements about the parameter $m$ and the corresponding null and alternative hypotheses. Similar statements can be made to test other population parameters, such as $p$, $\sigma$, or $\sigma^2$.

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<thead>
<tr>
<th>Verbal Statement $H_0$</th>
<th>Mathematical Statements</th>
<th>Verbal Statement $H_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>The mean is . . .</td>
<td></td>
<td>The mean is . . .</td>
</tr>
<tr>
<td>. . . greater than or equal to $k$.</td>
<td>$H_0: \mu \geq k, H_a: \mu &lt; k$</td>
<td>. . . less than $k$.</td>
</tr>
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<td>. . . below $k$.</td>
</tr>
<tr>
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<td></td>
<td>. . . fewer than $k$.</td>
</tr>
<tr>
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</tr>
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<td></td>
<td>. . . longer than $k$.</td>
</tr>
<tr>
<td>. . . equal to $k$.</td>
<td>$H_0: \mu = k, H_a: \mu \neq k$</td>
<td>. . . not equal to $k$.</td>
</tr>
<tr>
<td>. . . k.</td>
<td></td>
<td>. . . different from $k$.</td>
</tr>
<tr>
<td>. . . exactly $k$.</td>
<td></td>
<td>. . . not $k$.</td>
</tr>
<tr>
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<td></td>
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<tr>
<td>. . . k.</td>
<td></td>
<td>. . . different from $k$.</td>
</tr>
<tr>
<td>. . . exactly $k$.</td>
<td></td>
<td>. . . not $k$.</td>
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<tr>
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<td></td>
<td>. . . different from $k$.</td>
</tr>
<tr>
<td>. . . not changed from $k$.</td>
<td>$H_0: \mu = k, H_a: \mu \neq k$</td>
<td>. . . changed from $k$.</td>
</tr>
</tbody>
</table>
Note to Instructor

Begin with a hypothesis statement and ask students to state its logical complement. Some students will have difficulty with the fact that the complement of \( \mu \neq k \) is \( \mu = k \). Discuss the role of a double negative in English. The important point is that if you conclude that \( H_0 \) is false, then you are also concluding that \( H_a \) is true.

### EXAMPLE 1

**Stating the Null and Alternative Hypotheses**

Write each claim as a mathematical statement. State the null and alternative hypotheses, and identify which represents the claim.

1. A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%.
2. A car dealership announces that the mean time for an oil change is less than 15 minutes.
3. A company advertises that the mean life of its furnaces is more than 18 years.

**SOLUTION**

1. The claim “the proportion . . . is 61%” can be written as \( p = 0.61 \). Its complement is \( p \neq 0.61 \), as shown in the figure at the left. Because \( p = 0.61 \) contains the statement of equality, it becomes the null hypothesis. In this case, the null hypothesis represents the claim. You can write the null and alternative hypotheses as shown.

   \[
   H_0: p = 0.61 \quad \text{(Claim)}
   \]
   \[
   H_a: p \neq 0.61
   \]

2. The claim “the mean . . . is less than 15 minutes” can be written as \( \mu < 15 \). Its complement is \( \mu \geq 15 \), as shown in the figure at the left. Because \( \mu \geq 15 \) contains the statement of equality, it becomes the null hypothesis. In this case, the alternative hypothesis represents the claim. You can write the null and alternative hypotheses as shown.

   \[
   H_0: \mu \geq 15 \text{ minutes} \quad \text{(Claim)}
   \]
   \[
   H_a: \mu < 15 \text{ minutes}
   \]

3. The claim “the mean . . . is more than 18 years” can be written as \( \mu > 18 \). Its complement is \( \mu \leq 18 \), as shown in the figure at the left. Because \( \mu \leq 18 \) contains the statement of equality, it becomes the null hypothesis. In this case, the alternative hypothesis represents the claim. You can write the null and alternative hypotheses as shown.

   \[
   H_0: \mu \leq 18 \text{ years} \quad \text{(Claim)}
   \]
   \[
   H_a: \mu > 18 \text{ years}
   \]

In the three figures at the left, notice that each point on the number line is in either \( H_0 \) or \( H_a \), but no point is in both.

**TRY IT YOURSELF 1**

Write each claim as a mathematical statement. State the null and alternative hypotheses, and identify which represents the claim.

1. A consumer analyst reports that the mean life of a certain type of automobile battery is not 74 months.
2. An electronics manufacturer publishes that the variance of the life of its home theater systems is less than or equal to 2.7.
3. A realtor publicizes that the proportion of homeowners who feel their house is too small for their family is more than 24%.

Answer: Page A36

In Example 1, notice that the claim is represented by either the null hypothesis or the alternative hypothesis.
Types of Errors and Level of Significance

No matter which hypothesis represents the claim, you always begin a hypothesis test by assuming that the equality condition in the null hypothesis is true. So, when you perform a hypothesis test, you make one of two decisions:

1. reject the null hypothesis
   or
2. fail to reject the null hypothesis.

Because your decision is based on a sample rather than the entire population, there is always the possibility you will make the wrong decision.

For instance, you claim that a coin is not fair. To test your claim, you toss the coin 100 times and get 49 heads and 51 tails. You would probably agree that you do not have enough evidence to support your claim. Even so, it is possible that the coin is actually not fair and you had an unusual sample.

But then you toss the coin 100 times and get 21 heads and 79 tails. It would be a rare occurrence to get only 21 heads out of 100 tosses with a fair coin. So, you probably have enough evidence to support your claim that the coin is not fair. However, you cannot be 100% sure. It is possible that the coin is fair and you had an unusual sample.

Letting \( p \) represent the proportion of heads, the claim that “the coin is not fair” can be written as the mathematical statement \( p \neq 0.5 \). Its complement, “the coin is fair,” is written as \( p = 0.5 \), as shown in the figure.

So, the null hypothesis is

\[ H_0: p = 0.5 \]

and the alternative hypothesis is

\[ H_a: p \neq 0.5. \text{ (Claim)} \]

Remember, the only way to be absolutely certain of whether \( H_0 \) is true or false is to test the entire population. Because your decision—to reject \( H_0 \) or to fail to reject \( H_0 \)—is based on a sample, you must accept the fact that your decision might be incorrect. You might reject a null hypothesis when it is actually true. Or, you might fail to reject a null hypothesis when it is actually false. These types of errors are summarized in the next definition.

**Definition**

A **type I error** occurs if the null hypothesis is rejected when it is true.
A **type II error** occurs if the null hypothesis is not rejected when it is false.

The table shows the four possible outcomes of a hypothesis test.

<table>
<thead>
<tr>
<th>Truth of ( H_0 )</th>
<th>( H_0 ) is true.</th>
<th>( H_0 ) is false.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Decision</strong></td>
<td><strong>Do not reject ( H_0 )</strong></td>
<td><strong>Reject ( H_0 )</strong></td>
</tr>
<tr>
<td>Correct decision</td>
<td>Type II error</td>
<td>Type I error</td>
</tr>
<tr>
<td>Correct decision</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER 7 Hypothesis Testing with One Sample

Hypothesis testing is sometimes compared to the legal system used in the United States. Under this system, these steps are used.

1. A carefully worded accusation is written.
2. The defendant is assumed innocent \((H_0)\) until proven guilty. The burden of proof lies with the prosecution. If the evidence is not strong enough, then there is no conviction. A “not guilty” verdict does not prove that a defendant is innocent.
3. The evidence needs to be conclusive beyond a reasonable doubt. The system assumes that more harm is done by convicting the innocent (type I error) than by not convicting the guilty (type II error).

The table at the left shows the four possible outcomes.

<table>
<thead>
<tr>
<th>Verdict</th>
<th>innocent</th>
<th>guilty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not guilty</td>
<td>Justice</td>
<td>Type II error</td>
</tr>
<tr>
<td>Guilty</td>
<td>Type I error</td>
<td>Justice</td>
</tr>
</tbody>
</table>

### Identifying Type I and Type II Errors

The USDA limit for salmonella contamination for ground beef is 7.5%. A meat inspector reports that the ground beef produced by a company exceeds the USDA limit. You perform a hypothesis test to determine whether the meat inspector's claim is true. When will a type I or type II error occur? Which error is more serious? *(Source: U.S. Department of Agriculture)*

**Solution**

Let \(p\) represent the proportion of the ground beef that is contaminated. The meat inspector’s claim is “more than 7.5% is contaminated.” You can write the null hypothesis as

\[ H_0: p \leq 0.075 \quad \text{The proportion is less than or equal to 0.075.} \]

and the alternative hypothesis is

\[ H_a: p > 0.075. \quad \text{(Claim) The proportion is greater than 0.075.} \]

You can visualize the null and alternative hypotheses using a number line, as shown below.

```
0.055 0.065 0.075 0.085 0.095

H_0: p \leq 0.075
H_a: p > 0.075
```

A type I error will occur when the actual proportion of contaminated ground beef is less than or equal to 0.075, but you reject \(H_0\). A type II error will occur when the actual proportion of contaminated ground beef is greater than 0.075, but you do not reject \(H_0\). With a type I error, you might create a health scare and hurt the sales of ground beef producers who were actually meeting the USDA limits. With a type II error, you could be allowing ground beef that exceeded the USDA contamination limit to be sold to consumers. A type II error is more serious because it could result in sickness or even death.

**Try It Yourself 2**

A company specializing in parachute assembly states that its main parachute failure rate is not more than 1%. You perform a hypothesis test to determine whether the company’s claim is false. When will a type I or type II error occur? Which error is more serious?

*Answer: Page A36*
You will reject the null hypothesis when the sample statistic from the sampling distribution is unusual. You have already identified unusual events to be those that occur with a probability of 0.05 or less. When statistical tests are used, an unusual event is sometimes required to have a probability of 0.10 or less, 0.05 or less, or 0.01 or less. Because there is variation from sample to sample, there is always a possibility that you will reject a null hypothesis when it is actually true. In other words, although the null hypothesis is true, your sample statistic is determined to be an unusual event in the sampling distribution. You can decrease the probability of this happening by lowering the level of significance.

**DEFINITION**

In a hypothesis test, the **level of significance** is your maximum allowable probability of making a type I error. It is denoted by \( \alpha \), the lowercase Greek letter alpha.

The probability of a type II error is denoted by \( \beta \), the lowercase Greek letter beta.

By setting the level of significance at a small value, you are saying that you want the probability of rejecting a true null hypothesis to be small. Three commonly used levels of significance are

\[
\alpha = 0.10, \quad \alpha = 0.05, \quad \text{and} \quad \alpha = 0.01.
\]

**Statistical Tests and P-Values**

After stating the null and alternative hypotheses and specifying the level of significance, the next step in a hypothesis test is to obtain a random sample from the population and calculate the sample statistic (such as \( \bar{x} \), \( \hat{p} \), or \( s^2 \)) corresponding to the parameter in the null hypothesis (such as \( \mu \), \( p \), or \( \sigma^2 \)). This sample statistic is called the **test statistic**. With the assumption that the null hypothesis is true, the test statistic is then converted to a **standardized test statistic**, such as \( z \), \( t \), or \( \chi^2 \). The standardized test statistic is used in making the decision about the null hypothesis.

In this chapter, you will learn about several one-sample statistical tests. The table shows the relationships between population parameters and their corresponding test statistics and standardized test statistics.

<table>
<thead>
<tr>
<th>Population parameter</th>
<th>Test statistic</th>
<th>Standardized test statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>( \bar{x} )</td>
<td>( z ) (Section 7.2, ( \sigma ) known), ( t ) (Section 7.3, ( \sigma ) unknown)</td>
</tr>
<tr>
<td>( p )</td>
<td>( \hat{p} )</td>
<td>( z ) (Section 7.4)</td>
</tr>
<tr>
<td>( \sigma^2 )</td>
<td>( s^2 )</td>
<td>( \chi^2 ) (Section 7.5)</td>
</tr>
</tbody>
</table>

One way to decide whether to reject the null hypothesis is to determine whether the probability of obtaining the standardized test statistic (or one that is more extreme) is less than the level of significance.

**DEFINITION**

If the null hypothesis is true, then a **P-value** (or **probability value**) of a hypothesis test is the probability of obtaining a sample statistic with a value as extreme or more extreme than the one determined from the sample data.
The $P$-value of a hypothesis test depends on the nature of the test. There
are three types of hypothesis tests—left-tailed, right-tailed, and two-tailed.
The type of test depends on the location of the region of the sampling
distribution that favors a rejection of $H_0$. This region is indicated by the
alternative hypothesis.

1. If the alternative hypothesis $H_a$ contains the less-than inequality symbol
   ($<$), then the hypothesis test is a **left-tailed test**.

   \[
   H_0: \mu \geq k \\
   H_a: \mu < k
   \]

   $P$ is the area to the left of the standardized test statistic.

   ![Left-Tailed Test Diagram]

2. If the alternative hypothesis $H_a$ contains the greater-than inequality
   symbol ($>$), then the hypothesis test is a **right-tailed test**.

   \[
   H_0: \mu \leq k \\
   H_a: \mu > k
   \]

   $P$ is the area to the right of the standardized test statistic.

   ![Right-Tailed Test Diagram]

3. If the alternative hypothesis $H_a$ contains the not-equal-to symbol ($\neq$),
   then the hypothesis test is a **two-tailed test**. In a two-tailed test, each tail
   has an area of $\frac{1}{2}P$.

   \[
   H_0: \mu = k \\
   H_a: \mu \neq k
   \]

   The area to the left of the negative standardized test statistic is $\frac{1}{2}P$.
   The area to the right of the positive standardized test statistic is $\frac{1}{2}P$.

   ![Two-Tailed Test Diagram]

The smaller the $P$-value of the test, the more evidence there is to reject the
null hypothesis. A very small $P$-value indicates an unusual event. Remember,
however, that even a very low $P$-value does not constitute proof that the null
hypothesis is false, only that it is probably false.
SECTION 7.1 Introduction to Hypothesis Testing

**EXAMPLE 3**

**Identifying the Nature of a Hypothesis Test**

For each claim, state $H_0$ and $H_a$ in words and in symbols. Then determine whether the hypothesis test is a left-tailed test, right-tailed test, or two-tailed test. Sketch a normal sampling distribution and shade the area for the $P$-value.

1. A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%.

2. A car dealership announces that the mean time for an oil change is less than 15 minutes.

3. A company advertises that the mean life of its furnaces is more than 18 years.

**SOLUTION**

**In Symbols**

**In Words**

1. $H_0: p = 0.61$ The proportion of students who are involved in at least one extracurricular activity is 61%.

   $H_a: p \neq 0.61$ The proportion of students who are involved in at least one extracurricular activity is not 61%.

   Because $H_a$ contains the $\neq$ symbol, the test is a two-tailed hypothesis test. The figure at the left shows the normal sampling distribution with a shaded area for the $P$-value.

2. $H_0: \mu \geq 15 \text{ min}$ The mean time for an oil change is greater than or equal to 15 minutes.

   $H_a: \mu < 15 \text{ min}$ The mean time for an oil change is less than 15 minutes.

   Because $H_a$ contains the $<$ symbol, the test is a left-tailed hypothesis test. The figure at the left shows the normal sampling distribution with a shaded area for the $P$-value.

3. $H_0: \mu \leq 18 \text{ yr}$ The mean life of the furnaces is less than or equal to 18 years.

   $H_a: \mu > 18 \text{ yr}$ The mean life of the furnaces is more than 18 years.

   Because $H_a$ contains the $>$ symbol, the test is a right-tailed hypothesis test. The figure at the left shows the normal sampling distribution with a shaded area for the $P$-value.

**TRY IT YOURSELF 3**

For each claim, state $H_0$ and $H_a$ in words and in symbols. Then determine whether the hypothesis test is a left-tailed test, right-tailed test, or two-tailed test. Sketch a normal sampling distribution and shade the area for the $P$-value.

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2. An electronics manufacturer publishes that the variance of the life of its home theater systems is less than or equal to 2.7.

3. A realtor publicizes that the proportion of homeowners who feel their house is too small for their family is more than 24%.

Answer: Page A36
Making a Decision and Interpreting the Decision

To conclude a hypothesis test, you make a decision and interpret that decision. For any hypothesis test, there are two possible outcomes: (1) reject the null hypothesis or (2) fail to reject the null hypothesis. To decide to reject \( H_0 \) or fail to reject \( H_0 \), you can use the following decision rule.

**Decision Rule Based on \( P \)-Value**

To use a \( P \)-value to make a decision in a hypothesis test, compare the \( P \)-value with \( \alpha \).

1. If \( P \leq \alpha \), then reject \( H_0 \).
2. If \( P > \alpha \), then fail to reject \( H_0 \).

Failing to reject the null hypothesis does not mean that you have accepted the null hypothesis as true. It simply means that there is not enough evidence to reject the null hypothesis. To support a claim, state it so that it becomes the alternative hypothesis. To reject a claim, state it so that it becomes the null hypothesis. The table will help you interpret your decision.

<table>
<thead>
<tr>
<th>Claim</th>
<th>( \text{Claim is } H_0 )</th>
<th>( \text{Claim is } H_a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reject ( H_0 )</td>
<td>There is enough evidence to reject the claim.</td>
<td>There is enough evidence to support the claim.</td>
</tr>
<tr>
<td>Fail to reject ( H_0 )</td>
<td>There is not enough evidence to reject the claim.</td>
<td>There is not enough evidence to support the claim.</td>
</tr>
</tbody>
</table>

**EXAMPLE 4**

Interpreting a Decision

You perform a hypothesis test for each claim. How should you interpret your decision if you reject \( H_0 \)? If you fail to reject \( H_0 \)?

1. \( H_0 \) (Claim): A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%.
2. \( H_a \) (Claim): A car dealership announces that the mean time for an oil change is less than 15 minutes.

**SOLUTION**

1. The claim is represented by \( H_0 \). If you reject \( H_0 \), then you should conclude “there is enough evidence to reject the school’s claim that the proportion of students who are involved in at least one extracurricular activity is 61%.” If you fail to reject \( H_0 \), then you should conclude “there is not enough evidence to reject the school’s claim that the proportion of students who are involved in at least one extracurricular activity is 61%.”

2. The claim is represented by \( H_a \), so the null hypothesis is “the mean time for an oil change is greater than or equal to 15 minutes.” If you reject \( H_0 \), then you should conclude “there is enough evidence to support the dealership’s claim that the mean time for an oil change is less than 15 minutes.” If you fail to reject \( H_0 \), then you should conclude “there is not enough evidence to support the dealership’s claim that the mean time for an oil change is less than 15 minutes.”
TRY IT YOURSELF 4

You perform a hypothesis test for each claim. How should you interpret your decision if you reject $H_0$? If you fail to reject $H_0$?

1. A consumer analyst reports that the mean life of a certain type of automobile battery is not 74 months.

2. $H_a$ (Claim): A realtor publicizes that the proportion of homeowners who feel their house is too small for their family is more than 24%.

Answer: Page A36

The general steps for a hypothesis test using $P$-values are summarized below. Note that when performing a hypothesis test, you should always state the null and alternative hypotheses before collecting data. You should not collect the data first and then create a hypothesis based on something unusual in the data.

Steps for Hypothesis Testing

1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.
   
   $H_0: \ ?$  $H_a: \ ?$

2. Specify the level of significance.
   
   $\alpha = \ ?$

3. Determine the standardized sampling distribution and sketch its graph.

This sampling distribution is based on the assumption that $H_0$ is true.

4. Calculate the test statistic and its corresponding standardized test statistic. Add it to your sketch.

5. Find the $P$-value.

6. Use this decision rule.

   Is the $P$-value less than or equal to the level of significance?  

   Yes  
   No

   Fail to reject $H_0$.  
   Reject $H_0$.

7. Write a statement to interpret the decision in the context of the original claim.

In Step 4 above, the figure shows a right-tailed test. However, the same basic steps also apply to left-tailed and two-tailed tests.
Strategies for Hypothesis Testing

In a courtroom, the strategy used by an attorney depends on whether the attorney is representing the defense or the prosecution. In a similar way, the strategy that you will use in hypothesis testing should depend on whether you are trying to support or reject a claim. Remember that you cannot use a hypothesis test to support your claim when your claim is the null hypothesis. So, as a researcher, to perform a hypothesis test where the possible outcome will support a claim, word the claim so it is the alternative hypothesis. To perform a hypothesis test where the possible outcome will reject a claim, word it so the claim is the null hypothesis.

EXAMPLE 5

Writing the Hypotheses

A medical research team is investigating the benefits of a new surgical treatment. One of the claims is that the mean recovery time for patients after the new treatment is less than 96 hours.

1. How would you write the null and alternative hypotheses when you are on the research team and want to support the claim? How should you interpret a decision that rejects the null hypothesis?

2. How would you write the null and alternative hypotheses when you are on an opposing team and want to reject the claim? How should you interpret a decision that rejects the null hypothesis?

SOLUTION

1. To answer the question, first think about the context of the claim. Because you want to support this claim, make the alternative hypothesis state that the mean recovery time for patients is less than 96 hours. So, $H_a: \mu < 96$ hours. Its complement, $H_0: \mu \geq 96$ hours, would be the null hypothesis. If you reject $H_0$, then you will support the claim that the mean recovery time is less than 96 hours.

   $H_0: \mu \geq 96$ and $H_a: \mu < 96$ (Claim)

2. First think about the context of the claim. As an opposing researcher, you do not want the recovery time to be less than 96 hours. Because you want to reject this claim, make it the null hypothesis. So, $H_0: \mu \leq 96$ hours. Its complement, $H_\alpha: \mu > 96$ hours, would be the alternative hypothesis. If you reject $H_0$, then you will reject the claim that the mean recovery time is less than or equal to 96 hours.

   $H_0: \mu \leq 96$ (Claim) and $H_\alpha: \mu > 96$

TRY IT YOURSELF 5

1. You represent a chemical company that is being sued for paint damage to automobiles. You want to support the claim that the mean repair cost per automobile is less than $650. How would you write the null and alternative hypotheses? How should you interpret a decision that rejects the null hypothesis?

2. You are on a research team that is investigating the mean temperature of adult humans. The commonly accepted claim is that the mean temperature is about 98.6°F. You want to show that this claim is false. How would you write the null and alternative hypotheses? How should you interpret a decision that rejects the null hypothesis?

Answer: Page A36
7.1 EXERCISES

Building Basic Skills and Vocabulary

1. What are the two types of hypotheses used in a hypothesis test? How are they related?
2. Describe the two types of errors possible in a hypothesis test decision.
3. What are the two decisions that you can make from performing a hypothesis test?
4. Does failing to reject the null hypothesis mean that the null hypothesis is true? Explain.

True or False? In Exercises 5–10, determine whether the statement is true or false. If it is false, rewrite it as a true statement.

5. In a hypothesis test, you assume the alternative hypothesis is true.
6. A statistical hypothesis is a statement about a sample.
7. If you decide to reject the null hypothesis, then you can support the alternative hypothesis.
8. The level of significance is the maximum probability you allow for rejecting a null hypothesis when it is actually true.
9. A large P-value in a test will favor rejection of the null hypothesis.
10. To support a claim, state it so that it becomes the null hypothesis.

Stating Hypotheses In Exercises 11–16, the statement represents a claim. Write its complement and state which is
Write its complement and state which is 11. \( \mu \leq 645 \) (claim); \( H_a: \mu > 645 \)
12. \( \mu < 128 \)
13. \( \sigma 
eq 5 \)
14. \( \sigma^2 \geq 1.2 \)
15. \( p < 0.45 \)
16. \( p = 0.21 \)

Graphical Analysis In Exercises 17–20, match the alternative hypothesis with its graph. Then state the null hypothesis and sketch its graph.
17. \( H_a: \mu > 3 \)
18. \( H_a: \mu < 3 \)
19. \( H_a: \mu \neq 3 \)
20. \( H_a: \mu > 2 \)

Identifying a Test In Exercises 21–24, determine whether the hypothesis test is left-tailed, right-tailed, or two-tailed.
21. \( H_0: \mu \leq 8.0 \) \( H_a: \mu > 8.0 \)
22. \( H_0: \sigma \geq 5.2 \) \( H_a: \sigma < 5.2 \)
23. \( H_0: \sigma^2 = 142 \) \( H_a: \sigma^2 \neq 142 \)
24. \( H_0: p = 0.25 \) \( H_a: p 
eq 0.25 \)
25. \( \mu > 8 \)  
   \( H_0: \mu \leq 8; \ H_a: \mu > 8 \) (claim)  
26. \( \sigma < 3 \)  
   \( H_0: \sigma \geq 3; \ H_a: \sigma < 3 \) (claim)  
27. \( \sigma \leq 320 \)  
   \( H_0: \sigma \leq 320; \ H_a: \sigma > 320 \)  
28. \( \mu \geq 20,000 \)  
   \( H_0: \mu \geq 20,000; \ H_a: \mu < 20,000 \)  
29. \( p = 0.73 \)  
   \( H_0: p = 0.73; \ H_a: p \neq 0.73 \)  
30. \( p = 0.52 \)  
   \( H_0: p = 0.52; \ H_a: p \neq 0.52 \)  
31. A type I error will occur when the actual proportion of new customers who return to buy their next textbook is at least 0.60, but you reject \( H_0: p \geq 0.60 \).  
   A type II error will occur when the actual proportion of new customers who return to buy their next textbook is less than 0.60, but you fail to reject \( H_0: p \geq 0.60 \).  
32. A type I error will occur when the actual noontime mean traffic flow rate is 35 cars per minute, but you reject \( H_0: \mu = 35 \).  
   A type II error will occur when the actual noontime mean traffic flow rate is not 35 cars per minute, but you fail to reject \( H_0: \mu = 35 \).  
33. A type I error will occur when the actual standard deviation of the length of time to play a game is less than or equal to 12 minutes, but you reject \( H_0: \sigma \leq 12 \).  
   A type II error will occur when the actual standard deviation of the length of time to play a game is greater than 12 minutes, but you fail to reject \( H_0: \sigma \leq 12 \).  
34. See Selected Answers, page A99.  
35. See Odd Answers, page A68.  
37. See Odd Answers, page A68.  
38. See Selected Answers, page A100.  
39. See Odd Answers, page A68.  
40. See Selected Answers, page A100.  

Using and Interpreting Concepts

Stating the Null and Alternative Hypotheses  In Exercises 25–30, write the claim as a mathematical statement. State the null and alternative hypotheses, and identify which represents the claim.

25. Tablets  A tablet manufacturer claims that the mean life of the battery for a certain model of tablet is more than 8 hours.
26. Shipping Errors  As stated by a company’s shipping department, the number of shipping errors per million shipments has a standard deviation that is less than 3.
27. Base Price of an ATV  The standard deviation of the base price of an all-terrain vehicle is no more than $320.
28. Attendance  An amusement park claims that the mean daily attendance at the park is at least 20,000 people.
29. Paying for College  According to a recent survey, 73% of college students did not use student loans to pay for college. (Source: Sallie Mae)
30. Paying for College  According to a recent survey, 52% of college students used their own income or savings to pay for college. (Source: Sallie Mae)

Identifying Type I and Type II Errors  In Exercises 31–36, describe type I and type II errors for a hypothesis test of the indicated claim.

31. Repeat Customers  A used textbook selling website claims that at least 60% of its new customers will return to buy their next textbook.
32. Flow Rate  An urban planner claims that the noontime mean traffic flow rate on a busy downtown college campus street is 35 cars per minute.
33. Chess  A local chess club claims that the length of time to play a game has a standard deviation of more than 12 minutes.
34. Video Game Systems  A researcher claims that the percentage of adults in the United States who own a video game system is not 26%.
35. Security  A campus security department publicizes that at most 25% of applicants become campus security officers.
36. Phone Repairs  A cellphone repair shop advertises that the mean cost of repairing a phone screen is less than $75.

Identifying the Nature of a Hypothesis Test  In Exercises 37–42, state \( H_0 \) and \( H_a \) in words and in symbols. Then determine whether the hypothesis test is left-tailed, right-tailed, or two-tailed. Explain your reasoning. Sketch a normal sampling distribution and shade the area for the P-value.

37. Security Alarms  A security expert claims that at least 14% of all homeowners have a home security alarm.
38. Clocks  A manufacturer of grandfather clocks claims that the mean time its clocks lose is no more than 0.02 second per day.
39. Golf  A golf analyst claims that the standard deviation of the 18-hole scores for a golfer is less than 2.1 strokes.
40. Lung Cancer  A report claims that lung cancer accounts for 25% of all cancer diagnoses. (Source: American Cancer Society)
41. **High School Graduation Rate**  A high school claims that its mean graduation rate is more than 97%.
42. **Survey** A polling organization reports that the number of responses to a survey mailed to 100,000 U.S. residents is not 100,000.

**Interpreting a Decision** In Exercises 43–48, determine whether the claim represents the null hypothesis or the alternative hypothesis. If a hypothesis test is performed, how should you interpret a decision that

(a) rejects the null hypothesis?

(b) fails to reject the null hypothesis?

43. **Swans** A scientist claims that the mean incubation period for swan eggs is less than 40 days.
44. **Affording Basic Necessities** A report claims that more than 40% of households in a New York county struggle to afford basic necessities. *(Source: Niagara Frontier Publications)*
45. **Lawn Mowers** A researcher claims that the standard deviation of the life of a brand of lawn mower is at most 2.8 years.
46. **Gas Mileage** An automotive manufacturer claims that the standard deviation for the gas mileage of one of the vehicles it manufactures is 3.9 miles per gallon.
47. **Terrorism Convictions** A report claims that at least 65% of individuals convicted of terrorism or terrorism-related offenses in the United States are foreign born. *(Source: Hannity.com)*
48. **Minimum Wage** A marketing organization claims that none of its employees are paid minimum wage.

**Writing Hypotheses: Medicine** A medical research team is investigating the mean cost of a 30-day supply of a heart medication. A pharmaceutical company thinks that the mean cost is less than $60. You want to support this claim. How would you write the null and alternative hypotheses?

**Writing Hypotheses: Transportation Network Company** A transportation network company claims that the mean travel time between two destinations is about 16 minutes. You work for one of the company's competitors and want to reject this claim. How would you write the null and alternative hypotheses?

**Writing Hypotheses: Backpack Manufacturer** A backpack manufacturer claims that the mean life of its competitor's backpacks is less than 5 years. You are asked to perform a hypothesis test to test this claim. How would you write the null and alternative hypotheses when

(a) you represent the manufacturer and want to support the claim?

(b) you represent the competitor and want to reject the claim?

**Writing Hypotheses: Internet Provider** An Internet provider is trying to gain advertising deals and claims that the mean time a customer spends online per day is greater than 28 minutes. You are asked to test this claim. How would you write the null and alternative hypotheses when

(a) you represent the Internet provider and want to support the claim?

(b) you represent a competing advertiser and want to reject the claim?
53. If you decrease $\alpha$, then you are decreasing the probability that you will reject $H_0$. Therefore, you are increasing the probability of failing to reject $H_0$. This could increase $\beta$, the probability of failing to reject $H_0$ when $H_0$ is false.

54. If $\alpha = 0$, then the null hypothesis cannot be rejected and the hypothesis test is useless.

55. Yes; If the $P$-value is less than $\alpha = 0.05$, then it is also less than $\alpha = 0.10$.

56. Not necessarily; A $P$-value less than $\alpha = 0.10$ may or may not also be less than $\alpha = 0.05$.

57. (a) Fail to reject $H_0$ because the confidence interval includes values greater than 70.
(b) Reject $H_0$ because the confidence interval is located entirely to the left of 70.
(c) Fail to reject $H_0$ because the confidence interval includes values greater than 70.

58. (a) Fail to reject $H_0$ because the confidence interval includes values less than 54.
(b) Fail to reject $H_0$ because the confidence interval includes values less than 54.
(c) Reject $H_0$ because the confidence interval is located entirely to the right of 54.

59. (a) Reject $H_0$ because the confidence interval is located entirely to the right of 0.20.
(b) Fail to reject $H_0$ because the confidence interval includes values less than 0.20.
(c) Fail to reject $H_0$ because the confidence interval includes values less than 0.20.

60. (a) Fail to reject $H_0$ because the confidence interval includes values greater than 0.73.
(b) Reject $H_0$ because the confidence interval is located entirely to the left of 0.73.
(c) Fail to reject $H_0$ because the confidence interval includes values greater than 0.73.

### Extending Concepts

53. **Getting at the Concept** Why can decreasing the probability of a type I error cause an increase in the probability of a type II error?

54. **Getting at the Concept** Explain why a level of significance of $\alpha = 0$ is not used.

55. **Writing** A null hypothesis is rejected with a level of significance of 0.05. Is it also rejected at a level of significance of 0.10? Explain.

56. **Writing** A null hypothesis is rejected with a level of significance of 0.10. Is it also rejected at a level of significance of 0.05? Explain.

### Graphical Analysis

In Exercises 57–60, you are given a null hypothesis and three confidence intervals that represent three samplings. Determine whether each confidence interval indicates that you should reject $H_0$. Explain your reasoning.

57. **Hypothesis Testing with One Sample**
SECTION 7.2 Hypothesis Testing for the Mean (σ Known)

What You Should Learn

- How to find and interpret P-values
- How to use P-values for a z-test for a mean μ when σ is known
- How to find critical values and rejection regions in the standard normal distribution
- How to use rejection regions for a z-test for a mean μ when σ is known

Using P-Values to Make Decisions ■ Using P-Values for a z-Test ■ Rejection Regions and Critical Values ■ Using Rejection Regions for a z-Test

Using P-Values to Make Decisions

In Chapter 5, you learned that when the sample size is at least 30, the sampling distribution for \( \bar{x} \) (the sample mean) is normal. In Section 7.1, you learned that a way to reach a conclusion in a hypothesis test is to use a P-value for the sample statistic, such as \( \bar{x} \). Recall that when you assume the null hypothesis is true, a P-value (or probability value) of a hypothesis test is the probability of obtaining a sample statistic with a value as extreme or more extreme than the one determined from the sample data. The decision rule for a hypothesis test based on a P-value is shown below.

Decision Rule Based on P-Value

To use a P-value to make a decision in a hypothesis test, compare the P-value with \( \alpha \).

1. If \( P \leq \alpha \), then reject \( H_0 \).
2. If \( P > \alpha \), then fail to reject \( H_0 \).

EXAMPLE 1

Interpreting a P-Value

The P-value for a hypothesis test is \( P = 0.0237 \). What is your decision when the level of significance is (1) \( \alpha = 0.05 \) and (2) \( \alpha = 0.01 \)?

SOLUTION

1. Because 0.0237 < 0.05, you reject the null hypothesis.
2. Because 0.0237 > 0.01, you fail to reject the null hypothesis.

TRY IT YOURSELF 1

The P-value for a hypothesis test is \( P = 0.0745 \). What is your decision when the level of significance is (1) \( \alpha = 0.05 \) and (2) \( \alpha = 0.10 \)?

Answer: Page A37

The lower the P-value, the more evidence there is in favor of rejecting \( H_0 \). The P-value gives you the lowest level of significance for which the sample statistic allows you to reject the null hypothesis. In Example 1, you would reject \( H_0 \) at any level of significance greater than or equal to 0.0237.

Finding the P-Value for a Hypothesis Test

After determining the hypothesis test’s standardized test statistic and the standardized test statistic’s corresponding area, do one of the following to find the P-value.

a. For a left-tailed test, \( P = (\text{Area in left tail}) \).

b. For a right-tailed test, \( P = (\text{Area in right tail}) \).

c. For a two-tailed test, \( P = 2(\text{Area in tail of standardized test statistic}) \).
**EXAMPLE 2**

Finding a $P$-Value for a Left-Tailed Test

Find the $P$-value for a left-tailed hypothesis test with a standardized test statistic of $z = -2.23$. Decide whether to reject $H_0$ when the level of significance is $\alpha = 0.01$.

**SOLUTION**

The figure at the left shows the standard normal curve with a shaded area to the left of $z = -2.23$. For a left-tailed test,

$$P = \text{(Area in left tail)}.$$  

Using Table 4 in Appendix B, the area corresponding to $z = -2.23$ is 0.0129, which is the area in the left tail. So, the $P$-value for a left-tailed hypothesis test with a standardized test statistic of $z = -2.23$ is $P = 0.0129$. You can check your answer using technology, as shown below.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NORM.DIST(-2.23,0,1,TRUE)</td>
</tr>
<tr>
<td>2</td>
<td>0.012873721</td>
</tr>
</tbody>
</table>

**Interpretation** Because the $P$-value of 0.0129 is greater than 0.01, you fail to reject $H_0$.

**TRY IT YOURSELF 2**

Find the $P$-value for a left-tailed hypothesis test with a standardized test statistic of $z = -1.71$. Decide whether to reject $H_0$ when the level of significance is $\alpha = 0.05$.

*Answer: Page A37*

**EXAMPLE 3**

Finding a $P$-Value for a Two-Tailed Test

Find the $P$-value for a two-tailed hypothesis test with a standardized test statistic of $z = 2.14$. Decide whether to reject $H_0$ when the level of significance is $\alpha = 0.05$.

**SOLUTION**

The figure at the left shows the standard normal curve with shaded areas to the left of $z = -2.14$ and to the right of $z = 2.14$. For a two-tailed test,

$$P = 2(\text{Area in tail of standardized test statistic}).$$

Using Table 4, the area corresponding to $z = 2.14$ is 0.9838. The area in the right tail is $1 - 0.9838 = 0.0162$. So, the $P$-value for a two-tailed hypothesis test with a standardized test statistic of $z = 2.14$ is

$$P = 2(0.0162) = 0.0324.$$ 

**Interpretation** Because the $P$-value of 0.0324 is less than 0.05, you reject $H_0$.

**TRY IT YOURSELF 3**

Find the $P$-value for a two-tailed hypothesis test with a standardized test statistic of $z = 1.64$. Decide whether to reject $H_0$ when the level of significance is $\alpha = 0.10$.

*Answer: Page A37*
**SECTION 7.2  Hypothesis Testing for the Mean (σ Known)**

### Using P-Values for a z-Test

You will now learn how to perform a hypothesis test for a mean μ assuming the standard deviation σ is known. When σ is known, you can use a z-test for the mean. To use the z-test, you need to find the standardized value for the test statistic x. The standardized test statistic takes the form of

\[
z = \frac{(\text{Sample mean}) - (\text{Hypothesized mean})}{\text{Standard error}}.
\]

**z-Test for a Mean μ**

The **z-test for a mean μ** is a statistical test for a population mean. The **test statistic** is the sample mean \( \bar{x} \). The **standardized test statistic** is

\[
z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}
\]

when these conditions are met.

1. The sample is random.
2. At least one of the following is true: The population is normally distributed or \( n \geq 30 \).

Recall that \( \sigma / \sqrt{n} \) is the standard error of the mean, \( \sigma _{\bar{x}} \).

### GUIDELINES

**Using P-Values for a z-Test for a Mean μ (σ Known)**

<table>
<thead>
<tr>
<th>In Words</th>
<th>In Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Verify that σ is known, the sample is random, and either the population is normally distributed or ( n \geq 30 ).</td>
<td>State ( H_0 ) and ( H_a ).</td>
</tr>
<tr>
<td>2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.</td>
<td>Identify ( \alpha ).</td>
</tr>
<tr>
<td>3. Specify the level of significance.</td>
<td></td>
</tr>
<tr>
<td>4. Find the standardized test statistic.</td>
<td>[ z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} ]</td>
</tr>
<tr>
<td>5. Find the area that corresponds to ( z ).</td>
<td>Use Table 4 in Appendix B.</td>
</tr>
<tr>
<td>6. Find the P-value.</td>
<td></td>
</tr>
<tr>
<td>a. For a left-tailed test, ( P = (\text{Area in left tail}) ).</td>
<td></td>
</tr>
<tr>
<td>b. For a right-tailed test, ( P = (\text{Area in right tail}) ).</td>
<td></td>
</tr>
<tr>
<td>c. For a two-tailed test, ( P = 2(\text{Area in tail of standardized test statistic}) ).</td>
<td></td>
</tr>
<tr>
<td>7. Make a decision to reject or fail to reject the null hypothesis.</td>
<td>If ( P \leq \alpha ), then reject ( H_0 ). Otherwise, fail to reject ( H_0 ).</td>
</tr>
<tr>
<td>8. Interpret the decision in the context of the original claim.</td>
<td></td>
</tr>
</tbody>
</table>

With all hypothesis tests, it is helpful to sketch the sampling distribution. Your sketch should include the standardized test statistic.
In auto racing, a pit stop is where a racing vehicle stops for new tires, fuel, repairs, and other mechanical adjustments. The efficiency of a pit crew that makes these adjustments can affect the outcome of a race. A pit crew claims that its mean pit stop time (for 4 new tires and fuel) is less than 13 seconds. A random sample of 32 pit stop times has a sample mean of 12.9 seconds. Assume the population standard deviation is 0.19 second. Is there enough evidence to support the claim at $\alpha = 0.01$? Use a $P$-value.

**SOLUTION**

Because $\sigma$ is known ($\sigma = 0.19$), the sample is random, and $n = 32 \geq 30$, you can use the $z$-test. The claim is “the mean pit stop time is less than 13 seconds.” So, the null and alternative hypotheses are

$H_0: \mu \geq 13$ seconds \quad and \quad $H_a: \mu < 13$ seconds. \quad (Claim)

The level of significance is $\alpha = 0.01$. The standardized test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Because $\sigma$ is known and $n \geq 30$, use the $z$-test.

$$z = \frac{12.9 - 13}{0.19 / \sqrt{32}}$$

$$\approx -2.98.$$ \quad Round to two decimal places.

Using Table 4 in Appendix B, the area corresponding to $z = -2.98$ is $0.0014$. Because this test is a left-tailed test, the $P$-value is equal to the area to the left of $z = -2.98$, as shown in the figure at the left. So, $P = 0.0014$. Because the $P$-value is less than $\alpha = 0.01$, you reject the null hypothesis. You can check your answer using technology, as shown below. Note that the $P$-value differs slightly from the one you found due to rounding.

**Interpretation** \quad There is enough evidence at the 1% level of significance to support the claim that the mean pit stop time is less than 13 seconds.

**TRY IT YOURSELF 4**

Homeowners claim that the mean speed of automobiles traveling on their street is greater than the speed limit of 35 miles per hour. A random sample of 100 automobiles has a mean speed of 36 miles per hour. Assume the population standard deviation is 4 miles per hour. Is there enough evidence to support the claim at $\alpha = 0.05$? Use a $P$-value.

*Answer: Page A37*
Hypothesis Testing Using a P-Value

According to a study of U.S. homes that use heating equipment, the mean indoor temperature at night during winter is 68.3°F. You think this information is incorrect. You randomly select 25 U.S. homes that use heating equipment in the winter and find that the mean indoor temperature at night is 67.2°F. From past studies, the population standard deviation is known to be 3.5°F and the population is normally distributed. Is there enough evidence to support your claim at $\alpha = 0.05$? Use a P-value. (Adapted from U.S. Energy Information Administration)

**SOLUTION**

Because $\sigma$ is known ($\sigma = 3.5^\circ\text{F}$), the sample is random, and the population is normally distributed, you can use the $z$-test. The claim is “the mean is different from 68.3°F.” So, the null and alternative hypotheses are

$$H_0: \mu = 68.3^\circ\text{F} \quad \text{and} \quad H_a: \mu \neq 68.3^\circ\text{F}. \quad \text{(Claim)}$$

The level of significance is $\alpha = 0.05$. The standardized test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$= \frac{67.2 - 68.3}{3.5 / \sqrt{25}}$$

$$\approx -1.57.$$ 

Round to two decimal places.

In Table 4, the area corresponding to $z = -1.57$ is 0.0582. Because the test is a two-tailed test, the $P$-value is equal to twice the area to the left of $z = -1.57$, as shown in the figure.

So, the $P$-value is $P = 2(0.0582) = 0.1164$. Because the $P$-value is greater than $\alpha = 0.05$, you fail to reject the null hypothesis.

**Interpretation** There is not enough evidence at the 5% level of significance to support the claim that the mean indoor temperature at night during winter is different from 68.3°F for U.S. homes that use heating equipment.

**TRY IT YOURSELF 5**

According to a study of employed U.S. adults ages 18 and over, the mean number of workdays missed due to illness or injury in the past 12 months is 3.5 days. You randomly select 25 employed U.S. adults ages 18 and over and find that the mean number of workdays missed is 4 days. Assume the population standard deviation is 1.5 days and the population is normally distributed. Is there enough evidence to doubt the study’s claim at $\alpha = 0.01$? Use a $P$-value. (Adapted from U.S. National Center for Health Statistics)

Answer: Page A37
CHAPTER 7  Hypothesis Testing with One Sample

Left-Tailed Test

Right-Tailed Test

Two-Tailed Test

Using Technology to Find a $P$-Value

Use the TI-84 Plus displays to make a decision to reject or fail to reject the null hypothesis at a level of significance of $\alpha = 0.05$.

TI-84 PLUS

Inpt: Data

Stats

$\mu_0$: 6.2

$s$: 0.47

$x$: 6.07

$n$: 53

$\mu$: $\neq \mu_0 < \mu_0 > \mu_0$

Calculate

TI-84 PLUS

$z$-test

$\mu \neq 6.2$

$z = -2.013647416$

$p = 0.0440464253$

$x = 6.07$

$n = 53$

SOLUTION

The $P$-value for this test is 0.0440464253. Because the $P$-value is less than $\alpha = 0.05$, you reject the null hypothesis.

TRY IT YOURSELF 6

Repeat Example 6 using a level of significance of $\alpha = 0.01$.

Answer: Page A37

Rejection Regions and Critical Values

Another method to decide whether to reject the null hypothesis is to determine whether the standardized test statistic falls within a range of values called the rejection region of the sampling distribution.

DEFINITION

A rejection region (or critical region) of the sampling distribution is the range of values for which the null hypothesis is not probable. If a standardized test statistic falls in this region, then the null hypothesis is rejected. A critical value $z_0$ separates the rejection region from the nonrejection region.

GUIDELINES

Finding Critical Values in the Standard Normal Distribution

1. Specify the level of significance $\alpha$.
2. Determine whether the test is left-tailed, right-tailed, or two-tailed.
3. Find the critical value(s) $z_0$. When the hypothesis test is
   a. left-tailed, find the $z$-score that corresponds to an area of $\alpha$.
   b. right-tailed, find the $z$-score that corresponds to an area of $1 - \frac{1}{2}\alpha$.
   c. two-tailed, find the $z$-scores that correspond to $\frac{1}{2}\alpha$ and $1 - \frac{1}{2}\alpha$.
4. Sketch the standard normal distribution. Draw a vertical line at each critical value and shade the rejection region(s). (See the figures at the left.)

Note that a standardized test statistic that falls in a rejection region is considered an unusual event.
When you cannot find the exact area in Table 4, use the area that is closest. For an area that is exactly midway between two areas in the table, use the $z$-score midway between the corresponding $z$-scores.

**EXAMPLE 7**

**Finding a Critical Value for a Left-Tailed Test**

Find the critical value and rejection region for a left-tailed test with $\alpha = 0.01$.

**SOLUTION**

The figure shows the standard normal curve with a shaded area of 0.01 in the left tail. In Table 4, the $z$-score that is closest to an area of 0.01 is $-2.33$. So, the critical value is $z_0 = -2.33$.

The rejection region is to the left of this critical value. You can check your answer using technology, as shown below.

**EXCEL**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NORM.S.INV(0.01)</td>
</tr>
<tr>
<td>2</td>
<td>-2.32634787</td>
</tr>
</tbody>
</table>

**TRY IT YOURSELF 7**

Find the critical value and rejection region for a left-tailed test with $\alpha = 0.10$.

*Answer: Page A37*

Because normal distributions are symmetric, in a two-tailed test the critical values are opposites, as shown in the next example.

**EXAMPLE 8**

**Finding Critical Values for a Two-Tailed Test**

Find the critical values and rejection regions for a two-tailed test with $\alpha = 0.05$.

**SOLUTION**

The figure shows the standard normal curve with shaded areas of $\frac{1}{2}\alpha = 0.025$ in each tail. The area to the left of $-z_0$ is $\frac{1}{2}\alpha = 0.025$, and the area to the left of $z_0$ is $1 - \frac{1}{2}\alpha = 0.975$. In Table 4, the $z$-scores that correspond to the areas 0.025 and 0.975 are $-1.96$ and $1.96$, respectively. So, the critical values are $-z_0 = -1.96$ and $z_0 = 1.96$.

The rejection regions are to the left of $-1.96$ and to the right of $1.96$.

**TRY IT YOURSELF 8**

Find the critical values and rejection regions for a two-tailed test with $\alpha = 0.08$.

*Answer: Page A37*
Using Rejection Regions for a \( z \)-Test

To conclude a hypothesis test using rejection region(s), you make a decision and interpret the decision according to the next rule.

**Decision Rule Based on Rejection Region**

To use a rejection region to conduct a hypothesis test, calculate the standardized test statistic \( z \). If the standardized test statistic

1. is in the rejection region, then reject \( H_0 \).
2. is not in the rejection region, then fail to reject \( H_0 \).

\[
\begin{align*}
z < z_0 &: \text{Reject } H_0. \\
z > z_0 &: \text{Reject } H_0. \\
z < -z_0 &: \text{Reject } H_0. \\
z > -z_0 &: \text{Reject } H_0.
\end{align*}
\]

Remember, failing to reject the null hypothesis does not mean that you have accepted the null hypothesis as true. It simply means that there is not enough evidence to reject the null hypothesis.

**GUIDELINES**

**Using Rejection Regions for a \( z \)-Test for a Mean \( \mu \) (\( \sigma \) Known)**

**In Words**

1. Verify that \( \sigma \) is known, the sample is random, and either the population is normally distributed or \( n \geq 30 \).
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.
3. Specify the level of significance.
4. Determine the critical value(s).
5. Determine the rejection region(s).
6. Find the standardized test statistic and sketch the sampling distribution.
7. Make a decision to reject or fail to reject the null hypothesis.
8. Interpret the decision in the context of the original claim.

**In Symbols**

- State \( H_0 \) and \( H_a \).
- Identify \( \alpha \).
- Use Table 4 in Appendix B.
- \( z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \)
- If \( z \) is in the rejection region, then reject \( H_0 \). Otherwise, fail to reject \( H_0 \).
Hypothesis Testing for the Mean (σ Known)

Section 7.2

Employees at a construction and mining company claim that the mean salary of the company’s mechanical engineers is less than that of one of its competitors, which is $88,200. A random sample of 20 of the company’s mechanical engineers has a mean salary of $85,900. Assume the population standard deviation is $9500 and the population is normally distributed. At \( \alpha = 0.05 \), test the employees’ claim.

SOLUTION

Because \( \sigma \) is known, the sample is random, and the population is normally distributed, you can use the \( z \) test. The claim is “the mean salary is less than $88,200.” So, the null and alternative hypotheses can be written as

\[ H_0: \mu \geq 88,200 \quad \text{and} \quad H_a: \mu < 88,200. \]  

(Claim)

Because the test is a left-tailed test and the level of significance is \( \alpha = 0.05 \), the critical value is \( z_{0.05} = -1.645 \) and the rejection region is \( z < -1.645 \).

The standardized test statistic is

\[ z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \]

Because \( \sigma \) is known and the population is normally distributed, use the \( z \) test.

\[ = \frac{85,900 - 88,200}{9500 / \sqrt{20}} \]

\[ = -1.08. \]

Round to two decimal places.

The figure shows the location of the rejection region and the standardized test statistic \( z \). Because \( z \) is not in the rejection region, you fail to reject the null hypothesis.

There is enough evidence at the 1% level of significance to conclude that the mean mileage of the station wagon is greater than 49 miles per gallon on the highway.

EXAMPLE 9

Picturing the World

Each year, the Environmental Protection Agency (EPA) publishes reports of gas mileage for all makes and models of passenger vehicles. In a recent year, the small station wagon with an automatic transmission that posted the best mileage had a mean mileage of 52 miles per gallon (city) and 49 miles per gallon (highway). An auto manufacturer claims its station wagons exceed 49 miles per gallon on the highway. To support its claim, it tests 36 vehicles on highway driving and obtains a sample mean of 51.2 miles per gallon. Assume the population standard deviation is 4.8 miles per gallon. (Source: U.S. Department of Energy)

Is the evidence strong enough to support the claim that the station wagon’s highway miles per gallon exceeds the EPA estimate? Use a \( z \)-test with \( \alpha = 0.01 \).

There is enough evidence at the 1% level of significance to conclude that the mean mileage of the station wagon is greater than 49 miles per gallon on the highway.

TRY IT YOURSELF 9

The CEO of the company in Example 9 claims that the mean workday of the company’s mechanical engineers is less than 8.5 hours. A random sample of 25 of the company’s mechanical engineers has a mean workday of 8.2 hours. Assume the population standard deviation is 0.5 hour and the population is normally distributed. At \( \alpha = 0.01 \), test the CEO’s claim.

Answer: Page A37
Hypothesis Testing Using Rejection Regions

A researcher claims that the mean annual cost of raising a child (age 2 and under) by married-couple families in the U.S. is $14,050. In a random sample of married-couple families in the U.S., the mean annual cost of raising a child (age 2 and under) is $13,795. The sample consists of 500 children. Assume the population standard deviation is $2875. At $\alpha = 0.10$, is there enough evidence to reject the claim? (Adapted from U.S. Department of Agriculture Center for Nutrition Policy and Promotion)

**SOLUTION**

Because $\sigma$ is known ($\sigma = 2875$), the sample is random, and $n = 500 \geq 30$, you can use the $z$-test. The claim is “the mean annual cost is $14,050.” So, the null and alternative hypotheses are

$$H_0: \mu = 14,050 \quad (\text{Claim}) \quad \text{and} \quad H_a: \mu \neq 14,050.$$  

Because the test is a two-tailed test and the level of significance is $\alpha = 0.10$, the critical values are $-z_0 = -1.645$ and $z_0 = 1.645$. The rejection regions are $z < -1.645$ and $z > 1.645$. The standardized test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \quad \text{Because } \sigma \text{ is known and } n \geq 30, \text{ use the } z\text{-test.}$$

$$= \frac{13,795 - 14,050}{2875/\sqrt{500}} \quad \text{Assume } \mu = 14,050.$$ 

$$\approx -1.98. \quad \text{Round to two decimal places.}$$

The figure shows the location of the rejection regions and the standardized test statistic $z$. Because $z$ is in the rejection region, you reject the null hypothesis.

You can check your answer using technology, as shown below.

**MINITAB**

<table>
<thead>
<tr>
<th>One-Sample Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test of $\mu = 14050$ vs $\neq 14050$</td>
</tr>
<tr>
<td>The assumed standard deviation = 2875</td>
</tr>
<tr>
<td>N</td>
</tr>
<tr>
<td>----</td>
</tr>
<tr>
<td>500</td>
</tr>
</tbody>
</table>

**Interpretation** There is enough evidence at the 10% level of significance to reject the claim that the mean annual cost of raising a child (age 2 and under) by married-couple families in the U.S. is $14,050.

**TRY IT YOURSELF 10**

In Example 10, at $\alpha = 0.01$, is there enough evidence to reject the claim?

*Answer: Page A37*
7.2 EXERCISES

1. The z-test using a P-value compares the P-value with the level of significance \( \alpha \). In the z-test using rejection region(s), the test statistic is compared with critical values.

2. No; Both involve comparing the test statistic’s probability with the level of significance. The P-value method converts the standardized test statistic to a probability (P-value) and compares this with the level of significance, whereas the critical value method converts the level of significance to a z-score and compares this with the standardized test statistic.

3. (a) Fail to reject \( H_0 \).
   (b) Reject \( H_0 \).
   (c) Reject \( H_0 \).

4. (a) Fail to reject \( H_0 \).
   (b) Fail to reject \( H_0 \).
   (c) Reject \( H_0 \).

5. (a) Fail to reject \( H_0 \).
   (b) Fail to reject \( H_0 \).
   (c) Fail to reject \( H_0 \).

6. (a) Fail to reject \( H_0 \).
   (b) Reject \( H_0 \).
   (c) Reject \( H_0 \).

7. (a) Fail to reject \( H_0 \).
   (b) Fail to reject \( H_0 \).
   (c) Reject \( H_0 \).

8. (a) Reject \( H_0 \).
   (b) Reject \( H_0 \).
   (c) Reject \( H_0 \).

9. Left-tailed test
   \( z = -1.32 \)
   \( \alpha = 0.10 \)

10. Left-tailed test
    \( z = -1.55 \)
    \( \alpha = 0.05 \)

11. Right-tailed test
    \( z = 2.46 \)
    \( \alpha = 0.01 \)

12. Right-tailed test
    \( z = 1.23 \)
    \( \alpha = 0.10 \)

13. Two-tailed test
    \( z = -1.68 \)
    \( \alpha = 0.05 \)

14. Two-tailed test
    \( z = 1.95 \)
    \( \alpha = 0.08 \)

Graphical Analysis In Exercises 15 and 16, match each P-value with the graph that displays its area without performing any calculations. Explain your reasoning.

15. \( P = 0.0089 \) and \( P = 0.3050 \)

(a) \( P = 0.0089 \)
(b) \( P = 0.3050 \)

The larger P-value corresponds to the larger area.

16. \( P = 0.0688 \) and \( P = 0.2802 \)

(a) \( P = 0.2802 \)
(b) \( P = 0.0688 \)

The larger P-value corresponds to the larger area.

Building Basic Skills and Vocabulary

1. Explain the difference between the z-test for \( \mu \) using a P-value and the z-test for \( \mu \) using rejection region(s).

2. In hypothesis testing, does using the critical value method or the P-value method affect your conclusion? Explain.

Interpreting a P-Value In Exercises 3–8, the P-value for a hypothesis test is shown. Use the P-value to decide whether to reject \( H_0 \) when the level of significance is (a) \( \alpha = 0.01 \), (b) \( \alpha = 0.05 \), and (c) \( \alpha = 0.10 \).

3. \( P = 0.0461 \)
4. \( P = 0.0691 \)
5. \( P = 0.1271 \)
6. \( P = 0.0107 \)
7. \( P = 0.0838 \)
8. \( P = 0.0062 \)

Finding a P-Value In Exercises 9–14, find the P-value for the hypothesis test with the standardized test statistic \( z \). Decide whether to reject \( H_0 \) for the level of significance \( \alpha \).

9. \( P = 0.0934 \); Reject \( H_0 \).
10. \( P = 0.0606 \); Fail to reject \( H_0 \).
11. \( P = 0.0069 \); Reject \( H_0 \).
12. \( P = 0.1093 \); Fail to reject \( H_0 \).
13. \( P = 0.0930 \); Fail to reject \( H_0 \).
14. \( P = 0.0512 \); Reject \( H_0 \).

(a) \( P = 0.0089 \)
(b) \( P = 0.3050 \)

The larger P-value corresponds to the larger area.
17. Fail to reject $H_0$.
18. Reject $H_0$.
19. Critical value: $z_0 = -1.88$
    Rejection region: $z < -1.88$

20. Critical value: $z_0 = -1.34$
    Rejection region: $z < -1.34$

21. Critical value: $z_0 = 1.645$
    Rejection region: $z > 1.645$

22. See Selected Answers, page A100.
25. (a) Fail to reject $H_0$ because $z < 1.285$.
    (b) Fail to reject $H_0$ because $z < 1.285$.
    (c) Fail to reject $H_0$ because $z < 1.285$.
    (d) Reject $H_0$ because $z > 1.285$.
26. (a) Reject $H_0$ because $z > 1.96$.
    (b) Fail to reject $H_0$ because $-1.96 < z < 1.96$.
    (c) Fail to reject $H_0$ because $-1.96 < z < 1.96$.
    (d) Reject $H_0$ because $z < -1.96$.
27. Reject $H_0$. There is enough evidence at the 5% level of significance to reject the claim.
28. Fail to reject $H_0$. There is not enough evidence at the 7% level of significance to support the claim.
29. Fail to reject $H_0$. There is not enough evidence at the 3% level of significance to support the claim.
30. Reject $H_0$. There is enough evidence at the 1% level of significance to reject the claim.

In Exercises 17 and 18, use the TI-84 Plus displays to make a decision to reject or fail to reject the null hypothesis at the level of significance.

17. $\alpha = 0.05$

- Left-tailed test, $\alpha = 0.03$
- Right-tailed test, $\alpha = 0.05$
- Two-tailed test, $\alpha = 0.02$

Finding Critical Values and Rejection Regions In Exercises 19–24, find the critical value(s) and rejection region(s) for the type of $z$-test with level of significance $\alpha$. Include a graph with your answer.

19. Left-tailed test, $\alpha = 0.03$
20. Left-tailed test, $\alpha = 0.09$
21. Right-tailed test, $\alpha = 0.05$
22. Right-tailed test, $\alpha = 0.08$
23. Two-tailed test, $\alpha = 0.02$
24. Two-tailed test, $\alpha = 0.12$

Graphical Analysis In Exercises 25 and 26, state whether each standardized test statistic $z$ allows you to reject the null hypothesis. Explain your reasoning.

25. (a) $z = -1.301$
    (b) $z = 1.203$
    (c) $z = 1.280$
    (d) $z = 1.286$
26. (a) $z = 1.98$
    (b) $z = -1.89$
    (c) $z = 1.65$
    (d) $z = -1.99$

In Exercises 27–30, test the claim about the population mean $\mu$ at the level of significance $\alpha$. Assume the population is normally distributed.

27. Claim: $\mu = 40$; $\alpha = 0.05$; $\sigma = 1.97$
    Sample statistics: $\bar{x} = 39.2$, $n = 25$
28. Claim: $\mu \geq 1475$; $\alpha = 0.07$; $\sigma = 29$
    Sample statistics: $\bar{x} = 1468$, $n = 26$
29. Claim: $\mu \neq 5880$; $\alpha = 0.03$; $\sigma = 413$
    Sample statistics: $\bar{x} = 5771$, $n = 67$
30. Claim: $\mu \leq 22,500$; $\alpha = 0.01$; $\sigma = 1200$
    Sample statistics: $\bar{x} = 23,500$, $n = 45$
Using and Interpreting Concepts

Hypothesis Testing Using a P-Value  In Exercises 31–36,
(a) identify the claim and state $H_0$ and $H_a$.
(b) find the standardized test statistic $z$.
(c) find the corresponding P-value.
(d) decide whether to reject or fail to reject the null hypothesis.
(e) interpret the decision in the context of the original claim.

31. **MCAT Scores** A random sample of 100 medical school applicants at a university has a mean total score of 502 on the MCAT. According to a report, the mean total score for the school’s applicants is more than 499. Assume the population standard deviation is 10.6. At $\alpha = 0.01$, is there enough evidence to support the report’s claim? (Source: Association of American Medical Colleges)

32. **Sprinkler Systems** A manufacturer of sprinkler systems designed for fire protection claims that the average activating temperature is at least 135°F. To test this claim, you randomly select a sample of 32 systems and find the mean activation temperature to be 133°F. Assume the population standard deviation is 3.3°F. At $\alpha = 0.10$, do you have enough evidence to reject the manufacturer’s claim?

33. **Boston Marathon** A sports statistician claims that the mean winning times for Boston Marathon women’s open division champions is at least 2.68 hours. The mean winning time of a sample of 30 randomly selected Boston Marathon women’s open division champions is 2.60 hours. Assume the population standard deviation is 0.32 hour. At $\alpha = 0.05$, can you reject the claim? (Source: Boston Athletic Association)

34. **Acceleration Times** A consumer group claims that the mean acceleration time from 0 to 60 miles per hour for a sedan is 6.3 seconds. A random sample of 33 sedans has a mean acceleration time from 0 to 60 miles per hour of 6.2 seconds. Assume the population standard deviation is 0.32 second. At $\alpha = 0.09$, is there enough evidence to reject the null hypothesis?

35. **Roller Coasters** The heights (in feet) of 36 randomly selected top-rated roller coasters are listed. Assume the population standard deviation is 71.6 feet. At $\alpha = 0.05$, is there enough evidence to reject the claim that the mean height of top-rated roller coasters is 160 feet? (Source: POP World Media, LLC)

36. **Salaries** An analyst claims that the mean annual salary for intermediate level architects in Wichita, Kansas, is more than the national mean, $52,000. The annual salaries (in dollars) for a random sample of 21 intermediate level architects in Wichita are listed. Assume the population is normally distributed and the population standard deviation is $8000. At $\alpha = 0.09$, is there enough evidence to support the analyst’s claim? (Adapted from Salary.com)
37. (a) The claim is “the mean caffeine content per 12-ounce bottle of a population of caffeinated soft drinks is 37.7 milligrams.”

\[ H_0: \mu = 37.7 \quad \text{(claim)} \]
\[ H_a: \mu \neq 37.7 \]

(b) \[ z = -2.575, \quad z_0 = 2.575 \]

Rejection regions:
\[ z < -2.575, \quad z > 2.575 \]

(c) \(-0.72 \quad \text{(d) Fail to reject } H_0. \]

(e) There is not enough evidence at the 1% level of significance to reject the consumer research organization’s claim that the mean caffeine content per 12-ounce bottle of a population of caffeinated soft drinks is 37.7 milligrams.

38. (a) The claim is “the mean high school graduation rate per state in the United States is 80%.”

\[ H_0: \mu = 80 \quad \text{(claim)} \]
\[ H_a: \mu \neq 80 \]

(b) \[ z = -1.96, \quad z_0 = 1.96 \]

Rejection regions:
\[ z < -1.96, \quad z > 1.96 \]

(c) \(2.15 \quad \text{(d) Reject } H_0. \]

(e) There is enough evidence at the 5% level of significance to reject the education researcher’s claim that the mean high school graduation rate per state in the United States is 80%.

40. See Selected Answers, page A100.
41. See Odd Answers, page A69.

### Hypothesis Testing Using Rejection Region(s)

**In Exercises 37–42, (a) identify the claim and state \( H_0 \) and \( H_a \), (b) find the critical value(s) and identify the rejection region(s), (c) find the standardized test statistic \( z \), (d) decide whether to reject or fail to reject the null hypothesis, and (e) interpret the decision in the context of the original claim.**

37. **Caffeine Content** A consumer research organization states that the mean caffeine content per 12-ounce bottle of a population of caffeinated soft drinks is 37.7 milligrams. You want to test this claim. During your tests, you find that a random sample of thirty-six 12-ounce bottles of caffeinated soft drinks has a mean caffeine content of 36.4 milligrams. Assume the population standard deviation is 10.8 milligrams. At \( \alpha = 0.01 \), can you reject the research organization’s claim? *(Source: National Soft Drink Association)*

38. **High School Graduation Rate** An education researcher claims that the mean high school graduation rate per state in the United States is 80%. You want to test this claim. You find that a random sample of 30 states has a mean high school graduation rate of 82%. Assume the population standard deviation is 5.1%. At \( \alpha = 0.05 \), do you have enough evidence to support the researcher’s claim? *(Source: U.S. Department of Education)*

39. **Fast Food** A fast food restaurant estimates that the mean sodium content in one of its breakfast sandwiches is no more than 920 milligrams. A random sample of 44 breakfast sandwiches has a mean sodium content of 925 milligrams. Assume the population standard deviation is 18 milligrams. At \( \alpha = 0.10 \), do you have enough evidence to reject the restaurant’s claim?

40. **Light Bulbs** A light bulb manufacturer guarantees that the mean life of a certain type of light bulb is at least 750 hours. A random sample of 25 light bulbs has a mean life of 745 hours. Assume the population is normally distributed and the population standard deviation is 60 hours. At \( \alpha = 0.02 \), do you have enough evidence to reject the manufacturer’s claim?

41. **Fluorescent Lamps** A fluorescent lamp manufacturer guarantees that the mean life of a fluorescent lamp is at least 10,000 hours. You want to test this guarantee. To do so, you record the lives of a random sample of 32 fluorescent lamps. The results (in hours) are listed. Assume the population standard deviation is 1850 hours. At \( \alpha = 0.11 \), do you have enough evidence to reject the manufacturer’s claim?

### Carbon Dioxide Emissions

<table>
<thead>
<tr>
<th>Carbon dioxide emissions (in megatons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>340 76 46 44 75 1617</td>
</tr>
<tr>
<td>34 43 23 0.5 0.3 6</td>
</tr>
<tr>
<td>0.3 0.7 11 0.1 0.2 7.6</td>
</tr>
<tr>
<td>0.6 0.6 26 9.9 2.3 8.2</td>
</tr>
<tr>
<td>3.4 0.1 472 4.2 4.2 0</td>
</tr>
<tr>
<td>113 21 7.2 5 0.1 16</td>
</tr>
<tr>
<td>0.2 45 5.1 175 0 4.1</td>
</tr>
</tbody>
</table>

**TABLE FOR EXERCISE 42**

42. See Selected Answers, page A100.

43. Writing When \( P > \alpha \), does the standardized test statistic lie inside or outside of the rejection region(s)? Explain your reasoning.

44. Writing In a right-tailed test where \( P < \alpha \), does the standardized test statistic lie to the left or the right of the critical value? Explain your reasoning.
SECTION 7.3 Hypothesis Testing for the Mean (σ Unknown)

What You Should Learn

- How to find critical values in a t-distribution
- How to use the t-test to test a mean μ when σ is not known
- How to use technology to find P-values and use them with a t-test to test a mean μ when σ is not known

Critical Values in a t-Distribution

In Section 7.2, you learned how to perform a hypothesis test for a population mean when the population standard deviation is known. In many real-life situations, the population standard deviation is not known. When either the population has a normal distribution or the sample size is at least 30, you can still test the population mean μ. To do so, you can use the t-distribution with n − 1 degrees of freedom.

Critical Values in a t-Distribution

1. Specify the level of significance α.
2. Identify the degrees of freedom, d.f. = n − 1.
3. Find the critical value(s) using Table 5 in Appendix B in the row with n − 1 degrees of freedom. When the hypothesis test is
   a. left-tailed, use the “One Tail, α” column with a negative sign.
   b. right-tailed, use the “One Tail, α” column with a positive sign.
   c. two-tailed, use the “Two Tails, α” column with a negative and a positive sign.

GUIDELINES

Finding Critical Values in a t-Distribution

1. Specify the level of significance α.
2. Identify the degrees of freedom, d.f. = n − 1.
3. Find the critical value(s) using Table 5 in Appendix B in the row with n − 1 degrees of freedom. When the hypothesis test is
   a. left-tailed, use the “One Tail, α” column with a negative sign.
   b. right-tailed, use the “One Tail, α” column with a positive sign.
   c. two-tailed, use the “Two Tails, α” column with a negative and a positive sign.

See the figures below.

EXAMPLE 1

Finding a Critical Value for a Left-Tailed Test

Find the critical value t₀ for a left-tailed test with α = 0.05 and n = 21.

SOLUTION

The degrees of freedom are
d.f. = n − 1 = 21 − 1 = 20.

To find the critical value, use Table 5 in Appendix B with d.f. = 20 and α = 0.05 in the “One Tail, α” column. Because the test is left-tailed, the critical value is negative. So, t₀ = −1.725, as shown in the figure at the left.

TRY IT YOURSELF 1

Find the critical value t₀ for a left-tailed test with α = 0.01 and n = 14.

Answer: Page A37
CHAPTER 7
Hypothesis Testing with One Sample

EXAMPLE 2

Finding a Critical Value for a Right-Tailed Test
Find the critical value \( t_0 \) for a right-tailed test with \( \alpha = 0.01 \) and \( n = 17 \).

SOLUTION

The degrees of freedom are
\[
d.f. = n - 1 = 17 - 1 = 16.
\]

To find the critical value, use Table 5 with d.f. = 16 and \( \alpha = 0.01 \) in the “One Tail, \( \alpha \)” column. Because the test is right-tailed, the critical value is positive. So,
\[
t_0 = 2.583
\]
as shown in the figure.

TRY IT YOURSELF 2
Find the critical value \( t_0 \) for a right-tailed test with \( \alpha = 0.10 \) and \( n = 9 \).

Answer: Page A37

Because \( t \)-distributions are symmetric, in a two-tailed test the critical values are opposites, as shown in the next example.

EXAMPLE 3

Finding Critical Values for a Two-Tailed Test
Find the critical values \(-t_0\) and \(t_0\) for a two-tailed test with \( \alpha = 0.10 \) and \( n = 26 \).

SOLUTION

The degrees of freedom are
\[
d.f. = n - 1 = 26 - 1 = 25.
\]

To find the critical values, use Table 5 with d.f. = 25 and \( \alpha = 0.10 \) in the “Two Tails, \( \alpha \)” column. Because the test is two-tailed, one critical value is negative and one is positive. So,
\[
-t_0 = -1.708 \quad \text{and} \quad t_0 = 1.708
\]
as shown in the figure at the left. You can check your answer using technology, as shown below.

EXCEL

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: T.INV.2T(0.1,25)</td>
<td>1.708140761</td>
</tr>
</tbody>
</table>

TRY IT YOURSELF 3
Find the critical values \(-t_0\) and \(t_0\) for a two-tailed test with \( \alpha = 0.05 \) and \( n = 16 \).

Answer: Page A37
The \( t \)-Test for a Mean \( \mu \)

To test a claim about a mean \( \mu \) when \( \sigma \) is not known, you can use a \( t \)-sampling distribution. The standardized test statistic takes the form of

\[
 t = \frac{(\text{Sample mean}) - (\text{Hypothesized mean})}{\text{Standard error}}.
\]

Because \( \sigma \) is not known, the standardized test statistic is calculated using the sample standard deviation \( s \), as shown in the next definition.

**The \( t \)-Test for a Mean \( \mu \)**

The \( t \)-test for a mean \( \mu \) is a statistical test for a population mean. The test statistic is the sample mean \( \bar{x} \). The standardized test statistic is

\[
 t = \frac{\bar{x} - \mu}{s / \sqrt{n}}
\]

when these conditions are met.

1. The sample is random.
2. At least one of the following is true: The population is normally distributed or \( n \geq 30 \).

The degrees of freedom are \( \text{d.f.} = n - 1 \).

**GUIDELINES**

Using the \( t \)-Test for a Mean \( \mu \) (\( \sigma \) Unknown)

**In Words**

1. Verify that \( \sigma \) is not known, the sample is random, and either the population is normally distributed or \( n \geq 30 \).
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.
3. Specify the level of significance.
4. Identify the degrees of freedom.
5. Determine the critical value(s). Use Table 5 in Appendix B.
6. Determine the rejection region(s).
7. Find the standardized test statistic and sketch the sampling distribution.
8. Make a decision to reject or fail to reject the null hypothesis.
9. Interpret the decision in the context of the original claim.

In Step 8 of the guidelines, the decision rule uses rejection regions. You can also test a claim using \( P \)-values, as shown on page 382. Also, when the number of degrees of freedom you need is not in Table 5, use the closest number in the table that is less than the value you need (or use technology). For instance, for \( \text{d.f.} = 57 \), use 50 degrees of freedom.

**Describe the possible type I and type II errors of this situation.**

A type I error will occur when the actual amount of lead is less than or equal to 0.015 milligram per liter, but you reject \( H_0 \): \( \mu \leq 0.015 \). So, even though the water is safe, the water system will undertake actions that are not needed and possibly cause a public panic.

A type II error will occur when the actual amount of lead is greater than 0.015 milligram per liter, but you fail to reject \( H_0 \): \( \mu \leq 0.015 \). So, the water system will not undertake actions to protect the public from water that has too much lead, which could cause health problems.
CHAPTER 7  Hypothesis Testing with One Sample

Hypothesis Testing Using a Rejection Region

A used car dealer says that the mean price of used cars sold in the last 12 months is at least $21,000. You suspect this claim is incorrect and find that a random sample of 14 used cars sold in the last 12 months has a mean price of $19,189 and a standard deviation of $2950. Is there enough evidence to reject the dealer’s claim at \( \alpha = 0.05 \)? Assume the population is normally distributed. (Adapted from Edmunds.com)

**SOLUTION**

Because \( \sigma \) is unknown, the sample is random, and the population is normally distributed, you can use the \( t \)-test. The claim is “the mean price is at least $21,000.” So, the null and alternative hypotheses are

\[
H_0: \mu \geq 21,000 \quad \text{(Claim)}
\]

and

\[
H_a: \mu < 21,000.
\]

The test is a left-tailed test, the level of significance is \( \alpha = 0.05 \), and the degrees of freedom are

\[
d.f. = 14 - 1 = 13.
\]

So, using Table 5, the critical value is \( t_0 = -1.771 \). The rejection region is \( t < -1.771 \). The standardized test statistic is

\[
t = \frac{\bar{x} - \mu}{s/\sqrt{n}}
\]

Because \( \sigma \) is unknown and the population is normally distributed, use the \( t \)-test.

Assume \( \mu = 21,000 \).

Round to three decimal places.

The figure shows the location of the rejection region and the standardized test statistic \( t \). Because \( t \) is in the rejection region, you reject the null hypothesis.

**Interpretation** There is enough evidence at the 5% level of significance to reject the claim that the mean price of used cars sold in the last 12 months is at least $21,000.

**TRY IT YOURSELF 4**

An industry analyst says that the mean age of a used car sold in the last 12 months is less than 4.1 years. A random sample of 25 used cars sold in the last 12 months has a mean age of 3.7 years and a standard deviation of 1.3 years. Is there enough evidence to support the analyst’s claim at \( \alpha = 0.10 \)? Assume the population is normally distributed. (Adapted from Edmunds.com)

**Answer:** Page A37

Remember that when you make a decision, the possibility of a type I or a type II error exists. For instance, in Example 4, a type I error is possible when you reject \( H_0 \), because \( \mu \geq 21,000 \) may be true.
Hypothesis Testing Using Rejection Regions

An industrial company claims that the mean pH level of the water in a nearby river is 6.8. You randomly select 39 water samples and measure the pH of each. The sample mean and standard deviation are 6.7 and 0.35, respectively. Is there enough evidence to reject the company’s claim at $\alpha = 0.05$?

**SOLUTION**

Because $\sigma$ is unknown, the sample is random, and $n = 39 \geq 30$, you can use the $t$-test. The claim is “the mean pH level is 6.8.” So, the null and alternative hypotheses are

$H_0: \mu = 6.8$ (Claim) and $H_a: \mu \neq 6.8$.

The test is a two-tailed test, the level of significance is $\alpha = 0.05$, and the degrees of freedom are $d.f. = 39 - 1 = 38$. So, using Table 5, the critical values are $t_0 = -2.024$ and $t_0 = 2.024$. The rejection regions are $t < -2.024$ and $t > 2.024$. The standardized test statistic is

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Because $\sigma$ is unknown and $n \geq 30$, use the $t$-test.

$$= \frac{6.7 - 6.8}{0.35/\sqrt{39}}$$

Assume $\mu = 6.8$.

$$\approx -1.784.$$  
Round to three decimal places.

The figure shows the location of the rejection regions and the standardized test statistic $t$. Because $t$ is not in the rejection region, you fail to reject the null hypothesis. You can confirm this decision using technology, as shown below. Note that the standardized statistic $t$ differs from the one found using Table 5 due to rounding.

**MINITAB**

<table>
<thead>
<tr>
<th>One-Sample T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test of $\mu = 6.8$ vs $\neq 6.8$</td>
</tr>
<tr>
<td>N</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>39</td>
</tr>
</tbody>
</table>

**Interpretation** There is not enough evidence at the 5% level of significance to reject the claim that the mean pH level is 6.8.

**TRY IT YOURSELF 5**

The company in Example 5 claims that the mean conductivity of the river is 1890 milligrams per liter. The conductivity of a water sample is a measure of the total dissolved solids in the sample. You randomly select 39 water samples and measure the conductivity of each. The sample mean and standard deviation are 2350 milligrams per liter and 900 milligrams per liter, respectively. Is there enough evidence to reject the company’s claim at $\alpha = 0.01$?

Answer: Page A37
Using $P$-Values With $t$-Tests

You can also use $P$-values for a $t$-test for a mean $\mu$. For instance, consider finding a $P$-value given $t = 1.98$, 15 degrees of freedom, and a right-tailed test. Using Table 5 in Appendix B, you can determine that $P$ falls between

$$\alpha = 0.025$$ and $$\alpha = 0.05$$

but you cannot determine an exact value for $P$. In such cases, you can use technology to perform a hypothesis test and find exact $P$-values.

**EXAMPLE 6**

Using $P$-Values with a $t$-Test

A department of motor vehicles office claims that the mean wait time is less than 14 minutes. A random sample of 10 people has a mean wait time of 13 minutes with a standard deviation of 3.5 minutes. At $\alpha = 0.10$, test the office’s claim. Assume the population is normally distributed.

**SOLUTION**

Because $\sigma$ is unknown, the sample is random, and the population is normally distributed, you can use the $t$-test. The claim is “the mean wait time is less than 14 minutes.” So, the null and alternative hypotheses are

$$H_0: \mu \geq 14 \text{ minutes}$$

and

$$H_a: \mu < 14 \text{ minutes. (Claim)}$$

The TI-84 Plus display at the far left shows how to set up the hypothesis test. The two displays on the right show the possible results, depending on whether you select Calculate or Draw.

From the displays, you can see that

$$P \approx 0.1949.$$ 

Because the $P$-value is greater than $\alpha = 0.10$, you fail to reject the null hypothesis.

**Interpretation** There is not enough evidence at the 10% level of significance to support the office’s claim that the mean wait time is less than 14 minutes.

**TRY IT YOURSELF 6**

Another department of motor vehicles office claims that the mean wait time is at most 18 minutes. A random sample of 12 people has a mean wait time of 15 minutes with a standard deviation of 2.2 minutes. At $\alpha = 0.05$, test the office’s claim. Assume the population is normally distributed.

*Answer: Page A37*
### Building Basic Skills and Vocabulary

1. Explain how to find critical values for a *t*-distribution.
2. Explain how to use a *t*-test to test a hypothesized mean \( \mu \) when \( \sigma \) is unknown. What assumptions are necessary?

### Graphical Analysis

In Exercises 3–8, find the critical value(s) and rejection region(s) for the type of *t*-test with level of significance \( \alpha \) and sample size \( n \).

3. Left-tailed test, \( \alpha = 0.10, n = 20 \)
4. Left-tailed test, \( \alpha = 0.01, n = 35 \)
5. Right-tailed test, \( \alpha = 0.05, n = 23 \)
6. Right-tailed test, \( \alpha = 0.01, n = 31 \)
7. Two-tailed test, \( \alpha = 0.05, n = 27 \)
8. Two-tailed test, \( \alpha = 0.10, n = 38 \)

### Exercises

1. See Odd Answers, page A69.
2. See Selected Answers, page A100.
3. Critical value: \( t_\alpha = -1.328 \)
   Rejection region: \( t < -1.328 \)
4. Critical value: \( t_\alpha = -2.441 \)
   Rejection region: \( t < -2.441 \)
5. Critical value: \( t_\alpha = 1.717 \)
   Rejection region: \( t > 1.717 \)
6. Critical value: \( t_\alpha = 2.457 \)
   Rejection region: \( t > 2.457 \)
7. Critical values: \( -t_\alpha = -2.056, t_\alpha = 2.056 \)
   Rejection regions: \( t < -2.056, t > 2.056 \)
8. Critical values: \( -t_\alpha = -1.687, t_\alpha = 1.687 \)
   Rejection regions: \( t < -1.687, t > 1.687 \)

9. (a) Fail to reject \( H_0 \) because \( t > 2.086 \).
   (b) Fail to reject \( H_0 \) because \( t > -2.086 \).
   (c) Reject \( H_0 \) because \( t < -2.086 \).
10. (a) Fail to reject \( H_0 \) because \( t > 1.402 \).
    (b) Reject \( H_0 \) because \( t > 1.402 \).
    (c) Fail to reject \( H_0 \) because \( t < 1.402 \).
11. (a) Reject \( H_0 \) because \( t < -1.725 \).
    (b) Fail to reject \( H_0 \) because \( -1.725 < t < 1.725 \).
    (c) Reject \( H_0 \) because \( t > 1.725 \).
12. (a) Reject \( H_0 \) because \( t < -1.071 \).
    (b) Fail to reject \( H_0 \) because \( -1.071 < t < 1.071 \).
    (c) Reject \( H_0 \) because \( t > 1.071 \).
13. Fail to reject \( H_0 \). There is not enough evidence at the 1% level of significance to reject the claim.
15. See Odd Answers, page A69.
17. See Odd Answers, page A69.
Using and Interpreting Concepts

Hypothesis Testing Using Rejection Regions  In Exercises 19–26, (a) identify the claim and state \( H_0 \) and \( H_a \), (b) find the critical value(s) and identify the rejection region(s), (c) find the standardized test statistic \( t \), (d) decide whether to reject or fail to reject the null hypothesis, and (e) interpret the decision in the context of the original claim. Assume the population is normally distributed.

19. **Used Car Cost** A used car dealer says that the mean price of a three-year-old sport utility vehicle (in good condition) is $20,000. You suspect this claim is incorrect and find that a random sample of 22 similar vehicles has a mean price of $20,640 and a standard deviation of $1990. Is there enough evidence to reject the claim at \( \alpha = 0.05 \)?

20. **DMV Wait Times** A state Department of Transportation claims that the mean wait time for various services at its different locations is at most 6 minutes. A random sample of 34 services at different locations has a mean wait time of 10.3 minutes and a standard deviation of 8.0 minutes. Is there enough evidence to reject the claim at \( \alpha = 0.01 \)?

21. **Credit Card Debt** A credit reporting agency claims that the mean credit card debt by state is greater than $5500 per person. You want to test this claim. You find that a random sample of 30 states has a mean credit card debt of $5594 per person and a standard deviation of $597 per person. At \( \alpha = 0.05 \), can you support the claim? (Adapted from TransUnion)

22. **Battery Life** A company claims that the mean battery life of their MP3 player is at least 30 hours. You suspect this claim is incorrect and find that a random sample of 18 MP3 players has a mean battery life of 28.5 hours and a standard deviation of 1.7 hours. Is there enough evidence to reject the claim at \( \alpha = 0.01 \)?

23. **Carbon Monoxide Levels** As part of your work for an environmental awareness group, you want to test a claim that the mean amount of carbon monoxide in the air in U.S. cities is less than 2.34 parts per million. You find that the mean amount of carbon monoxide in the air for a random sample of 64 U.S. cities is 2.37 parts per million and the standard deviation is 2.11 parts per million. At \( \alpha = 0.10 \), can you support the claim? (Adapted from U.S. Environmental Protection Agency)

24. **Lead Levels** As part of your work for an environmental awareness group, you want to test a claim that the mean amount of lead in the air in U.S. cities is less than 0.036 microgram per cubic meter. You find that the mean amount of lead in the air for a random sample of 56 U.S. cities is 0.039 microgram per cubic meter and the standard deviation is 0.069 microgram per cubic meter. At \( \alpha = 0.01 \), can you support the claim? (Adapted from U.S. Environmental Protection Agency)

25. **Annual Salary** An employment information service claims the mean annual salary for senior level product engineers is $98,000. The annual salaries (in dollars) for a random sample of 16 senior level product engineers are shown in the table at the left. At \( \alpha = 0.05 \), test the claim that the mean salary is $98,000. (Adapted from Salary.com)

26. **Annual Salary** An employment information service claims the mean annual salary for home care physical therapists is more than $80,000. The annual salaries (in dollars) for a random sample of 12 home care physical therapists are shown in the table at the left. At \( \alpha = 0.10 \), is there enough evidence to support the claim that the mean salary is more than $80,000? (Adapted from Salary.com)
27. (a) The claim is “the mean minimum time it takes for a sedan to travel a quarter mile is greater than 14.7 seconds.”
   \[ H_0: \mu \leq 14.7 \]
   \[ H_a: \mu > 14.7 \] (claim)
   (b) 0.0664 (c) Reject \( H_0 \).
   (d) There is enough evidence at the 10% level of significance to support the consumer group’s claim that the mean minimum time it takes for a sedan to travel a quarter mile is greater than 14.7 seconds.

28. (a) The claim is “the mean dive duration of a North Atlantic right whale is 11.5 minutes.”
   \[ H_0: \mu = 11.5 \] (claim)
   \[ H_a: \mu \neq 11.5 \]
   (b) 0.0725 (c) Reject \( H_0 \).
   (d) There is enough evidence at the 10% level of significance to reject the oceanographer’s claim that the mean dive duration of a North Atlantic right whale is 11.5 minutes.

29. You receive a brochure from a large university. The brochure indicates that the mean class size for full-time faculty is fewer than 32 students. You want to test this claim. You randomly select 18 classes taught by full-time faculty and determine the class size of each. The results are shown in the table at the left. At \( \alpha = 0.05 \), can you support the university’s claim?

30. The dean of a university estimates that the mean number of classroom hours per week for full-time faculty is 11.0. As a member of the student council, you want to test this claim. A random sample of the number of classroom hours for eight full-time faculty for one week is shown in the table at the left. At \( \alpha = 0.01 \), can you reject the dean’s claim?

31. A car company claims that the mean gas mileage for its luxury sedan is at least 23 miles per gallon. You believe the claim is incorrect and find that a random sample of 5 cars has a mean gas mileage of 22 miles per gallon and a standard deviation of 4 miles per gallon. At \( \alpha = 0.05 \), test the company’s claim. Assume the population is normally distributed.

32. An education publication claims that the mean in-state tuition and fees at public four-year institutions by state is more than $9000 per year. A random sample of 30 states has a mean in-state tuition and fees at public four-year institutions of $9231 per year. Assume the population standard deviation is $2380. At \( \alpha = 0.01 \), test the publication’s claim. (Adapted from The College Board)

33. You are testing a claim and incorrectly use the standard normal sampling distribution instead of the \( t \)-sampling distribution. Does this make it more or less likely to reject the null hypothesis? Is this result the same no matter whether the test is left-tailed, right-tailed, or two-tailed? Explain your reasoning.
Hypothesis Tests for a Mean

7.3  
ACTIVITY
APPLET
You can find the interactive applet for this activity within MyLab Statistics or at www.pearsonhighered.com/mathstatsresources.

The hypothesis tests for a mean applet allows you to visually investigate hypothesis tests for a mean. You can specify the sample size $n$, the shape of the distribution (Normal or Right skewed), the true population mean (Mean), the true population standard deviation (Std. Dev.), the null value for the mean (Null mean), and the alternative for the test (Alternative). When you click SIMULATE, 100 separate samples of size $n$ will be selected from a population with these population parameters. For each of the 100 samples, a hypothesis test based on the T statistic is performed, and the results from each test are displayed in the plots at the right. The test statistic for each test is shown in the top plot and the $P$-value is shown in the bottom plot. The green and blue lines represent the cutoffs for rejecting the null hypothesis with the 0.05 and 0.01 level tests, respectively. Additional simulations can be carried out by clicking SIMULATE multiple times. The cumulative number of times that each test rejects the null hypothesis is also shown. Press CLEAR to clear existing results and start a new simulation.

**EXPLORE**

Step 1  Specify a value for $n$.
Step 2  Specify a distribution.
Step 3  Specify a value for the mean.
Step 4  Specify a value for the standard deviation.
Step 5  Specify a value for the null mean.
Step 6  Specify an alternative hypothesis.
Step 7  Click SIMULATE to generate the hypothesis tests.

**DRAW CONCLUSIONS**

1. Set $n = 15$, Mean = 40, Std. Dev. = 5, and the distribution to “Normal.” Test the claim that the mean is equal to 40. What are the null and alternative hypotheses? Run the simulation so that at least 1000 hypothesis tests are run. Compare the proportion of null hypothesis rejections for the 0.05 level and the 0.01 level. Is this what you would expect? Explain.

2. Suppose a null hypothesis is rejected at the 0.01 level. Will it be rejected at the 0.05 level? Explain. Suppose a null hypothesis is rejected at the 0.05 level. Will it be rejected at the 0.01 level? Explain.

3. Set $n = 25$, Mean = 25, Std. Dev. = 3, and the distribution to “Normal.” Test the claim that the mean is at least 27. What are the null and alternative hypotheses? Run the simulation so that at least 1000 hypothesis tests are run. Compare the proportion of null hypothesis rejections for the 0.05 level and the 0.01 level. Is this what you would expect? Explain.
In an article in the *Journal of Statistics Education* (vol. 4, no. 2), Allen Shoemaker describes a study that was reported in the Journal of the American Medical Association (JAMA).* It is generally accepted that the mean body temperature of an adult human is 98.6°F. In his article, Shoemaker uses the data from the JAMA article to test this hypothesis. Here is a summary of his test.

**Claim:** The body temperature of adults is 98.6°F.

\[ H_0: \mu = 98.6°F \quad (\text{Claim}) \quad H_a: \mu \neq 98.6°F \]

**Sample Size:** \( n = 130 \)

**Population:** Adult human temperatures (Fahrenheit)

**Distribution:** Approximately normal

**Test Statistics:** \( \bar{x} \approx 98.25, s \approx 0.73 \)

* Data for the JAMA article were collected from healthy men and women, ages 18 to 40, at the University of Maryland Center for Vaccine Development, Baltimore.

---

**Men’s Temperatures**  
(in degrees Fahrenheit)

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<th>Frequency</th>
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</tr>
<tr>
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<td>7</td>
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<tr>
<td>100</td>
<td>100</td>
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Key: 96|3 = 96.3

**Women’s Temperatures**  
(in degrees Fahrenheit)

<table>
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</tr>
<tr>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

Key: 96|4 = 96.4

---

**EXERCISES**

1. Complete the hypothesis test for all adults (men and women) by performing the following steps. Use a level of significance of \( \alpha = 0.05 \).
   
   (a) Sketch the sampling distribution.
   
   (b) Determine the critical values and add them to your sketch.
   
   (c) Determine the rejection regions and shade them in your sketch.
   
   (d) Find the standardized test statistic. Plot and label it in your sketch.
   
   (e) Make a decision to reject or fail to reject the null hypothesis.
   
   (f) Interpret the decision in the context of the original claim.

2. If you lower the level of significance to \( \alpha = 0.01 \), does your decision change? Explain your reasoning.

3. Test the hypothesis that the mean temperature of men is 98.6°F. What can you conclude at a level of significance of \( \alpha = 0.01 \)?

4. Test the hypothesis that the mean temperature of women is 98.6°F. What can you conclude at a level of significance of \( \alpha = 0.01 \)?

5. Use the sample of 130 temperatures to form a 99% confidence interval for the mean body temperature of adult humans.

6. The conventional “normal” body temperature was established by Carl Wunderlich over 100 years ago. What were possible sources of error in Wunderlich’s sampling procedure?
Hypothesis Testing for Proportions

In Sections 7.2 and 7.3, you learned how to perform a hypothesis test for a population mean $\mu$. In this section, you will learn how to test a population proportion $p$.

Hypothesis tests for proportions can be used when politicians want to know the proportion of their constituents who favor a certain bill or when quality assurance engineers test the proportion of parts that are defective.

If $np \geq 5$ and $nq \geq 5$ for a binomial distribution, then the sampling distribution for $\hat{p}$ is approximately normal with a mean of $\mu_\hat{p} = p$ and a standard error of

$$\sigma_\hat{p} = \sqrt{pq/n}.$$

### z-Test for a Proportion $p$

The **z-test for a proportion $p$** is a statistical test for a population proportion. The $z$-test can be used when a binomial distribution is given such that $np \geq 5$ and $nq \geq 5$. The **test statistic** is the sample proportion $\hat{p}$ and the **standardized test statistic** is

$$z = \frac{\hat{p} - \mu_\hat{p}}{\sigma_\hat{p}} = \frac{\hat{p} - p}{\sqrt{pq/n}}.$$

### GUIDELINES

**Using a z-Test for a Proportion $p$**

**In Words**

1. Verify that the sampling distribution of $\hat{p}$ can be approximated by a normal distribution.
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.
3. Specify the level of significance.
4. Determine the critical value(s).
5. Determine the rejection region(s).
6. Find the standardized test statistic and sketch the sampling distribution.
7. Make a decision to reject or fail to reject the null hypothesis.
8. Interpret the decision in the context of the original claim.

**In Symbols**

- $np \geq 5, nq \geq 5$
- $H_0$ and $H_a$
- $\alpha$
- Use Table 4 in Appendix B.
- $z = \frac{\hat{p} - p}{\sqrt{pq/n}}$

If $z$ is in the rejection region, then reject $H_0$. Otherwise, fail to reject $H_0$.

In Step 7 of the guidelines, the decision rule uses rejection regions. You can also test a claim using $P$-values, as shown in the Study Tip at the left.

Study Tip

A hypothesis test for a proportion $p$ can also be performed using $P$-values. Use the guidelines on page 365 for using $P$-values for a $z$-test for a mean $\mu$, but in Step 4 find the standardized test statistic by using the formula

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}}.$$

The other steps in the test are the same.
Hypothesis Test for a Proportion

A researcher claims that less than 45% of U.S. adults use passwords that are less secure because complicated ones are too hard to remember. In a random sample of 100 adults, 41% say they use passwords that are less secure because complicated ones are too hard to remember. At $\alpha = 0.01$, is there enough evidence to support the researcher’s claim? (Adapted from Pew Research Center)

**SOLUTION**

The products $np = 100(0.45) = 45$ and $nq = 100(0.55) = 55$ are both greater than 5. So, you can use a $z$-test. The claim is “less than 45% of U.S. adults use passwords that are less secure because complicated ones are too hard to remember.” So, the null and alternative hypotheses are

$$H_0: p \geq 0.45 \quad \text{and} \quad H_a: p < 0.45. \quad \text{(Claim)}$$

Because the test is a left-tailed test and the level of significance is $\alpha = 0.01$, the critical value is $z_0 = -2.33$ and the rejection region is $z < -2.33$. The standardized test statistic is

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

Because $np \geq 5$ and $n \geq 5$, you can use the $z$-test.

$$= \frac{0.41 - 0.45}{\sqrt{(0.45)(0.55)/100}}$$

$$= -0.80.$$ 

Assume $p = 0.45$.

Round to two decimal places.

The figure shows the location of the rejection region and the standardized test statistic $z$. Because $z$ is not in the rejection region, you fail to reject the null hypothesis.

**Interpretation** There is not enough evidence at the 1% level of significance to support the claim that less than 45% of U.S. adults use passwords that are less secure because complicated ones are too hard to remember.

**TRY IT YOURSELF 1**

A researcher claims that more than 90% of U.S. adults have access to a smartphone. In a random sample of 150 adults, 87% say they have access to a smartphone. At $\alpha = 0.01$, is there enough evidence to support the researcher’s claim? (Adapted from Nielsen Mobile Insights)

**Answer:** Page A37

To use a $P$-value to perform the hypothesis test in Example 1, you can use technology, as shown at the right, or you can use Table 4. Using Table 4, the area corresponding to $z = -0.80$ is 0.2119. Because this is a left-tailed test, the $P$-value is equal to the area to the left of $z = -0.80$. So, $P = 0.2119$. (This value differs from the one found using technology due to rounding.) Because the $P$-value is greater than $\alpha = 0.01$, you fail to reject the null hypothesis. Note that this is the same result obtained in Example 1.
Recall from Section 6.3 that when the sample proportion is not given, you can find it using the formula

\[ \hat{p} = \frac{x}{n} \]

**Sample proportion**

where \( x \) is the number of successes in the sample and \( n \) is the sample size.

### Example 2

**Hypothesis Test for a Proportion**

A researcher claims that 51% of U.S. adults believe, incorrectly, that antibiotics are effective against viruses. In a random sample of 2202 adults, 1161 say antibiotics are effective against viruses. At \( \alpha = 0.10 \), is there enough evidence to support the researcher’s claim? *(Source: HealthDay/Harris Poll)*

**Solution**

The products \( np = 2202(0.51) = 1123 \) and \( nq = 2202(0.49) = 1079 \) are both greater than 5. So, you can use a \( z \)-test. The claim is “51% of U.S. adults believe, incorrectly, that antibiotics are effective against viruses.” So, the null and alternative hypotheses are

\[ H_0: p = 0.51 \quad \text{(Claim)} \quad \text{and} \quad H_a: p \neq 0.51. \]

Because the test is a two-tailed test and the level of significance is \( \alpha = 0.10 \), the critical values are \( z_{0.05} = -1.645 \) and \( z_{0.05} = 1.645 \). The rejection regions are \( z < -1.645 \) and \( z > 1.645 \). Because the number of successes is \( x = 1161 \) and \( n = 2202 \), the sample proportion is

\[ \hat{p} = \frac{x}{n} = \frac{1161}{2202} \approx 0.527. \]

The standardized test statistic is

\[ z = \frac{\hat{p} - p}{\sqrt{pq/n}} \]

\[ = \frac{0.527 - 0.51}{\sqrt{(0.51)(0.49)/2202}} \]

\[ \approx 1.60. \]

The figure shows the location of the rejection regions and the standardized test statistic \( z \). Because \( z \) is not in the rejection region, you fail to reject the null hypothesis.

**Interpretation**

There is not enough evidence at the 10% level of significance to reject the claim that 51% of U.S. adults believe, incorrectly, that antibiotics are effective against viruses.

### Try It Yourself 2

A researcher claims that 67% of U.S. adults believe that doctors prescribing antibiotics for viral infections for which antibiotics are not effective is a significant cause of drug-resistant superbugs. (Superbugs are bacterial infections that are resistant to many or all antibiotics.) In a random sample of 1768 adults, 1150 say they believe that doctors prescribing antibiotics for viral infections for which antibiotics are not effective is a significant cause of drug-resistant superbugs. At \( \alpha = 0.10 \), is there enough evidence to support the researcher’s claim? *(Source: HealthDay/Harris Poll)*

Answer: Page A37
7.4 EXERCISES

Building Basic Skills and Vocabulary

1. Explain how to determine whether a normal distribution can be used to approximate a binomial distribution.

2. Explain how to test a population proportion \( p \).

In Exercises 3–6, determine whether a normal sampling distribution can be used. If it can be used, test the claim.

3. Claim: \( p < 0.12; \alpha = 0.01 \). Sample statistics: \( \hat{p} = 0.10, n = 40 \)

4. Claim: \( p \geq 0.48; \alpha = 0.08 \). Sample statistics: \( \hat{p} = 0.40, n = 90 \)

5. Claim: \( p \neq 0.15; \alpha = 0.05 \). Sample statistics: \( \hat{p} = 0.12, n = 500 \)

6. Claim: \( p > 0.70; \alpha = 0.04 \). Sample statistics: \( \hat{p} = 0.64, n = 225 \)

Using and Interpreting Concepts

Hypothesis Testing Using Rejection Regions In Exercises 7–12, (a) identify the claim and state \( H_0 \) and \( H_a \), (b) find the critical value(s) and identify the rejection region(s), (c) find the standardized test statistic \( z \), (d) decide whether to reject or fail to reject the null hypothesis, and (e) interpret the decision in the context of the original claim.

7. Vaccination Requirement A medical researcher says that less than 80% of U.S. adults think that healthy children should be required to be vaccinated. In a random sample of 200 U.S. adults, 82% think that healthy children should be required to be vaccinated. At \( \alpha = 0.05 \), is there enough evidence to support the researcher’s claim? (Adapted from Pew Research Center)

8. Internal Revenue Service Audits A research center claims that at least 27% of U.S. adults think that the IRS will audit their taxes. In a random sample of 1000 U.S. adults in a recent year, 23% say they are concerned that the IRS will audit their taxes. At \( \alpha = 0.01 \), is there enough evidence to reject the center’s claim? (Source: Rasmussen Reports)

9. Student Employment An education researcher claims that at most 3% of working college students are employed as teachers or teaching assistants. In a random sample of 200 working college students, 4% are employed as teachers or teaching assistants. At \( \alpha = 0.01 \), is there enough evidence to reject the researcher’s claim? (Adapted from Sallie Mae)

10. Working Students An education researcher claims that 57% of college students work year-round. In a random sample of 300 college students, 171 say they work year-round. At \( \alpha = 0.10 \), is there enough evidence to support the researcher’s claim? (Adapted from Sallie Mae)

11. Zika Virus A researcher claims that 85% percent of Americans think they are unlikely to contract the Zika virus. In a random sample of 250 Americans, 225 think they are unlikely to contract the Zika virus. At \( \alpha = 0.05 \), is there enough evidence to reject the researcher’s claim? (Adapted from Gallup)

12. Changing Jobs A research center claims that more than 29% of U.S. employees have changed jobs in the past three years. In a random sample of 180 U.S. employees, 63 have changed jobs in the past three years. At \( \alpha = 0.10 \), is there enough evidence to support the center’s claim? (Adapted from Gallup)
Hypothesis Testing Using a P-Value  In Exercises 13–16, (a) identify the claim and state $H_0$ and $H_a$, (b) use technology to find the P-value, (c) decide whether to reject or fail to reject the null hypothesis, and (d) interpret the decision in the context of the original claim.

13. **Space Travel**  A research center claims that 27% of U.S. adults would travel into space on a commercial flight if they could afford it. In a random sample of 1000 U.S. adults, 30% say that they would travel into space on a commercial flight if they could afford it. At $\alpha = 0.05$, is there enough evidence to reject the research center’s claim?  
(Source: Rasmussen Reports)

14. **Purchasing Food Online**  A research center claims that at most 18% of U.S. adults’ online food purchases are for snacks. In a random sample of 1995 U.S. adults, 20% say their online food purchases are for snacks. At $\alpha = 0.10$, is there enough evidence to support the center’s claim?  
(Source: The Harris Poll)

15. **Pet Ownership**  A humane society claims that less than 67% of U.S. households own a pet. In a random sample of 600 U.S. households, 390 say they own a pet. At $\alpha = 0.10$, is there enough evidence to support the society’s claim?  
(Adapted from The Humane Society of the United States)

16. **Stray dogs**  A humane society claims that 5% of U.S. households have taken in a stray dog. In a random sample of 200 U.S. households, 12 say they have taken in a stray dog. At $\alpha = 0.05$, is there enough evidence to reject the society’s claim?  
(Adapted from The Humane Society of the United States)

17. **Are People Concerned About Protecting the Environment?**  You interview a random sample of 100 adults. The results of the survey show that 59% of the adults said they live in ways that help protect the environment some of the time. At $\alpha = 0.05$, can you reject the claim that at least 63% of adults make an effort to live in ways that help protect the environment some of the time?

18. **What Are People’s Attitudes About Protecting the Environment?**  Use your conclusion from Exercise 17 to write a paragraph on people’s attitudes about protecting the environment.

**Extending Concepts**

**Alternative Formula**  In Exercises 19 and 20, use the information below. When you know the number of successes $x$, the sample size $n$, and the population proportion $p$, it can be easier to use the formula

$$z = \frac{x - np}{\sqrt{npq}}$$

To find the standardized test statistic when using a z-test for a population proportion $p$.

19. **Rework Exercise 7** using the alternative formula and verify that the results are the same.

20. The alternative formula is derived from the formula

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{(x/n) - p}{\sqrt{pq/n}}.$$

Use this formula to derive the alternative formula. Justify each step.
The *hypothesis tests for a proportion* applet allows you to visually investigate hypothesis tests for a population proportion. You can specify the sample size $n$, the population proportion (True $p$), the null value for the proportion (Null $p$), and the alternative for the test (Alternative). When you click SIMULATE, 100 separate samples of size $n$ will be selected from a population with a proportion of successes equal to True $p$. For each of the 100 samples, a hypothesis test based on the Z statistic is performed, and the results from each test are displayed in plots at the right. The standardized test statistic for each test is shown in the top plot and the $P$-value is shown in the bottom plot. The green and blue lines represent the cutoffs for rejecting the null hypothesis with the 0.05 and 0.01 level tests, respectively. Additional simulations can be carried out by clicking SIMULATE multiple times. The cumulative number of times that each test rejects the null hypothesis is also shown. Press CLEAR to clear existing results and start a new simulation.

**EXPLORE**

**Step 1** Specify a value for $n$.
**Step 2** Specify a value for True $p$.
**Step 3** Specify a value for Null $p$.
**Step 4** Specify an alternative hypothesis.
**Step 5** Click SIMULATE to generate the hypothesis tests.

**DRAW CONCLUSIONS**

1. Set $n = 25$ and True $p = 0.35$. Test the claim that the proportion is equal to 35%. What are the null and alternative hypotheses? Run the simulation so that at least 1000 tests are run. Compare the proportion of null hypothesis rejections for the 0.05 and 0.01 levels. Is this what you would expect? Explain.

2. Set $n = 50$ and True $p = 0.6$. Test the claim that the proportion is at least 40%. What are the null and alternative hypotheses? Run the simulation so that at least 1000 tests are run. Compare the proportion of null hypothesis rejections for the 0.05 and 0.01 levels. Perform a hypothesis test for each level. Use the results of the hypothesis tests to explain the results of the simulation.
What You Should Learn

- How to find critical values for a chi-square test
- How to use the chi-square test to test a variance \( \sigma^2 \) or a standard deviation \( \sigma \)

Critical Values for a Chi-Square Test

In real life, it is important to produce consistent, predictable results. For instance, consider a company that manufactures golf balls. The manufacturer must produce millions of golf balls, each having the same size and the same weight. There is a very low tolerance for variation. For a normally distributed population, you can test the variance and standard deviation of the process using the chi-square distribution with \( n - 1 \) degrees of freedom. Before learning how to do the test, you must know how to find the critical values, as shown in the guidelines.

GUIDELINES

Finding Critical Values for a Chi-Square Test

1. Specify the level of significance \( \alpha \).
2. Identify the degrees of freedom, \( \text{d.f.} = n - 1 \).
3. The critical values for the chi-square distribution are found in Table 6 in Appendix B. To find the critical value(s) for a
   a. right-tailed test, use the value that corresponds to d.f. and \( \alpha \).
   b. left-tailed test, use the value that corresponds to d.f. and \( 1 - \alpha \).
   c. two-tailed test, use the values that correspond to d.f. and \( \frac{1}{2} \alpha \), and d.f. and \( 1 - \frac{1}{2} \alpha \).

See the figures at the left.

EXAMPLE 1

Finding a Critical Value for a Right-Tailed Test

Find the critical value \( \chi^2_0 \) for a right-tailed test when \( n = 26 \) and \( \alpha = 0.10 \).

SOLUTION

The degrees of freedom are \( \text{d.f.} = n - 1 = 26 - 1 = 25 \). The figure below shows a chi-square distribution with 25 degrees of freedom and a shaded area of \( \alpha = 0.10 \) in the right tail. Using Table 6 in Appendix B with \( \text{d.f.} = 25 \) and \( \alpha = 0.10 \), the critical value is \( \chi^2_0 = 34.382 \).

TRY IT YOURSELF 1

Find the critical value \( \chi^2_0 \) for a right-tailed test when \( n = 18 \) and \( \alpha = 0.01 \).

Answer: Page A37
SECTION 7.5  Hypothesis Testing for Variance and Standard Deviation

Finding a Critical Value for a Left-Tailed Test

Find the critical value $x^2$ for a left-tailed test when $n = 11$ and $\alpha = 0.01$.

**SOLUTION**

The degrees of freedom are

$$d.f. = n - 1 = 11 - 1 = 10.$$ 

The figure at the left shows a chi-square distribution with 10 degrees of freedom and a shaded area of $\alpha = 0.01$ in the left tail. The area to the right of the critical value is

$$1 - \alpha = 1 - 0.01 = 0.99.$$ 

Using Table 6 with d.f. = 10 and the area 0.99, the critical value is $x^2_{0.01} = 2.558$.

You can check your answer using technology, as shown below.

![MINITAB](image)

**TRY IT YOURSELF 2**

Find the critical value $x^2$ for a left-tailed test when $n = 30$ and $\alpha = 0.05$.

Answer: Page A37

Note that because chi-square distributions are not symmetric (like normal or $t$-distributions), in a two-tailed test the two critical values are not opposites. Each critical value must be calculated separately, as shown in the next example.

**EXAMPLE 3**

Finding Critical Values for a Two-Tailed Test

Find the critical values $x^2_L$ and $x^2_R$ for a two-tailed test when $n = 9$ and $\alpha = 0.05$.

**SOLUTION**

The degrees of freedom are

$$d.f. = n - 1 = 9 - 1 = 8.$$ 

The figure shows a chi-square distribution with 8 degrees of freedom and a shaded area of $\frac{1}{2}\alpha = 0.025$ in each tail. The area to the right of $x^2_L$ is $\frac{1}{2}\alpha = 0.025$, and the area to the right of $x^2_R$ is $1 - \frac{1}{2}\alpha = 0.975$. Using Table 6 with d.f. = 8 and the areas 0.025 and 0.975, the critical values are $x^2_L = 17.535$ and $x^2_R = 2.180$. You can check you answer using technology, as shown at the left.

**TRY IT YOURSELF 3**

Find the critical values $x^2_L$ and $x^2_R$ for a two-tailed test when $n = 51$ and $\alpha = 0.01$.

Answer: Page A37
The Chi-Square Test

To test a variance \( \sigma^2 \) or a standard deviation \( \sigma \) of a population that is normally distributed, you can use the chi-square test. The chi-square test for a variance or standard deviation is not as robust as the tests for the population mean \( \mu \) or the population proportion \( p \). So, it is essential in performing a chi-square test for a variance or standard deviation that the population be normally distributed. The results can be misleading when the population is not normal.

**Chi-Square Test for a Variance \( \sigma^2 \) or Standard Deviation \( \sigma \)**

The *chi-square test for a variance \( \sigma^2 \) or standard deviation \( \sigma \)* is a statistical test for a population variance or standard deviation. The chi-square test can only be used when the population is normal. The *test statistic* is \( s^2 \) and the *standardized test statistic* 

\[
\chi^2 = \frac{(n - 1)s^2}{\sigma^2}
\]

follows a chi-square distribution with degrees of freedom 

\[\text{d.f.} = n - 1.\]

In Step 8 of the guidelines below, the decision rule uses rejection regions. You can also test a claim using \( P \)-values (see Exercises 31-34).

**GUIDELINES**

**Using the Chi-Square Test for a Variance \( \sigma^2 \) or a Standard Deviation \( \sigma \)**

<table>
<thead>
<tr>
<th>In Words</th>
<th>In Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Verify that the sample is random and the population is normally distributed.</td>
<td>State ( H_0 ) and ( H_a ).</td>
</tr>
<tr>
<td>2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.</td>
<td>Identify ( \alpha ).</td>
</tr>
<tr>
<td>3. Specify the level of significance.</td>
<td>d.f. = ( n - 1 )</td>
</tr>
<tr>
<td>4. Identify the degrees of freedom.</td>
<td>Use Table 6 in Appendix B.</td>
</tr>
<tr>
<td>5. Determine the critical value(s).</td>
<td>[ \chi^2 = \frac{(n - 1)s^2}{\sigma^2} ]</td>
</tr>
<tr>
<td>6. Determine the rejection region(s).</td>
<td>If ( \chi^2 ) is in the rejection region, then reject ( H_0 ). Otherwise, fail to reject ( H_0 ).</td>
</tr>
<tr>
<td>7. Find the standardized test statistic and sketch the sampling distribution.</td>
<td></td>
</tr>
<tr>
<td>8. Make a decision to reject or fail to reject the null hypothesis.</td>
<td></td>
</tr>
<tr>
<td>9. Interpret the decision in the context of the original claim.</td>
<td></td>
</tr>
</tbody>
</table>

For Step 5 of the guidelines, in addition to using Table 6 in Appendix B, you can use technology to find the critical value(s). Also, some technology tools allow you to perform a hypothesis test for a variance (or a standard deviation) using only the descriptive statistics.
Using a Hypothesis Test for the Population Variance

A dairy processing company claims that the variance of the amount of fat in the whole milk processed by the company is no more than 0.25. You suspect this is wrong and find that a random sample of 41 milk containers has a variance of 0.27. At $\alpha = 0.05$, is there enough evidence to reject the company’s claim? Assume the population is normally distributed.

**SOLUTION**

Because the sample is random and the population is normally distributed, you can use the chi-square test. The claim is “the variance is no more than 0.25.” So, the null and alternative hypotheses are

$$H_0: \sigma^2 \leq 0.25 \quad \text{(Claim)} \quad \text{and} \quad H_a: \sigma^2 > 0.25.$$  

The test is a right-tailed test, the level of significance is $\alpha = 0.05$, and the degrees of freedom are $d.f. = 41 - 1 = 40$. So, using Table 6, the critical value is

$$x_{0.05}^2 = 55.758.$$  

The rejection region is $x^2 > 55.758$. The standardized test statistic is

$$x^2 = \frac{(n - 1)s^2}{\sigma^2} = \frac{(41 - 1)(0.27)}{0.25} = 43.2.$$  

Assume $\sigma^2 = 0.25$.

The figure at the left shows the location of the rejection region and the standardized test statistic $x^2$. Because $x^2$ is not in the rejection region, you fail to reject the null hypothesis. You can check your answer using technology, as shown below. Note that the test statistic, 43.2, is the same as what you found above.

**STATCRUNCH**

<table>
<thead>
<tr>
<th>Hypothesis Test Results:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance</td>
</tr>
<tr>
<td>$\sigma^2$</td>
</tr>
</tbody>
</table>

**Interpretation** There is not enough evidence at the 5% level of significance to reject the company’s claim that the variance of the amount of fat in the whole milk is no more than 0.25.

**TRY IT YOURSELF 4**

A bottling company claims that the variance of the amount of sports drink in a 12-ounce bottle is no more than 0.40. A random sample of 31 bottles has a variance of 0.75. At $\alpha = 0.01$, is there enough evidence to reject the company’s claim? Assume the population is normally distributed.

*Answer: Page A37*
Using a Hypothesis Test for the Standard Deviation

A company claims that the standard deviation of the lengths of time it takes an incoming telephone call to be transferred to the correct office is less than 1.4 minutes. A random sample of 25 incoming telephone calls has a standard deviation of 1.1 minutes. At $\alpha = 0.10$, is there enough evidence to support the company’s claim? Assume the population is normally distributed.

**SOLUTION**

Because the sample is random and the population is normally distributed, you can use the chi-square test. The claim is “the standard deviation is less than 1.4 minutes.” So, the null and alternative hypotheses are

$H_0: \sigma \geq 1.4$ minutes \hspace{1cm} and \hspace{1cm} $H_a: \sigma < 1.4$ minutes. (Claim)

The test is a left-tailed test, the level of significance is $\alpha = 0.10$, and the degrees of freedom are

$$d.f. = 25 - 1 = 24.$$ 

So, using Table 6, the critical value is

$$\chi^2_0 = 15.659.$$ 

The rejection region is $\chi^2 < 15.659$. The standardized test statistic is

$$\chi^2 = \frac{(n - 1)s^2}{\sigma^2} \quad \text{Use the chi-square test.}$$

$$\approx \frac{(25 - 1)(1.1)^2}{(1.4)^2} \quad \text{Assume } \sigma = 1.4.$$ 

$$\approx 14.816. \quad \text{Round to three decimal places.}$$

The figure below shows the location of the rejection region and the standardized test statistic $\chi^2$. Because $\chi^2$ is in the rejection region, you reject the null hypothesis.

**Interpretation**

There is enough evidence at the 10% level of significance to support the claim that the standard deviation of the lengths of time it takes an incoming telephone call to be transferred to the correct office is less than 1.4 minutes.

**TRY IT YOURSELF 5**

A police chief claims that the standard deviation of the lengths of response times is less than 3.7 minutes. A random sample of 9 response times has a standard deviation of 3.0 minutes. At $\alpha = 0.05$, is there enough evidence to support the police chief’s claim? Assume the population is normally distributed.

*Answer: Page A37*
Using a Hypothesis Test for the Population Variance

A sporting goods manufacturer claims that the variance of the strengths of a certain fishing line is 15.9. A random sample of 15 fishing line spools has a variance of 21.8. At $\alpha = 0.05$, is there enough evidence to reject the manufacturer’s claim? Assume the population is normally distributed.

**SOLUTION**

Because the sample is random and the population is normally distributed, you can use the chi-square test. The claim is “the variance is 15.9.” So, the null and alternative hypotheses are

$$H_0: \sigma^2 = 15.9 \quad \text{(Claim)} \quad \text{and} \quad H_a: \sigma^2 \neq 15.9.$$

The test is a two-tailed test, the level of significance is $\alpha = 0.05$, and the degrees of freedom are

$$d.f. = 15 - 1 = 14.$$

Using Table 6, the critical values are $\chi^2_L = 5.629$ and $\chi^2_R = 26.119$. The rejection regions are

$$\chi^2 < 5.629 \quad \text{and} \quad \chi^2 > 26.119.$$

The standardized test statistic is

$$\chi^2 = \frac{(n - 1)s^2}{\sigma^2} \quad \text{Use the chi-square test.}$$

Substitute the given values:

$$\chi^2 = \frac{(15 - 1)(21.8)}{15.9} \approx 19.195. \quad \text{Assume } \sigma^2 = 15.9.$$

Round to three decimal places.

The figure below shows the location of the rejection regions and the standardized test statistic $\chi^2$. Because $\chi^2$ is not in the rejection regions, you fail to reject the null hypothesis.

**Interpretation** There is not enough evidence at the 5% level of significance to reject the claim that the variance of the strengths of the fishing line is 15.9.

**TRY IT YOURSELF 6**

A company that offers dieting products and weight loss services claims that the variance of the weight losses of their users is 25.5. A random sample of 13 users has a variance of 10.8. At $\alpha = 0.10$, is there enough evidence to reject the company’s claim? Assume the population is normally distributed.

*Answer: Page A37*
7.5 Exercises

Building Basic Skills and Vocabulary

1. Explain how to find critical values in a chi-square distribution.
2. Can a critical value for the chi-square test be negative? Explain.
3. How do the requirements for a chi-square test for a variance or standard deviation differ from a z-test or a t-test for a mean?
4. Explain how to test a population variance or a population standard deviation.

In Exercises 5–12, find the critical value(s) and rejection region(s) for the type of chi-square test with sample size \( n \) and level of significance \( \alpha \).

5. Right-tailed test, 
   \( n = 27, \alpha = 0.05 \)
6. Right-tailed test, 
   \( n = 10, \alpha = 0.10 \)
7. Left-tailed test, 
   \( n = 7, \alpha = 0.01 \)
8. Left-tailed test, 
   \( n = 24, \alpha = 0.05 \)
9. Two-tailed test, 
   \( n = 81, \alpha = 0.10 \)
10. Two-tailed test, 
    \( n = 61, \alpha = 0.01 \)
11. Right-tailed test, 
    \( n = 30, \alpha = 0.01 \)
12. Two-tailed test, 
    \( n = 31, \alpha = 0.05 \)

Graphical Analysis  In Exercises 13 and 14, state whether each standardized test statistic \( \chi^2 \) allows you to reject the null hypothesis. Explain.

13. (a) \( \chi^2 = 2.091 \)  
    (b) \( \chi^2 = 0 \)  
    (c) \( \chi^2 = 1.086 \)  
    (d) \( \chi^2 = 6.3471 \)
14. (a) \( \chi^2 = 22.302 \)  
    (b) \( \chi^2 = 23.309 \)  
    (c) \( \chi^2 = 8.457 \)  
    (d) \( \chi^2 = 8.577 \)

In Exercises 15–22, test the claim about the population variance \( \sigma^2 \) or standard deviation \( \sigma \) at the level of significance \( \alpha \). Assume the population is normally distributed.

15. Claim: \( \sigma^2 = 0.52; \alpha = 0.05 \). Sample statistics: \( s^2 = 0.508, n = 18 \)
16. Claim: \( \sigma^2 \geq 8.5; \alpha = 0.05 \). Sample statistics: \( s^2 = 7.45, n = 23 \)
17. Claim: \( \sigma^2 \leq 17.6; \alpha = 0.01 \). Sample statistics: \( s^2 = 28.33, n = 41 \)
18. Claim: \( \sigma^2 > 19; \alpha = 0.1 \). Sample statistics: \( s^2 = 28, n = 17 \)
19. Claim: \( \sigma^2 \neq 32.8; \alpha = 0.1 \). Sample statistics: \( s^2 = 40.9, n = 101 \)
20. Claim: \( \sigma^2 = 63; \alpha = 0.01 \). Sample statistics: \( s^2 = 58, n = 29 \)
21. Claim: \( \sigma < 40; \alpha = 0.01 \). Sample statistics: \( s = 40.8, n = 12 \)
22. Claim: \( \sigma = 24.9; \alpha = 0.10 \). Sample statistics: \( s = 29.1, n = 51 \)
Using and Interpreting Concepts

Hypothesis Testing Using Rejection Regions  In Exercises 23–30, (a) identify the claim and state $H_0$ and $H_a$, (b) find the critical value(s) and identify the rejection region(s), (c) find the standardized test statistic $\chi^2$, (d) decide whether to reject or fail to reject the null hypothesis, and (e) interpret the decision in the context of the original claim. Assume the population is normally distributed.

23. Tires  A tire manufacturer claims that the variance of the diameters in a tire model is 8.6. A random sample of 10 tires has a variance of 4.3. At $\alpha = 0.01$, is there enough evidence to reject the claim?

24. Gas Mileage  An auto manufacturer claims that the variance of the gas mileages in a model of hybrid vehicle is 0.16. A random sample of 30 vehicles has a variance of 0.26. At $\alpha = 0.05$, is there enough evidence to reject the claim?  (Adapted from Green Hybrid)

25. Mathematics Assessment Tests  A school administrator claims that the standard deviation for grade 12 students on a mathematics assessment test is less than 35 points. A random sample of 28 grade 12 test scores has a standard deviation of 34 points. At $\alpha = 0.10$, is there enough evidence to support the claim?  (Adapted from National Center for Educational Statistics)

26. Vocabulary Assessment Tests  A school administrator claims that the standard deviation for grade 12 students on a vocabulary assessment test is greater than 45 points. A random sample of 25 grade 12 test scores has a standard deviation of 46 points. At $\alpha = 0.01$, is there enough evidence to support the claim?  (Adapted from National Center for Educational Statistics)

27. Waiting Times  A hospital claims that the standard deviation of the waiting times for patients in its emergency department is no more than 0.5 minute. A random sample of 25 waiting times has a standard deviation of 0.7 minute. At $\alpha = 0.10$, is there enough evidence to reject the claim?

28. Hotel Room Rates  A travel analyst claims that the standard deviation of the room rates for two adults at three-star hotels in Denver is at least $68. A random sample of 18 three-star hotels has a standard deviation of $40. At $\alpha = 0.01$, is there enough evidence to reject the claim?  (Adapted from Expedia)

29. Salaries  The annual salaries (in dollars) of 15 randomly chosen senior level graphic design specialists are shown in the table at the left. At $\alpha = 0.05$, is there enough evidence to support the claim that the standard deviation of the annual salaries is different from $10,300?  (Adapted from Salary.com)

30. Salaries  The annual salaries (in dollars) of 12 randomly chosen nursing supervisors are shown in the table at the left. At $\alpha = 0.10$, is there enough evidence to reject the claim that the standard deviation of the annual salaries is $16,500?  (Adapted from Salary.com)

Extending Concepts

$P$-Values  You can calculate the $P$-value for a chi-square test using technology. After calculating the standardized test statistic, use the cumulative distribution function (CDF) to calculate the area under the curve. From Example 4 on page 397, $\chi^2 = 43.2$. Using a TI-84 Plus (choose 8 from the DISTR menu), enter 0 for the lower bound, 43.2 for the upper bound, and 40 for the degrees of freedom, as shown at the left. Because it is a right-tailed test, the $P$-value is approximately $1 - 0.6638 = 0.3362$. Because $P > \alpha = 0.05$, fail to reject $H_0$. In Exercises 31–34, use the $P$-value method to perform the hypothesis test for the indicated exercise.


TABLE FOR EXERCISE 29

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<tbody>
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<td>65,876</td>
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<td>48,337</td>
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TABLE FOR EXERCISE 30

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<td>78,975</td>
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<td>74,644</td>
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<td>107,817</td>
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<td>71,090</td>
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<tr>
<td>78,975</td>
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<tr>
<td>98,221</td>
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</tbody>
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TI-84 PLUS

$\chi^2\text{cdf}(0, 43.2, 40) = 0.663776867$
A Summary of Hypothesis Testing

With hypothesis testing, perhaps more than any other area of statistics, it can be difficult to see the forest for all the trees. To help you see the forest—the overall picture—a summary of what you studied in this chapter is provided.

Writing the Hypotheses
- You are given a claim about a population parameter $\mu$, $p$, $\sigma^2$, or $\sigma$.
- Rewrite the claim and its complement using $\leq$, $\geq$, $=$ and $>$, $<$, $\neq$.

- Identify the claim. Is it $H_0$ or $H_a$?

Specifying a Level of Significance
- Specify $\alpha$, the maximum acceptable probability of rejecting a valid $H_0$ (a type I error).

Specifying the Sample Size
- Specify your sample size $n$.

Choosing the Test
- Normally distributed population
- Any population

- Mean: $H_0$ describes a hypothesized population mean $\mu$.
  - Use a $z$-test when $\sigma$ is known and the population is normal.
  - Use a $z$-test for any population when $\sigma$ is known and $n \geq 30$.
  - Use a $t$-test when $\sigma$ is not known and the population is normal.
  - Use a $t$-test for any population when $\sigma$ is not known and $n \geq 30$.

- Proportion: $H_0$ describes a hypothesized population proportion $p$.
  - Use a $z$-test for any population when $np \geq 5$ and $nq \geq 5$.

- Variance or Standard Deviation: $H_0$ describes a hypothesized population variance $\sigma^2$ or standard deviation $\sigma$.
  - Use a chi-square test when the population is normal.

Sketching the Sampling Distribution
- Use $H_a$ to decide whether the test is left-tailed, right-tailed, or two-tailed.

Finding the Standardized Test Statistic
- Take a random sample of size $n$ from the population.
- Compute the test statistic $\bar{x}$, $\hat{p}$, or $s^2$.
- Find the standardized test statistic $z$, $t$, or $\chi^2$.

Making a Decision

Option 1. Decision based on rejection region
- Use $\alpha$ to find the critical value(s) $z_0$, $t_0$, or $\chi^2_0$ and rejection region(s).
- Decision Rule:
  - Reject $H_0$ when the standardized test statistic is in the rejection region.
  - Fail to reject $H_0$ when the standardized test statistic is not in the rejection region.

Option 2. Decision based on $P$-value
- Use the standardized test statistic or technology to find the $P$-value.
- Decision Rule:
  - Reject $H_0$ when $P \leq \alpha$.
  - Fail to reject $H_0$ when $P > \alpha$.

Study Tip
Large sample sizes will usually increase the cost and effort of testing a hypothesis, but they also tend to make your decision more reliable.
**A Summary of Hypothesis Testing**

---

**Chi-Square Test for a Hypothesized Variance \( \sigma^2 \) or Standard Deviation \( \sigma \) (Section 7.5)**

**Test statistic:** \( s^2 \)

**Critical value:** \( \chi^2_{0.01} \) (Use Table 6.)

Sampling distribution is approximated by a chi-square distribution with d.f. = \( n - 1 \).

---

**t-Test for a Hypothesized Mean \( \mu \) (\( \sigma \) Unknown) (Section 7.3)**

**Test statistic:** \( t \)

**Critical value:** \( t_{0.01} \) (Use Table 5.)

Sampling distribution of sample means is approximated by a t-distribution with d.f. = \( n - 1 \).

---

**z-Test for a Hypothesized Mean \( \mu \) (\( \sigma \) Known) (Section 7.2)**

**Test statistic:** \( z \)

**Critical value:** \( z_{0.01} \) (Use Table 4.)

Sampling distribution of sample means is a normal distribution.

---

**z-Test for a Hypothesized Proportion \( p \) (Section 7.4)**

**Test statistic:** \( \hat{p} \)

**Critical value:** \( z_{0.01} \) (Use Table 4.)

Sampling distribution of sample proportions is a normal distribution.

---

**Chi-Square Test for a Hypothesized Variance \( \sigma^2 \) or Standard Deviation \( \sigma \) (Section 7.5)**

**Test statistic:** \( \chi^2 \)

**Critical value:** \( \chi^2_{0.01} \) (Use Table 6.)

Sampling distribution is approximated by a chi-square distribution with d.f. = \( n - 1 \).
Uses

Hypothesis testing is important in many different fields because it gives a scientific procedure for assessing the validity of a claim about a population. Some of the concepts in hypothesis testing are intuitive, but some are not. For instance, the American Journal of Clinical Nutrition suggests that eating dark chocolate can help prevent heart disease. A random sample of healthy volunteers were assigned to eat 3.5 ounces of dark chocolate each day for 15 days. After 15 days, the mean systolic blood pressure of the volunteers was 6.4 millimeters of mercury lower. A hypothesis test could show whether this drop in systolic blood pressure is significant or simply due to sampling error.

Careful inferences must be made concerning the results. The study only examined the effects of dark chocolate, so the inference of health benefits cannot be extended to all types of chocolate. You also would not infer that you should eat large quantities of chocolate because the benefits must be weighed against known risks, such as weight gain and acid reflux.

Abuses

Not Using a Random Sample  The entire theory of hypothesis testing is based on the fact that the sample is randomly selected. If the sample is not random, then you cannot use it to infer anything about a population parameter.

Attempting to Prove the Null Hypothesis  When the $P$-value for a hypothesis test is greater than the level of significance, you have not proven the null hypothesis is true—only that there is not enough evidence to reject it. For instance, with a $P$-value higher than the level of significance, a researcher could not prove that there is no benefit to eating dark chocolate—only that there is not enough evidence to support the claim that there is a benefit.

Making Type I or Type II Errors  Remember that a type I error is rejecting a null hypothesis that is true and a type II error is failing to reject a null hypothesis that is false. You can decrease the probability of a type I error by lowering the level of significance $\alpha$. Generally, when you decrease the probability of making a type I error, you increase the probability $\beta$ of making a type II error. Which error is more serious? It depends on the situation. In a criminal trial, a type I error is considered worse, as explained on page 352. If you are testing a person for a disease and they are assumed to be disease-free ($H_0$), then a type II error is more serious because you would fail to detect the disease even though the person has it. You can decrease the chance of making both types of errors by increasing the sample size.

Do You Consider the Amount of Federal Income Tax You Pay as Too High, About Right, or Too Low?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Too low</td>
<td>6%</td>
</tr>
<tr>
<td>About right</td>
<td>37%</td>
</tr>
<tr>
<td>Too high</td>
<td>57%</td>
</tr>
</tbody>
</table>

EXERCISES

In Exercises 1–3, assume that you work for the Internal Revenue Service. You are asked to write a report about the claim that 57% of U.S. adults think the amount of federal income tax they pay is too high. (Source: Gallup)

1. What is the null hypothesis in this situation? Describe how your report could be incorrect by trying to prove the null hypothesis.

2. Describe how your report could make a type I error.

3. Describe how your report could make a type II error.
## What Did You Learn?

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<tr>
<td>How to identify type I and type II errors</td>
<td>2</td>
<td>7–10</td>
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<tr>
<td>How to know whether to use a one-tailed or a two-tailed statistical test and find a $P$-value</td>
<td>3</td>
<td>7–10</td>
</tr>
<tr>
<td>How to interpret a decision based on the results of a statistical test</td>
<td>4</td>
<td>7–10</td>
</tr>
<tr>
<td>How to write a claim for a hypothesis test</td>
<td>5</td>
<td>7–10</td>
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<td>How to use rejection regions for a $z$-test for a mean $\mu$ when $\sigma$ is known</td>
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<td>How to use the $t$-test to test a mean $\mu$ when $\sigma$ is not known</td>
<td>4, 5</td>
<td>35–42</td>
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<td>How to use technology to find $P$-values and use them with a $t$-test to test a mean $\mu$ when $\sigma$ is not known</td>
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<td>51–54</td>
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<td>How to use the chi-square test to test a variance $\sigma^2$ or a standard deviation $\sigma$</td>
<td>4–6</td>
<td>55–62</td>
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Review Exercises

1. $H_0: \mu \leq 375$ (claim); $H_a: \mu > 375$
2. $H_0: \mu = 82$ (claim); $H_a: \mu \neq 82$
3. $H_0: p \geq 0.205$
   $H_a: p < 0.205$ (claim)
4. $H_0: \mu = 150,020$
   $H_a: \mu \neq 150,020$ (claim)
5. $H_0: \sigma \leq 1.9$; $H_a: \sigma > 1.9$ (claim)
6. $H_0: p \geq 0.64$ (claim); $H_a: p < 0.64$
7. See Odd Answers, page A72.
11. $0.1736; \text{Fail to reject } H_0.$
12. $0.0102; \text{Reject } H_0.$
15. See Odd Answers, page A72.
17. Fail to reject $H_0$ because $-1.645 < z < 1.645.$
18. Reject $H_0$ because $z > 1.645.$
19. Fail to reject $H_0$ because $-1.645 < z < 1.645.$
20. Reject $H_0$ because $z < -1.645.$
21. Fail to reject $H_0.$ There is not enough evidence at the 5% level of significance to reject the claim.

Section 7.1

In Exercises 1–6, the statement represents a claim. Write its complement and state which is $H_0$ and which is $H_a.$

1. $\mu \leq 375$
2. $\mu = 82$
3. $p < 0.205$
4. $\mu \neq 150,020$
5. $\sigma > 1.9$
6. $p \geq 0.64$

In Exercises 7–10, (a) state the null and alternative hypotheses and identify which represents the claim, (b) describe type I and type II errors for a hypothesis test of the claim, (c) explain whether the hypothesis test is left-tailed, right-tailed, or two-tailed, (d) explain how you should interpret a decision that rejects the null hypothesis, and (e) explain how you should interpret a decision that fails to reject the null hypothesis.

7. A poll of U.S. adults who have volunteered their time or donated money to help clean up the environment finds that 65% support the claim. (Source: Rasmussen Reports)
8. An agricultural cooperative guarantees that the mean shelf life of a type of dried fruit is at least 400 days.
9. A nonprofit consumer organization says that the standard deviation of the fuel economies of its top-rated vehicles for a recent year is no more than 9.5 miles per gallon. (Adapted from Consumer Reports)
10. An energy bar maker claims that the mean number of grams of carbohydrates in one bar is less than 25.

Section 7.2

In Exercises 11 and 12, find the $P$-value for the hypothesis test with the standardized test statistic $z.$ Decide whether to reject $H_0$ for the level of significance $\alpha.$

11. Left-tailed test, $z = -0.94, \alpha = 0.05$
12. Two-tailed test, $z = 2.57, \alpha = 0.10$

In Exercises 13–16, find the critical value(s) and rejection region(s) for the type of $z$-test with level of significance $\alpha.$ Include a graph with your answer.

13. Left-tailed test, $\alpha = 0.02$
14. Two-tailed test, $\alpha = 0.005$
15. Right-tailed test, $\alpha = 0.025$
16. Two-tailed test, $\alpha = 0.03$

In Exercises 17–20, state whether the standardized test statistic $z$ allows you to reject the null hypothesis. Explain your reasoning.

17. $z = 1.631$
18. $z = 1.723$
19. $z = -1.464$
20. $z = -1.655$

In Exercises 21–24, test the claim about the population mean $\mu$ at the level of significance $\alpha.$ Assume the population is normally distributed.

21. Claim: $\mu \leq 45; \alpha = 0.05; \sigma = 6.7.$ Sample statistics: $\bar{x} = 47.2, n = 22$
22. Claim: $\mu \neq 8.45; \alpha = 0.03; \sigma = 1.75.$ Sample statistics: $\bar{x} = 7.88, n = 60$
23. Claim: $\mu < 5.500; \alpha = 0.01; \sigma = 0.011.$ Sample statistics: $\bar{x} = 5.497, n = 36$
24. Claim: $\mu = 7450; \alpha = 0.10; \sigma = 243.$ Sample statistics: $\bar{x} = 7495, n = 27$

FIGURE FOR EXERCISES 17–20

22. Reject $H_0.$ There is enough evidence at the 3% level of significance to support the claim.
23. Fail to reject $H_0.$ There is not enough evidence at the 1% level of significance to support the claim.
25. (a) The claim is “the mean annual production of cotton is 3.5 million bales per country.”

\[ H_0: \mu = 3.5 \quad \text{(claim)} \]
\[ H_a: \mu \neq 3.5 \]

(b) \(-2.06\)  
(c) 0.0394

(d) Reject \( H_0 \).

(e) There is enough evidence at the 5% level of significance to reject the researcher’s claim that the mean annual production of cotton is 3.5 million bales per country.

In Exercises 25 and 26, (a) identify the claim and state \( H_0 \) and \( H_a \), (b) find the standardized test statistic \( z \), (c) find the corresponding \( P \)-value, (d) decide whether to reject or fail to reject the null hypothesis, and (e) interpret the decision in the context of the original claim.

25. Cotton Production  A researcher claims that the mean annual production of cotton is 3.5 million bales per country. A random sample of 44 countries has a mean annual production of 2.1 million bales. Assume the population standard deviation is 4.5 million bales. At \( \alpha = 0.05 \), can you reject the claim?  \( \text{(Source: U.S. Department of Agriculture)} \)

26. Cotton Consumption  A researcher claims that the mean annual consumption of cotton is greater than 1.1 million bales per country. A random sample of 67 countries has a mean annual consumption of 1.0 million bales. Assume the population standard deviation is 4.3 million bales. At \( \alpha = 0.01 \), can you support the claim?  \( \text{(Source: U.S. Department of Agriculture)} \)

27. An environmental researcher claims that the mean amount of sulfur dioxide in the air in U.S. cities is 1.15 parts per billion. In a random sample of 134 U.S. cities, the mean amount of sulfur dioxide in the air is 0.93 parts per billion. Assume the population standard deviation is 2.62 parts per billion. At \( \alpha = 0.01 \), is there enough evidence to reject the claim?  \( \text{(Source: U.S. Environmental Protection Agency)} \)

28. A travel analyst claims that the mean price of a round trip flight from New York City to Los Angeles is less than $507. In a random sample of 55 round trip flights from New York City to Los Angeles, the mean price is $502. Assume the population standard deviation is $111. At \( \alpha = 0.05 \), is there enough evidence to support the travel analyst’s claim?  \( \text{(Adapted from Expedia)} \)

Section 7.3

In Exercises 29–34, find the critical value(s) and rejection region(s) for the type of \( t \)-test with level of significance \( \alpha \) and sample size \( n \).

29. Two-tailed test, \( \alpha = 0.05 \), \( n = 20 \)

30. Right-tailed test, \( \alpha = 0.01 \), \( n = 33 \)

31. Right-tailed test, \( \alpha = 0.02 \), \( n = 63 \)

32. Left-tailed test, \( \alpha = 0.05 \), \( n = 48 \)

33. Left-tailed test, \( \alpha = 0.005 \), \( n = 15 \)

34. Two-tailed test, \( \alpha = 0.02 \), \( n = 12 \)

In Exercises 35–40, test the claim about the population mean \( \mu \) at the level of significance \( \alpha \). Assume the population is normally distributed.

35. Claim: \( \mu > 12,700; \alpha = 0.005 \)

Sample statistics: \( \bar{x} = 12,855, s = 248, n = 21 \)

36. Claim: \( \mu \geq 0; \alpha = 0.10 \). Sample statistics: \( \bar{x} = -0.45, s = 2.38, n = 31 \)

37. Claim: \( \mu \leq 51; \alpha = 0.01 \). Sample statistics: \( \bar{x} = 52, s = 2.5, n = 40 \)

38. Claim: \( \mu < 850; \alpha = 0.025 \). Sample statistics: \( \bar{x} = 875, s = 25, n = 14 \)

39. Claim: \( \mu = 195; \alpha = 0.10 \). Sample statistics: \( \bar{x} = 190, s = 36, n = 101 \)

40. Claim: \( \mu \neq 3,330,000; \alpha = 0.05 \)

Sample statistics: \( \bar{x} = 3,293,995, s = 12,801, n = 35 \)

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41. (a) The claim is “the mean monthly cost of joining a health club is $25.”

\[ H_0: \mu = 25 \text{ (claim)} \]

\[ H_a: \mu \neq 25 \]

(b) \(-t_0 = -1.740, t_0 = 1.740\)

Rejection regions:
\[ t < -1.740, t > 1.740 \]

(c) 1.64 (d) Fail to reject \( H_0 \).

(e) There is not enough evidence at the 10% level of significance to reject the advertisement’s claim that the mean monthly cost of joining a health club is $25.

42. (a) The claim is “the mean cost of a yoga session is no more than $14.”

\[ H_0: \mu \leq 14 \text{ (claim)} \]

\[ H_a: \mu > 14 \]

(b) \( t_0 = 2.040 \)

Rejection region: \( t > 2.040 \)

(c) 3.46 (d) Reject \( H_0 \).

(e) There is enough evidence at the 2.5% level of significance to reject the magazine’s claim that the mean cost of a yoga session is no more than $14.

43. (a) The claim is “the mean score for grade 12 students on a science achievement test is more than 145.”

\[ H_0: \mu \leq 145 \text{ (claim)} \]

\[ H_a: \mu > 145 \]

(b) 0.0824 (c) Reject \( H_0 \).

(d) There is enough evidence at the 10% level of significance to support the education publication’s claim that the mean score for grade 12 students on a science achievement test is more than 145.

44. See Selected Answers, page A102.

45. See Odd Answers, page A72.

46. Can use normal distribution. Reject \( H_0 \). There is enough evidence at the 3% level of significance to reject the claim.

47. Can use normal distribution. Reject \( H_0 \). There is enough evidence at the 1% level of significance to support the claim.

48. Cannot use normal distribution.

In Exercises 41 and 42, (a) identify the claim and state \( H_0 \) and \( H_a \), (b) find the critical value(s) and identify the rejection region(s), (c) find the standardized test statistic \( t \), (d) decide whether to reject or fail to reject the null hypothesis, and (e) interpret the decision in the context of the original claim. Assume the population is normally distributed.

41. A fitness magazine advertises that the mean monthly cost of joining a health club is $25. You want to test this claim. You find that a random sample of 18 clubs has a mean monthly cost of $26.25 and a standard deviation of $3.23. At \( \alpha = 0.10 \), do you have enough evidence to reject the advertisement’s claim?

42. A fitness magazine claims that the mean cost of a yoga session is no more than $14. You want to test this claim. You find that a random sample of 32 yoga sessions has a mean cost of $15.59 and a standard deviation of $2.60. At \( \alpha = 0.025 \), do you have enough evidence to reject the magazine’s claim?

In Exercises 43 and 44, (a) identify the claim and state \( H_0 \) and \( H_a \), (b) use technology to find the P-value, (c) decide whether to reject or fail to reject the null hypothesis, and (d) interpret the decision in the context of the original claim. Assume the population is normally distributed.

43. An education publication claims that the mean score for grade 12 students on a science achievement test is more than 145. You want to test this claim. You randomly select 36 grade 12 test scores. The results are listed below. At \( \alpha = 0.1 \), can you support the publication’s claim? (Adapted from National Center for Education Statistics)

```
188  80  175  195  201  143  119  81  118  119 165  222
109 134  200  110  199  181  79  135  124  205  90  120
216 167  198  183  173  187  143  166  147  219  206  97
```

44. An education researcher claims that the overall average score of 15-year-old students on an international mathematics literacy test is 494. You want to test this claim. You randomly select the average scores of 33 countries. The results are listed below. At \( \alpha = 0.05 \), do you have enough evidence to reject the researcher’s claim? (Source: National Center for Education Statistics)

```
561  554  536  531  523  518  515  511  506  500  499
493  490  489  485  482  482  479  477  466  453  448
439  432  423  421  413  407  394  388  386  376  368
```

Section 7.4

In Exercises 45–48, determine whether a normal sampling distribution can be used to approximate the binomial distribution. If it can, test the claim.

45. Claim: \( p = 0.15; \alpha = 0.05 \)

Sample statistics: \( \hat{p} = 0.09, n = 40 \)

46. Claim: \( p = 0.65; \alpha = 0.03 \)

Sample statistics: \( \hat{p} = 0.76, n = 116 \)

47. Claim: \( p < 0.70; \alpha = 0.01 \)

Sample statistics: \( \hat{p} = 0.50, n = 68 \)

48. Claim: \( p \geq 0.04; \alpha = 0.10 \)

Sample statistics: \( \hat{p} = 0.03, n = 30 \)
49. (a) The claim is “over 40% of U.S. adults say they are less likely to travel to Europe in the next six months for fear of terrorist attacks.”

\[ H_0: p \leq 0.40 \]
\[ H_a: p > 0.40 \ (\text{claim}) \]

(b) \( z_0 = 2.33 \)

Rejection region: \( z > 2.33 \)

(c) 1.29

(d) Fail to reject \( H_0 \).

(e) There is not enough evidence at the 1% level of significance to support the polling agency’s claim that over 40% of U.S. adults say they are less likely to travel to Europe in the next six months for fear of terrorist attacks.

In Exercises 49 and 50, (a) identify the claim and state \( H_0 \) and \( H_a \), (b) find the critical value(s) and identify the rejection region(s), (c) find the standardized test statistic \( z \), (d) decide whether to reject or fail to reject the null hypothesis, and (e) interpret the decision in the context of the original claim.

49. A polling agency reports that over 40% of U.S. adults say they are less likely to travel to Europe in the next six months for fear of terrorist attacks. In a random sample of 1000 U.S. adults, 42% said they are less likely to travel to Europe in the next six months for fear of terrorist attacks. At \( \alpha = 0.01 \), is there enough evidence to support the agency’s claim? (Adapted from Rasmussen Reports)

50. A labor researcher claims that 6% of U.S. employees say it is likely they will be laid off in the next year. In a random sample of 547 U.S. employees, 44 said it is likely they will be laid off in the next year. At \( \alpha = 0.05 \), is there enough evidence to reject the researcher’s claim? (Adapted from Gallup)

Section 7.5

In Exercises 51–54, find the critical value(s) and rejection region(s) for the type of chi-square test with sample size \( n \) and level of significance \( \alpha \).

51. Right-tailed test, \( n = 20, \alpha = 0.05 \)

52. Two-tailed test, \( n = 14, \alpha = 0.01 \)

53. Two-tailed test, \( n = 41, \alpha = 0.10 \)

54. Left-tailed test, \( n = 6, \alpha = 0.05 \)

In Exercises 55–58, test the claim about the population variance \( \sigma^2 \) or standard deviation \( \sigma \) at the level of significance \( \alpha \). Assume the population is normally distributed.

55. Claim: \( \sigma^2 \geq 2; \alpha = 0.10 \). Sample statistics: \( s^2 = 2.95, n = 18 \)

56. Claim: \( \sigma^2 \leq 60; \alpha = 0.025 \). Sample statistics: \( s^2 = 72.7, n = 15 \)

57. Claim: \( \sigma = 1.25; \alpha = 0.05 \). Sample statistics: \( s = 1.03, n = 6 \)

58. Claim: \( \sigma \neq 0.035; \alpha = 0.01 \). Sample statistics: \( s = 0.026, n = 16 \)

In Exercises 59 and 60, (a) identify the claim and state \( H_0 \) and \( H_a \), (b) find the critical value(s) and identify the rejection region(s), (c) find the standardized test statistic \( \chi^2 \), (d) decide whether to reject or fail to reject the null hypothesis, and (e) interpret the decision in the context of the original claim. Assume the population is normally distributed.

59. A bolt manufacturer makes a type of bolt to be used in airtight containers. The manufacturer claims that the variance of the bolt widths is at most 0.01. A random sample of 28 bolts has a variance of 0.064. At \( \alpha = 0.005 \), is there enough evidence to reject the claim?

60. A restaurant claims that the standard deviation of the lengths of serving times is 3 minutes. A random sample of 27 serving times has a standard deviation of 3.9 minutes. At \( \alpha = 0.01 \), is there enough evidence to reject the claim?

61. In Exercise 59, is there enough evidence to reject the claim at the \( \alpha = 0.01 \) level? Explain.

62. In Exercise 60, is there enough evidence to reject the claim at the \( \alpha = 0.05 \) level? Explain.
Chapter Quiz

1. (a) The claim is “the mean hat size for a male is at least 7.25.”
   
   \[ H_0: \mu = 7.25 \quad \text{(claim)} \]
   \[ H_a: \mu < 7.25 \]

   (b) Left-tailed because the alternative hypothesis contains <; z-test because \( \sigma \) is known and the population is normally distributed.

   (c) Sample answer: \( z_0 = -2.33 \); rejection region: \( z < -2.33 \; \text{or} \; -1.28 \)

   (d) Fail to reject \( H_0 \).

   (e) There is not enough evidence at the 1% level of significance to reject the organization’s claim that the mean hat size for a male is at least 7.25.

2. (a) The claim is “the mean daily base price for renting a full-size or less expensive vehicle in Vancouver, Washington, is more than $36.”
   
   \[ H_0: \mu = 36 \]
   \[ H_a: \mu > 36 \]

   (b) Right-tailed because the alternative hypothesis contains \( >; \) z-test because \( \sigma \) is known and \( n \geq 30 \).

   (c) Sample answer: \( z_0 = 1.28 \); rejection region: \( z > 1.28 \; \text{or} \; 1.997 \)

   (d) Reject \( H_0 \).

   (e) There is enough evidence at the 10% level of significance to support the travel analyst’s claim that the mean daily base price for renting a full-size or less expensive vehicle in Vancouver, Washington, is more than $36.

3. See Odd Answers, page A73.
4. See Odd Answers, page A73.
5. See Odd Answers, page A73.
6. See Odd Answers, page A73.

Take this quiz as you would take a quiz in class. After you are done, check your work against the answers given in the back of the book.

For each exercise, perform the steps below.

(a) Identify the claim and state \( H_0 \) and \( H_a \).

(b) Determine whether the hypothesis test is left-tailed, right-tailed, or two-tailed, and whether to use a z-test, a t-test, or a chi-square test. Explain your reasoning.

(c) Choose one of the options.

Option 1: Find the critical value(s), identify the rejection region(s), and find the appropriate standardized test statistic.

Option 2: Find the appropriate standardized test statistic and the P-value.

(d) Decide whether to reject or fail to reject the null hypothesis.

(e) Interpret the decision in the context of the original claim.

1. A hat company claims that the mean hat size for a male is at least 7.25. A random sample of 12 hat sizes has a mean of 7.15. At \( \alpha = 0.01 \), can you reject the company’s claim? Assume the population is normally distributed and the population standard deviation is 0.27.

2. A travel analyst claims the mean daily base price for renting a full-size or less expensive vehicle in Vancouver, Washington, is more than $36. You want to test this claim. In a random sample of 40 full-size or less expensive vehicles available to rent in Vancouver, Washington, the mean daily base price is $42. Assume the population standard deviation is $19. At \( \alpha = 0.10 \), do you have enough evidence to support the analyst’s claim? (Adapted from Expedia)

3. A government agency reports that the mean amount of earnings for full-time workers ages 18 to 24 with a bachelor’s degree in a recent year is $47,254. In a random sample of 15 full-time workers ages 18 to 24 with a bachelor’s degree, the mean amount of earnings is $50,781 and the standard deviation is $5290. At \( \alpha = 0.05 \), is there enough evidence to support the claim? Assume the population is normally distributed. (Adapted from U.S. Census Bureau)

4. A weight loss program claims that program participants have a mean weight loss of at least 10.5 pounds after 1 month. The weight losses after 1 month (in pounds) of a random sample of 40 program participants are listed below. At \( \alpha = 0.01 \), is there enough evidence to reject the program’s claim?

   4.7  6.0  7.2  8.3  9.2  10.1  14.0  11.7  12.8  10.8
   11.0  7.2  8.0  4.7  11.8  10.7  6.1  8.8  7.7  8.5
   9.5  10.2  5.6  6.9  7.9  8.6  10.5  9.6  5.7  9.6
   12.6  12.9  6.8  12.0  5.1  14.0  9.7  10.8  9.1  12.9

5. A nonprofit consumer organization says that less than 18% of the vehicles the organization rated in a recent year have an overall score of 78 or more. In a random sample of 90 vehicles the organization rated in a recent year, 20% have an overall score of 78 or more. At \( \alpha = 0.05 \), can you support the organization’s claim? (Adapted from Consumer Reports)

6. In Exercise 5, the nonprofit consumer organization says that the standard deviation of the vehicle rating scores is 11.90. A random sample of 90 vehicle rating scores has a standard deviation of 11.96. At \( \alpha = 0.10 \), is there enough evidence to reject the organization’s claim? Assume the population is normally distributed. (Adapted from Consumer Reports)
Chapter Test

1. (a) The claim is “more than 30% of adults have purchased a meal kit in a recent year.”
   \[ H_0: p \leq 0.30 \]
   \[ H_A: p > 0.30 \] (claim)
   (b) Right-tailed because the alternative hypothesis contains >; \( z \)-test because \( np \geq 5 \) and \( nq \geq 5 \).
   (c) Sample answer: \( z_0 = 1.28 \); Rejection region: \( z > 1.28 \); \(-0.65\)
   (d) Fail to reject \( H_0 \).
   (e) There is not enough evidence at the 10% level of significance to support the retail grocery chain’s claim that more than 30% of adults have purchased a meal kit in a recent year.

2. (a) The claim is “the mean of the room rates for two adults at three-star hotels in Salt Lake City is $134.”
   \[ H_0: \mu = 134 \] (claim)
   \[ H_A: \mu \neq 134 \]
   (b) Two-tailed because the alternative hypothesis contains \( \neq \); \( z \)-test because \( \sigma \) is known and \( n \geq 30 \).
   (c) Sample answer:
   \[ z_0 = -1.645, z_0 = 1.645; \]
   Rejection regions:
   \( z < -1.645, z > 1.645; 1.82 \)
   (d) Reject \( H_0 \).
   (e) There is enough evidence at the 10% level of significance to reject the travel analyst’s claim that the mean of the room rates for two adults at three-star hotels in Salt Lake City is $134.

5. See Selected Answers, page A103.
7. See Selected Answers, page A103.

Take this test as you would take a test in class.

For each exercise, perform the steps below.
(a) Identify the claim and state \( H_0 \) and \( H_A \).
(b) Determine whether the hypothesis test is left-tailed, right-tailed, or two-tailed, and whether to use a \( z \)-test, a \( t \)-test, or a chi-square test. Explain your reasoning.
(c) Choose one of the options.
   Option 1: Find the critical value(s), identify the rejection region(s), and find the appropriate standardized test statistic.
   Option 2: Find the appropriate standardized test statistic and the \( P \)-value.
(d) Decide whether to reject or fail to reject the null hypothesis.
(e) Interpret the decision in the context of the original claim.

1. A retail grocery chain owner claims that more than 30% of adults have purchased a meal kit in a recent year. In a random sample of 36 adults, 25% have purchased a meal kit in a recent year. At \( \alpha = 0.10 \), is there enough evidence to support the owner’s claim? \( \text{(Adapted from Harris Interactive)} \)

2. A travel analyst claims that the mean of the room rates for two adults at three-star hotels in Salt Lake City is $134. In a random sample of 37 three-star hotels in Salt Lake City, the mean room rate for two adults is $143. Assume the population standard deviation is $30. At \( \alpha = 0.10 \), is there enough evidence to reject the analyst’s claim? \( \text{(Adapted from Expedia)} \)

3. A travel analyst says that the mean price of a meal for a family of 4 in a recent year is at most $100. A random sample of 36 meal prices has a mean of $110 and a standard deviation of $19. At \( \alpha = 0.05 \), is there enough evidence to reject the analyst’s claim?

4. A research center claims that more than 80% of U.S. adults think that mothers should have paid maternity leave. In a random sample of 50 U.S. adults, 82% think that mothers should have paid maternity leave. At \( \alpha = 0.05 \), is there enough evidence to support the center’s claim? \( \text{(Adapted from Pew Research Center)} \)

5. A nutrition bar manufacturer claims that the standard deviation of the number of grams of carbohydrates in a bar is 1.11 grams. A random sample of 26 bars has a standard deviation of 1.19 grams. At \( \alpha = 0.05 \), is there enough evidence to reject the manufacturer’s claim? Assume the population is normally distributed.

6. A nonprofit consumer organization says that the mean price of the vehicles the organization rated in a recent year is at least $41,000. In a random sample of 150 vehicles the organization rated in a recent year, the mean price is $40,600 and the standard deviation is $17,300. At \( \alpha = 0.01 \), is there enough evidence to reject the organization’s claim? \( \text{(Adapted from Consumer Reports)} \)

7. A researcher claims that the mean age of the residents of a small town is more than 38 years. The ages (in years) of a random sample of 30 residents are listed below. At \( \alpha = 0.10 \), is there enough evidence to support the researcher’s claim? Assume the population standard deviation is 9 years.

\[
41 \quad 44 \quad 40 \quad 30 \quad 29 \quad 46 \quad 42 \quad 53 \quad 21 \quad 29 \quad 43 \quad 46 \quad 39 \quad 35 \quad 33 \\
42 \quad 35 \quad 43 \quad 35 \quad 24 \quad 21 \quad 29 \quad 24 \quad 25 \quad 85 \quad 56 \quad 82 \quad 87 \quad 72 \quad 31
\]
The charts show results of studies on four-year colleges in the United States. You want to portray your college in a positive light for an advertising campaign designed to attract high school students. You decide to use hypothesis tests to show that your college is better than the average in certain aspects.

**EXERCISES**

1. **What Would You Test?**
   What claims could you test if you wanted to convince a student to come to your college? Suppose the student you are trying to convince is mainly concerned with (a) affordability, (b) having a good experience, and (c) graduating and starting a career. List one claim for each case. State the null and alternative hypotheses for each claim.

2. **Choosing a Random Sample**
   Classmates suggest conducting the following sampling techniques to test various claims. Determine whether the sample will be random. If not, suggest an alternative.
   
   (a) Survey all the students you have class with and ask about the average time they spend daily on different activities.
   
   (b) Randomly select former students from a list of recent graduates and ask whether they are employed.
   
   (c) Randomly select students from a directory, ask how much debt money they borrowed to pay for college this year, and multiply by four.

3. **Supporting a Claim**
   You want your test to support a positive claim about your college, not just fail to reject one. Should you state your claim so that the null hypothesis contains the claim or the alternate hypothesis contains the claim? Explain.

4. **Testing a Claim**
   You want to claim that students at your college graduate with an average debt of less than $25,000. A random sample of 40 recent graduates has a mean amount borrowed of $23,475 and a standard deviation of $8000. At $\alpha = 0.05$, is there enough evidence to support your claim?

5. **Testing a Claim**
   You want to claim that your college has a freshmen retention rate of at least 80%. You take a random sample of 60 of last year’s freshmen and find that 54 of them still attend your college. At $\alpha = 0.05$, is there enough evidence to reject your claim?

6. **Conclusion**
   Test one of the claims you listed in Exercise 1 and interpret the results. Discuss any limits of your sampling process.
The Case of the Vanishing Women

From 1966 to 1968, Dr. Benjamin Spock and others were tried for conspiracy to violate the Selective Service Act by encouraging resistance to the Vietnam War. By a series of three selections, no women ended up being on the jury. In 1969, Hans Zeisel wrote an article in *The University of Chicago Law Review* using statistics and hypothesis testing to argue that the jury selection was biased against Dr. Spock. Dr. Spock was a well-known pediatrician and author of books about raising children. Millions of mothers had read his books and followed his advice. Zeisel argued that, by keeping women off the jury, the court prejudiced the verdict.

The jury selection process for Dr. Spock’s trial is shown at the right.

Stage 1. The clerk of the Federal District Court selected 350 people “at random” from the Boston City Directory. The directory contained several hundred names, 53% of whom were women. However, only 102 of the 350 people selected were women.

Stage 2. The trial judge, Judge Ford, selected 100 people “at random” from the 350 people. This group was called a venire and it contained only nine women.

Stage 3. The court clerk assigned numbers to the members of the venire and, one by one, they were interrogated by the attorneys for the prosecution and defense until 12 members of the jury were chosen. At this stage, only one potential female juror was questioned, and she was eliminated by the prosecutor under his quota of peremptory challenges (for which he did not have to give a reason).

**EXERCISES**

1. The Minitab display below shows a hypothesis test for a claim that the proportion of women in the city directory is \( p = 0.53 \). In the test, \( n = 350 \) and \( \hat{p} = 0.291 \). Should you reject the claim? What is the level of significance? Explain.

2. In Exercise 1, you rejected the claim that \( p = 0.53 \). But this claim was true. What type of error is this?

3. When you reject a true claim with a level of significance that is virtually zero, what can you infer about the randomness of your sampling process?

4. Describe a hypothesis test for Judge Ford’s “random” selection of the venire. Use a claim of \( p = \frac{102}{350} = 0.291 \).

   (a) Write the null and alternative hypotheses.

   (b) Use technology to perform the test.

   (c) Make a decision.

   (d) Interpret the decision in the context of the original claim. Could Judge Ford’s selection of 100 venire members have been random?

---

**Minitab**

Test and CI for One Proportion

Test of \( p = 0.53 \) vs \( p \neq 0.53 \)

<table>
<thead>
<tr>
<th>Sample</th>
<th>X</th>
<th>N</th>
<th>Sample p</th>
<th>99 % CI</th>
<th>Z-Value</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>102</td>
<td>350</td>
<td>0.291429</td>
<td>(0.228862, 0.353995)</td>
<td>-8.94</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Using the normal approximation.
Using Technology to Perform Hypothesis Tests

Here are some Minitab and TI-84 Plus printouts for some of the examples in this chapter.

See Example 5, page 367.

**Minitab**

**One-Sample Z**

Test of $\mu = 68.3$ vs $\neq 68.3$
The assumed standard deviation = 3.5

<table>
<thead>
<tr>
<th>N</th>
<th>Mean</th>
<th>SE Mean</th>
<th>95% CI</th>
<th>Z</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>67.200</td>
<td>0.700</td>
<td>(65.828, 68.572)</td>
<td>-1.57</td>
<td>0.116</td>
</tr>
</tbody>
</table>

See Example 4, page 380.

**Minitab**

**One-Sample T**

Test of $\mu = 21000$ vs $< 21000$

<table>
<thead>
<tr>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE Mean</th>
<th>95% Upper Bound</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>19189</td>
<td>2950</td>
<td>788</td>
<td>20585</td>
<td>-2.30</td>
<td>0.019</td>
</tr>
</tbody>
</table>

See Example 2, page 390.

**Minitab**

**Test and CI for One Proportion**

Test of $p = 0.51$ vs $p \neq 0.51$

<table>
<thead>
<tr>
<th>Sample</th>
<th>X</th>
<th>N</th>
<th>Sample p</th>
<th>90% CI</th>
<th>Z-Value</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1161</td>
<td>2202</td>
<td>0.527248</td>
<td>(0.509748, 0.544748)</td>
<td>1.62</td>
<td>0.105</td>
</tr>
</tbody>
</table>

Using the normal approximation.
Using Technology to Perform Hypothesis Tests

See Example 9, page 371.

See Example 5, page 381.

See Example 1, page 389.