ELEMENTARY & MIDDLE SCHOOL MATHEMATICS

TEACHING DEVELOPMENTALLY, 5/E

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CHAPTER

9

Developing Early Number Concepts and Number Sense

Number is a complex and multifaceted concept. A rich understanding of number, a relational understanding, involves many different ideas, relationships, and skills. Children come to school with many ideas about number. These ideas should be built upon as we work with children and help them develop new relationships. It is sad to see the large number of students in grades 4, 5, and above who essentially know little more about number than how to count. It takes time and lots of experiences for children to develop a full understanding of number that will grow and enhance all of the further number-related concepts of the school years.

This chapter looks at the development of number ideas for numbers up to about 20. These foundational ideas can all be extended to larger numbers, operations, basic facts, and computation.

Big Ideas

1. Counting tells how many things are in a set. When counting a set of objects, the last word in the counting sequence names the quantity for that set.
2. Numbers are related to each other through a variety of number relationships. The number 7, for example, is more than 4, two less than 9, composed of 3 and 4 as well as 2 and 5, is three away from 10, and can be quickly recognized in several patterned arrangements of dots. These ideas further extend to an understanding of 17, 57, and 370.
3. Number concepts are intimately tied to the world around us. Application of number relationships to the real world marks the beginning of making sense of the world in a mathematical manner.

Content Connections

Early number development is related to other areas in the curriculum in two ways: content that interacts with and enhances the development of number and content that is directly affected by how well early number concepts have been developed. Measurement, data, and operation meanings fall in the first category. Basic facts, place value, and computation fall in the second.

- **Operations** (Chapter 10): As children solve story problems for any of the four operations, they count on, count back, make and count groups, and make comparisons. In the process, they form new relationships and methods of working with numbers.
- **Measurement** (Chapter 19): The determination of measures of length, height, size, or weight is an important use of number for the young child. Measurement involves meaningful counting and comparing (number relationships) and connects number to the world in which the child lives.
- **Data** (Chapter 21): Data, like measurement, involve counts and comparisons to both aid in developing number and connecting it to the real world.
- **Basic Facts** (Chapter 11): A strategy or way of thinking about nearly every one of the addition and subtraction facts can be directly traced to number relationships.
- **Place Value and Computation** (Chapters 12 and 13): Many of the ideas that contribute to computational fluency and flexibility with numbers are clear extensions of how numbers are related to ten and how numbers can be taken apart and recombined in different ways.

Number Development in Pre-K and Kindergarten

Parents help children count their fingers, toys, people at the table, and other small sets of objects. Questions concerning "who has more?" or "are there enough?" are part of the daily lives of children as young as 2 or 3 years of age. Considerable evidence indicates that these children have some understanding of the concepts of number and counting (Baroody, 1987; Fuson, 1988; Gelman & Gallistel, 1978; Gelman & Meck, 1986; Ginsburg, 1977).

The Relationships of More, Less, and Same

The concepts of "more," "less," and "same" are basic relationships contributing to the overall concept of number. Children begin to develop these ideas before they begin school. An entering kindergarten child can almost always choose the set that is *more* if presented with two sets that are quite obviously different in number. In fact, Baroody (1987) states, "A child unable to use 'more' in this intuitive manner is at considerable educational risk" (p. 29). Classroom activities should help children build on this basic notion and refine it.

Though the concept of less is logically equivalent to the concept of more (selecting the set with more is the same as *not* selecting the set with less), the word *less* proves to be more difficult for children than *more*. A possible explanation is that children have many opportunities to use the word *more* but have limited exposure to the word *less*. To help children with the concept of less, frequently pair it with the word *more* and make a conscious effort to ask "which is less?" questions as well as "which is more?" questions. For example, suppose that your class has correctly selected the set that has more from two that are given. Immediately follow with the question "Which is less?" In this way, the less familiar term and concept can be connected with the better-known idea.

For all three concepts (more, less, and same), children should construct sets using counters as well as make comparisons or choices between two given sets. The activities described here include both types. These activities should be conducted in a spirit of inquiry followed whenever possible with requests for explanations. "Why do you think this set has less?"

ACTIVITY 9.1

Make Sets of More/Less/Same

At a workstation or table, provide about eight cards with sets of 4 to 12 objects, a set of small counters or blocks, and some word cards labeled *More, Less,* and *Same.* Next to each card have students make three collections of counters: a set that is more, one that is less, and one that is the same. The appropriate labels are placed on the sets (see Figure 9.1).

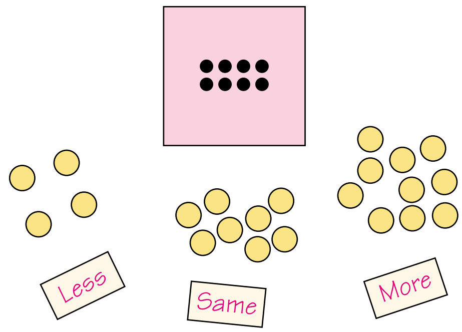


FIGURE 9.1 **Making sets that are more, less, and the same.**

In Activity 9.1, students create a set with counters, which gives them the opportunity to reflect on the sets and adjust them as they work. The next activity is done without counters. Although it addresses the same basic ideas, it provides a different problem situation.

ACTIVITY 9.2

Find the Same Amount

Give children a collection of cards with sets on them. Dot cards are one possibility (see the Blackline Masters). Have the children pick up any card in the collection and then find another card with the same amount to form a pair. Continue to find other pairs.

Activity 9.2 can be altered to have children find dot cards that are "less" or "more."

Observe children as they do this task. Children whose number ideas are completely tied to counting and nothing more will select cards at random and count each dot. Others will begin by selecting a card that appears to have about the same number of dots. This is a significantly higher level of understanding. Also observe how the dots are counted. Are the counts made accurately? Is each counted only once? A significant milestone for children occurs when they begin recognizing small patterned sets without counting.

STOP You have begun to see some of the early foundational ideas about number. Stop now and make a list of all of the important ideas that you think children should *know about 8* by the time they finish first grade. (The number 8 is used as an example. The list could be about any number from, say, 6 to 12.) Put your thoughts aside and we will revisit these ideas later.

Early Counting

Meaningful counting activities can begin in preschool. Generally, children at midyear in kindergarten should have a fair understanding of counting, but children must construct this idea.

It cannot be forced. Only the counting sequence is a rote procedure. The *meaning* attached to counting is the key conceptual idea on which all other number concepts are developed.

The Development of Counting Skills

Counting involves at least two separate skills. First, one must be able to produce the standard list of counting words in order: "One, two, three, four, ..." Second, one must be able to connect this sequence in a one-to-one manner with the items in the set being counted. Each item must get one and only one count.

Experience and guidance are the major factors in the development of these counting skills. Many children come to kindergarten able to count sets of ten or beyond. At the same time, children from impoverished backgrounds may require considerable practice to make up their experience deficit. The size of the set is also a factor related to success in counting. Obviously, longer number strings require more practice to learn. The first 12 counts involve no pattern or repetition, and many children do not recognize a pattern in the teens. Children still learning the skills of counting—that is, matching oral number words with objects—should be given sets of blocks or counters that they can move or pictures of sets that are arranged for easy counting.

Meaning Attached to Counting

Fosnot and Dolk (2001) make it very clear that an understanding of cardinality and the connection to counting is not a simple matter for 4-year-olds. Children will learn *how* to count (matching counting words with objects) before they understand that the last count word indicates the *amount* of the set or the *cardinality* of the set. Children who have made this connection are said to have the *cardinality principle,* which is a refinement of their early ideas about quantity. Most, but certainly not all, children by age $4\frac{1}{2}$ have made this connection (Fosnot & Dolk, 2001; Fuson & Hall, 1983).

To determine if young children have the cardinality rule, listen to how they respond when you discuss counting tasks with them. You ask, "How many are here?" The child counts correctly, and says, "Nine." Ask, "Are there nine?" Before developing cardinality, children may count again or will hesitate. Children with cardinality are apt to emphasize the last count, will explain that there are nine "Because I just counted them," and can use counting to find a matching set. Fosnot and Dolk discuss a class of 4-year-olds in which children who knew there were 17 children in the class were unsure how many milk cartons they should get so that each could have one.

To develop their understanding of counting, engage children in almost any game or activity that involves counts and comparisons. The following is a simple suggestion.

ACTIVITY 9.3

Fill the Chutes

Create a simple game board with four "chutes." Each consists of a column of about twelve 1-inch squares with a star at the top. Children take turns rolling a die and collecting the indicated number of counters. They then place these counters in one of the chutes. The object is to fill all of the chutes with counters. As an option, require that the chutes be filled exactly. A roll of 5 cannot be used to fill a chute with four spaces.

This "game" provides opportunities for you to talk with children about number and assess their thinking. Watch how the children count the dots on the die. Ask, "How do you know you have the right number of counters?" and "How many counters did you put in the chute? How many more do you need to fill the chute?"

Activities 9.1 and 9.2 also provide opportunities for diagnosis. Regular classroom activities, such as counting how many napkins are needed at snack time, are additional opportunities for children to learn about number and for teachers to listen to their students' ideas.

Numeral Writing and Recognition

Helping children read and write single-digit numerals is similar to teaching them to read and write letters of the alphabet. Neither has anything to do with number concepts. Traditionally, instruction has involved various forms of repetitious practice. Children trace over pages of numerals, repeatedly write the numbers from 0 to 10, make the numerals from clay, trace them in sand, write them on the chalkboard or in the air, and so on.

The calculator is a good instructional tool for numeral recognition. In addition to helping children with numerals, early activities can help develop familiarity with the calculator so that more complex activities are possible.

ACTIVITY 9.4

Find and Press

Every child should have a calculator. Always begin by having the children press the clear key. Then you say a number, and the children press that number on the calculator. If you have an overhead calculator, you can then show the children the correct key so that they can confirm their responses, or you can write the number on the board for children to check. Begin with single-digit numbers. Later, progress to two or three numbers called in succession. For example, call, "Three, seven, one." Children press the complete string of numbers as called.

Perhaps the most common kindergarten exercises have children match sets with numerals. Children are given pictured sets and asked to write or match the number that tells how many. Alternatively, they may be given a number and told to make or draw a set with that many objects. Many teacher resource books describe cute learning center activities where children put a numeral with the correct-sized set—numbered frogs on lily pads (with dots), for example. It is important to note that these frequently overworked activities involve only the skills of counting sets and numeral recognition or writing. When children are successful with these activities, little is gained by continuing to do them.

Technology Note

Computer software that allows children to create sets on the screen with the click of a mouse is quite common. *Unifix Software* (Hickey, 1996) is an electronic version of the popular Unifix cubes, plastic cubes that snap together to make bars. The software allows the teacher to add features to counting activities that are not available with the cubes alone. In its most basic form, children can click to create as many single cubes as they wish. They can link cubes to make bars of cubes, break the bars, move them around, add sounds to each cube, and more. The teacher can choose to have a numeral appear on each bar showing the total. From one to four loops can be created, with the loop total another option. Not only can students count to specified numbers and have the numerals appear for reinforcement, but also they can informally begin to explore the idea that two quantities can form a larger amount.

Do not let these computer tools become toys. It is important to keep the task problem based. For example, in early kindergarten, students could make a set of single blocks in one loop that is just as many as (or more or less than) a bar the teacher has made in the first loop. (Files can be prepared ahead of time.) Later students can explore different combinations of two bars that would equal a third bar. ■

Counting On and Counting Back

Although the forward sequence of numbers is relatively familiar to most young children, counting on and counting back are difficult skills for many. Frequent short practice drills are recommended.

ACTIVITY 9.5

Up and Back Counting

Counting up to and back from a target number in a rhythmic fashion is an important counting exercise. For example, line up five children and five chairs in front of the class. As the whole class counts from 1 to 5, the children sit down one at a time. When the target number, 5, is reached, it is repeated; the child who sat on 5 now stands, and the count goes back to 1. As the count goes back, the children stand up one at a time, and so on, "1, 2, 3, 4, 5, 5, 4, 3, 2, 1, 1, 2, ..." Kindergarten and first-grade children find exercises such as this both fun and challenging. Any movement (clapping, turning around, doing jumping jacks) can be used as the count goes up and back in a rhythmic manner.

The calculator provides an excellent counting exercise for young children because they see the numerals as they count.

ACTIVITY 9.6

Calculator Up and Back

Have each child press $\boxed{+}$ 1 $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$. The display will go from 1 to 5 with each $\boxed{=}$ press. The count should also be made out loud in a rhythm as in the other exercises. To start over, press the clear key and repeat. Counting up and back is also possible, but the end numbers will not be repeated. The following illustrates the key presses and what the children would say in rhythm:

$\boxed{+}$ 1 $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{-}$ 1 $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{+}$ 1 $\boxed{=}$ $\boxed{=}$ . . .

"1, 2, 3, 4, 5, minus 1, 4, 3, 2, 1, plus 1, 2, 3, . . . "

The last two activities are designed only to help students become fluent with the number words in both forward and reverse order and to begin counts with numbers other than 1. Although not at all easy for young students, these activities do not address counting on or counting back in a meaningful manner. Fosnot and Dolk (2001) describe the ability to count on as a "landmark" on the path to number sense. The next two activities are designed for that purpose.

ACTIVITY 9.7

Counting On with Counters

Give each child a collection of 10 or 12 small counters that the children line up left to right on their desks. Tell them to count four counters and push them under their left hands (see Figure 9.2). Then say, "Point to your hand. How many are there?" (Four.) "So let's count like this: f-o-u-r (pointing to their hand), five, six, ..." Repeat with other numbers under the hand.

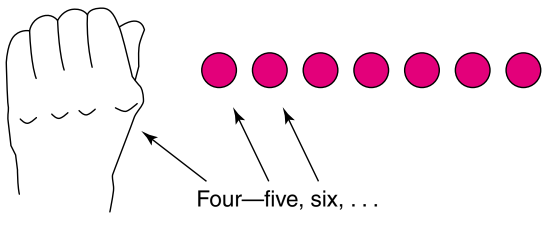


FIGURE 9.2 **Counting on: "Hide four. Count, starting from the number of counters hidden."**

The following activity addresses the same concept in a bit more problem-based manner.

ACTIVITY 9.8

Real Counting On

This "game" for two children requires a deck of cards with numbers 1 to 7, a die, a paper cup, and some counters. The first player turns over the top number card and places the indicated number of counters in the cup. The card is placed next to the cup as a reminder of how many are there. The second child rolls the die and places that many counters next to the cup. (See Figure 9.3.) Together they decide how many counters in all. A record sheet with columns for "In the Cup," "On the Side," and "In All" is an option. The largest number in the card deck can be adjusted if needed.

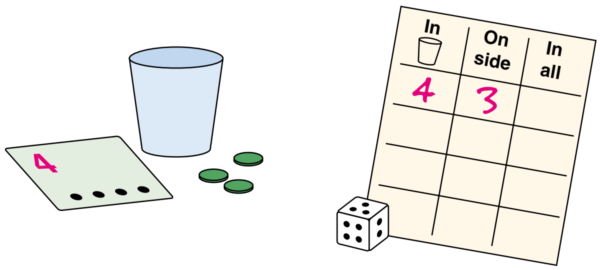


FIGURE 9.3 **How many in all? How do children count to tell the total? Dump the counters? Count up from 1 without dumping the counters? Count on?**

Watch how children determine the total amounts in this last activity. Children who are not yet counting on may want to dump the counters from the cup or will count up from one without dumping out the counters. Be sure to permit these strategies. As children continue to play, they will eventually count on as that strategy becomes meaningful and useful.

Early Number Sense

Number sense was a term that became popular in the late 1980s, even though terms such as this have somewhat vague definitions. Howden (1989) described number sense as a "good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms" (p. 11). This may still be the best definition.

NCTM Standards

In *Principles and Standards,* the term *number sense* is used freely throughout the Number and Operation standard. This example is from the K–2 level: "As students work with numbers, they gradually develop flexibility in thinking about numbers, which is a hallmark of number sense. . . . Number sense develops as students understand the size of numbers, develop multiple ways of thinking about and representing numbers, use numbers as referents, and develop accurate perceptions about the effects of operations on numbers" (p. 80).

The discussion of number sense begins in this book with the remainder of this chapter as we look at the kinds of relationships and connections children should be making about smaller numbers up to about 20. But "good intuition about numbers" does not end with these smaller whole numbers. Children continue to develop number sense as they begin to use numbers in operations, build an understanding of place value, and devise flexible methods of computing and making estimates involving large numbers. Flexible, intuitive thinking with numbers—number sense—should continue to be developed throughout the school years as fractions, decimals, and percents are added to students' repertoire of number ideas.

The early number ideas that have been discussed to this point in the chapter are the rudimentary aspects of number. Unfortunately, too many traditional programs move directly from these beginning ideas to addition and subtraction, leaving students with a very limited collection of ideas about number to bring to these new topics. The result is often that children continue to count by ones to solve simple story problems and have difficulty mastering basic facts. Early number sense development should demand significantly more attention than it is given in most traditional K–2 programs.

Relationships Among Numbers 1 Through 10

Once children have acquired a concept of cardinality and can meaningfully use their counting skills, little more is to be gained from the kinds of counting activities described so far. More relationships must be created for children to develop number sense, a flexible concept of number not completely tied to counting.

A Collection of Number Relationships

Figure 9.4 illustrates the four different types of relationships that children can and should develop with numbers:

- *Spatial relationships:* Children can learn to recognize sets of objects in patterned arrangements and tell how many without counting. For most numbers, there are several common patterns. Patterns can also be made up of two or more easier patterns for smaller numbers.
- *One and two more, one and two less:* The two-more-than and two-less-than relationships involve more than just the ability to count on two or count back two. Children should know that 7, for example, is 1 more than 6 and also 2 less than 9.

- *Anchors or "benchmarks" of 5 and 10:* Since 10 plays such a large role in our numeration system and because two fives make up 10, it is very useful to develop relationships for the numbers 1 to 10 to the important anchors of 5 and 10.
- *Part-part-whole relationships:* To conceptualize a number as being made up of two or more parts is the most important relationship that can be developed about numbers. For example, 7 can be thought of as a set of 3 and a set of 4 or a set of 2 and a set of 5.

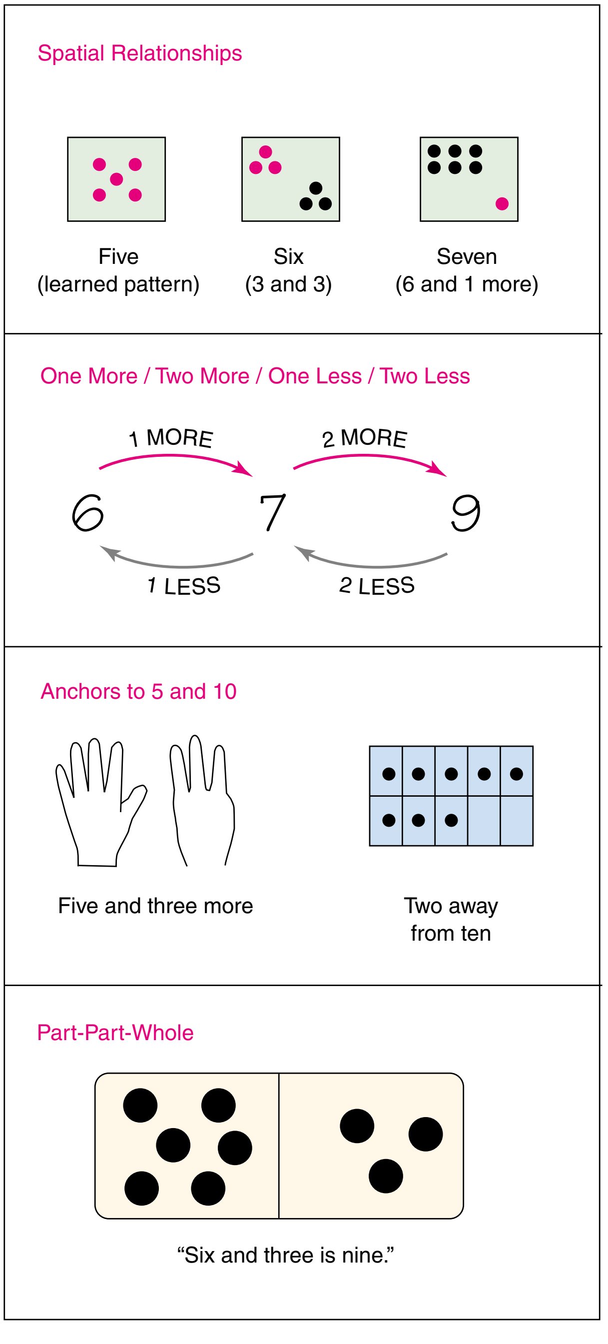


FIGURE 9.4 **Four relationships to be developed involving small numbers.**

The principal tool that children will use as they construct these relationships is the one number tool they possess: counting. Initially, then, you will notice a lot of counting, and you may wonder if you are making progress. Have patience! Counting will become less and less necessary as children construct these new relationships and begin to use the more powerful ideas.

Spatial Relationships: Patterned Set Recognition

Many children learn to recognize the dot arrangements on standard dice due to the many games they have played that use dice. Similar instant recognition can be developed for other patterns as well. The activities suggested here encourage reflective thinking about the patterns so that the relationships will be constructed. Quantities up to 10 can be known and named without the routine of counting. This can then aid in counting on (from a known patterned set) or learning combinations of numbers (seeing a pattern of two known smaller patterns).

A good set of materials to use in pattern recognition activities is a set of dot plates. These can be made using small paper plates and the peel-off dots commonly available in office supply stores. A reasonable collection of patterns is shown in Figure 9.5.

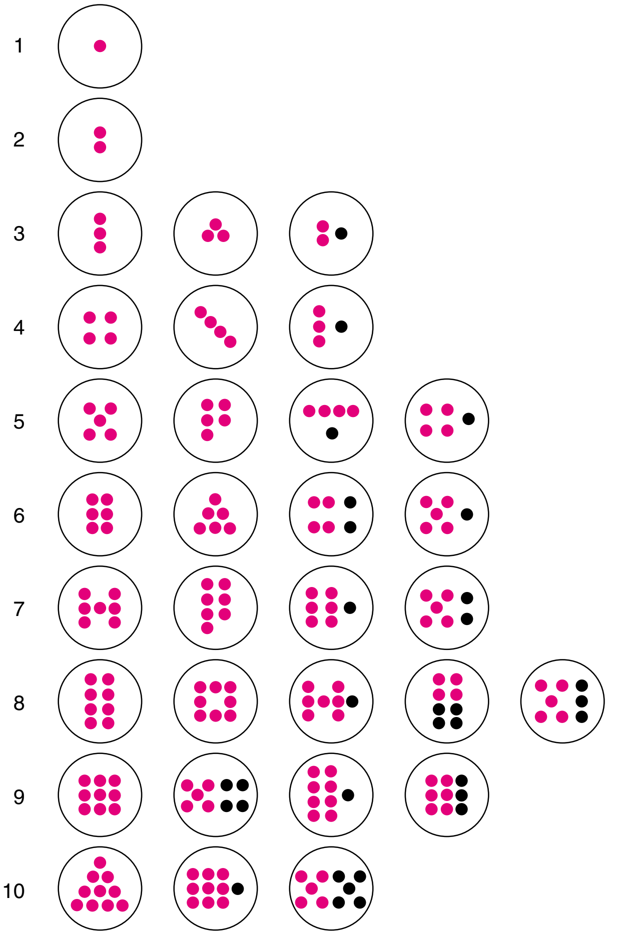


FIGURE 9.5 **A useful collection of dot patterns for "dot plates."**

Note that some patterns are combinations of two smaller patterns or a pattern with one or two additional dots. These should be made in two colors. Keep the patterns compact. If the dots are spread out, the patterns are hard to see.

ACTIVITY 9.9

Learning Patterns

To introduce the patterns, provide each student with about ten counters and a piece of construction paper as a mat. Hold up a dot plate for about 3 seconds. “Make the pattern you saw using the counters on the mat. How many dots did you see? How did you see them?” Spend some time discussing the configuration of the pattern and how many dots. Do this with a few new patterns each day.

ACTIVITY 9.10

Dot Plate Flash

Hold up a dot plate for only 1 to 3 seconds. “How many? How did you see it?” Children like to see how quickly they can recognize and say how many dots. Include lots of easy patterns and a few with more dots as you build their confidence. Students can also flash the dot plates to each other as a workstation activity.

ACTIVITY 9.11

Dominoes

Make a set of dominoes out of poster board and put a dot pattern on each end. The dominoes can be about 2 inches by 4 inches. The same patterns can appear on lots of dominoes with different pairs of patterns making up each one. Let the children play dominoes in the regular way, matching up the ends. As a speed activity, spread out all of the dominoes and see how fast the children play all of the dominoes or play until no more can be played. Regular dominoes could also be used, but there are not as many patterns.

The instant recognition activities with the plates are exciting and can be done in 5 minutes at any time of day or between lessons. There is value in using them at any primary grade level and at any time of year.

One and Two More, One and Two Less

When children count, they have no reason to reflect on the way one number is related to another. The goal is only to match number words with objects until they reach the end of the count. To learn that 6 and 8 are related by the twin relationships of “two more than” and “two less than” requires reflection on these ideas within tasks that permit counting. Counting on (or back) one or two counts is a useful tool in constructing these ideas.

ACTIVITY 9.12

One-Less-Than Dominoes

Use the dot-pattern dominoes or a standard set to play “one-less-than” dominoes. Play in the usual way, but instead of matching ends, a new domino can be added if it has an end that is one less than the end on the board. A similar game can be played for two less, one more, or two more.

The following activities can be done for any of the four relationships; each will be described for only one.

ACTIVITY 9.13

Make a Two-More-Than Set

Provide students with about six dot cards. Their task is to construct a set of counters that is two more than the set shown on the card. Similarly, spread out eight to ten dot cards, and find another card for each that is two less than the card shown. (Omit the 1 and 2 cards for two less than, and so on.)

In activities where children find a set or make a set, they can add a numeral card (a small card with a number written on it) to all of the sets involved. They can also be encouraged to take turns reading a number sentence to their partner. If, for example, a set has been made that is two more than a set of four, the child can read this by saying the number sentence, “Two more than four is six.”

The calculator can be an exciting device to practice the relationships of one more than, two more than, one less than, and two less than.

ACTIVITY 9.14

A Calculator Two-More-Than Machine

Teach children how to make a two-more-than machine. Press 0 + 2 =. This makes the calculator a two-more-than machine. Now press any number—for example, 5. Children hold their finger over the = key and predict the number that is two more than 5. Then they press = to confirm. If they do not press any of the operation keys (+, −, ×, ÷) the “machine” will continue to perform in this way.

What is really happening in the two-more-than machine is that the calculator "remembers" or stores the last operation, in this case "+2," and adds that to whatever number is in the window when the $\boxed{=}$ key is pressed. If the child continues to press $\boxed{=}$, the calculator will count by twos. At any time, a new number can be pressed followed by the equal key. To make a two-less-than machine, press 2 $\boxed{-}$ 2 $\boxed{=}$. (The first press of 2 is to avoid a negative number.) In the beginning, students forget and press operation keys, which change what their calculator is doing. Soon, however, they get the hang of using the calculator as a machine.

Anchoring Numbers to 5 and 10

Here again, we want to help children relate a given number to other numbers, specifically 5 and 10. These relationships are especially useful in thinking about various combinations of numbers. For example, in each of the following, consider how the knowledge of 8 as "5 and 3 more" and as "2 away from 10" can play a role: $5 + 3$, $8 + 6$, $8 - 2$, $8 - 3$, $8 - 4$, $13 - 8$. (It may be worth stopping here to consider the role of 5 and 10 in each of these examples.) Later similar relationships can be used in the development of mental computation skills on larger numbers such as $68 + 7$.

The most common and perhaps most important model for this relationship is the ten-frame. Generally attributed to Robert Wirtz (1974), the ten-frame is simply a 2×5 array in which counters or dots are placed to illustrate numbers (see Figure 9.6). Ten-frames can be simply drawn on a full sheet of construction paper (or use the Blackline Master). Nothing fancy is required, and each child can have one. The ten-frame has been incorporated into a variety of activities in this book and is now popular in standard textbooks for children.

For children in kindergarten or early first grade who have not yet explored a ten-frame, it is a good idea to begin with a five-frame. This row of five sections is also drawn on a sheet of construction paper (or use the Blackline Master). Provide children with about ten counters that will fit in the five-frame sections, and conduct the following activity.

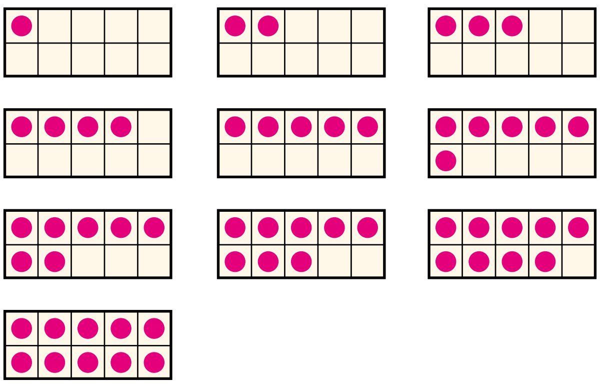


FIGURE 9.6 **Ten-frames.**

ACTIVITY 9.15

Five-Frame Tell-About

Explain that only one counter is permitted in each section of the five-frame. No other counters are allowed on the five-frame mat. Have the children show 3 on their five-frame. "What can you tell us about 3 from looking at your mat?" After hearing from several children, try other numbers from 0 to 5. Children may place their counters on the five-frame in any manner. What they observe will differ a great deal from child to child. For example, with four counters, a child with two on each end may say, "It has a space in the middle" or "It's two and two." There are no wrong answers. Focus attention on how many more counters are needed to make 5 or how far away from 5 a number is. Next try numbers between 5 and 10. The rule of one counter per section still holds. As shown in Figure 9.7, numbers greater than 5 are shown with a full five-frame and additional counters on the mat but not in the frame. In discussion, focus attention on these larger numbers as 5 and some more: "Eight is five and three more."

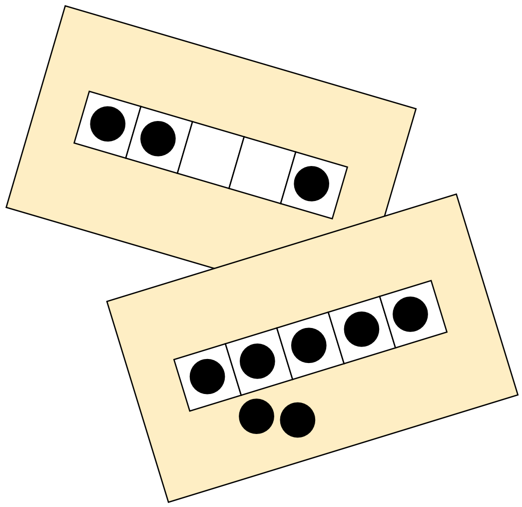


FIGURE 9.7 **A five-frame focuses on the 5 anchor. Counters are placed one to a section, and students tell how they see their number in the frame.**

Notice that the five-frame really focuses on the relationship to 5 as an anchor for numbers but does not anchor numbers to 10. When five-frames have been used for a week or so, introduce ten-frames. You may want to play a ten-frame version of a "Five-Frame Tell-About" but soon introduce the following rule for showing numbers on the ten-frame: *Always fill the top row first, starting on the left, the same way you read. When the top row is full, counters can be placed in the bottom row, also from the left.* This will produce the "standard" way to show numbers on the ten-frame as in Figure 9.6.

For a while, many children will count every counter on their ten-frame. Some will take all counters off and begin each number from a blank frame. Others will soon learn to adjust numbers by adding on or taking off only what is required, often capitalizing on a row of five without counting. Do not pressure students. With continued practice, all students will grow. How they are using the ten-frame provides insight into students' current number concept development.

ACTIVITY 9.16

Crazy Mixed-Up Numbers

This activity is adapted from *Mathematics Their Way* (Baratta-Lorton, 1976). All children make their ten-frame show the same number. The teacher then calls out random numbers between 0 and 10. After each number, the children change their ten-frames to show the new number. Children can play this game independently by preparing lists of about 15 "crazy mixed-up numbers." One child plays "teacher," and the rest use the ten-frames. Children like to make up their own number lists.

"Crazy Mixed-Up Numbers" is much more of a problem than it first appears. How do you decide how to change your ten-frame? Some children will wipe off the entire frame and start over with each number. Others will have learned what each number looks like. To add another dimension, have the children tell, *before changing their ten-frames,* how many more counters need to be added ("plus") or removed ("minus"). They then call out plus or minus whatever amount is appropriate. If, for example, the frames showed 6, and the teacher called out "four," the children would respond, "Minus two!" and then change their ten-frames accordingly. A discussion of how they know what to do is valuable.

Ten-frame flash cards are an important variation of ten-frames. Make cards from poster board about the size of a small index card, with a ten-frame on each and dots drawn in the frames. A set of 20 cards consists of a 0 card, a 10 card, and two each of the numbers 1 to 9. The cards allow for simple drill activities to reinforce the 5 and 10 anchors.

ACTIVITY 9.17

Ten-Frame Flash

Flash ten-frame cards to the class or group, and see how fast the children can tell how many dots are shown. This activity is fast-paced, takes only a few minutes, can be done at any time, and is a lot of fun if you encourage speed.

Important variations of "Ten-Frame Flash" include

- Saying the number of spaces on the card instead of the number of dots
- Saying one more than the number of dots (or two more, and also less than)
- Saying the "ten fact"—for example, "Six and four make ten"

Ten-frame tasks are surprisingly problematic for students. Students must reflect on the two rows of five, the spaces remaining, and how a particular number is more or less than 5 and how far away from 10. The early discussions of how numbers are seen on the five-frames or ten-frames are examples of brief "after" activities in which students learn from one another.

Part-Part-Whole Relationships

STOP Before reading on, get some counters or coins. Count out a set of eight counters in front of you as if you were a first- or second-grade child counting them.

Any child who has learned how to count meaningfully can count out eight objects as you just did. What is significant about the experience is what it did *not* cause you to think about. Nothing in counting a set of eight objects will cause a child to focus on the fact that it could be made of two parts. For example, separate the counters you just set out into two piles and reflect on the combination. It might be 2 and 6 or 7 and 1 or 4 and 4. Make a change in your two piles of counters and say the new combination to yourself. Focusing on a quantity in terms of its parts has important implications for developing number sense. A noted researcher in children's number concepts, Lauren Resnick (1983) states:

> Probably the major conceptual achievement of the early school years is the interpretation of numbers in terms of part and whole relationships. With the application of a Part-Whole schema to quantity, it becomes possible for children to think about numbers as compositions of other numbers. This enrichment of number understanding permits forms of mathematical problem solving and interpretation that are not available to younger children. (p. 114)

A study of kindergarten children examined the effects of part-part-whole activities on number concepts (Fischer, 1990). With only 20 days of instruction to develop the part-part-whole structure, children showed significantly higher achievement than the control group on number concepts, word problems, and place-value concepts.

Basic Ingredients of Part-Part-Whole Activities

Most part-part-whole activities focus on a single number for the entire activity. Thus, a child or group of children working together might work on the number 7 throughout the activity. Either children build the designated quantity in two or more parts, using a wide variety of materials and formats, or else they start with the full amount and separate it into two or more parts. A group of two or three children may work on one number in one activity for 5 to 20 minutes. Kindergarten children will usually begin these activities working on the number 4 or 5. As concepts develop, the children can extend their work to numbers 6 to 12. It is not unusual to find children in the second grade who have not developed firm part-part-whole constructs for numbers in the 7-to-12 range.

When children do these activities, have them say or "read" the parts aloud or write them down on some form of recording sheet (or do both). Reading or writing the combinations serves as a means of encouraging reflective thought focused on the part-whole relationship. Writing can be in the form of drawings, numbers written in blanks (________ and ________), or addition equations if these have been introduced ($3 + 5 = 8$). There is a clear connection between part-part-whole concepts and addition and subtraction ideas.

A special and important variation of part-part-whole activities is referred to as *missing-part* activities. In a missing-part activity, children know the whole amount and use their already developed knowledge of the parts of that whole to try to tell what the covered or hidden part is. If they do not know or are unsure, they simply uncover the unknown part and say the full combination as they would normally. Missing-part activities provide maximum reflection on the combinations for a number. They also serve as the forerunner to subtraction concepts. With a whole of 8 but with only 3 showing, the child can later learn to write "$8 - 3 = 5$."

Part-Part-Whole Activities

The following activity and its variations may be considered the "basic" part-part-whole activity.

ACTIVITY 9.18

Build It in Parts

Provide children with one type of material, such as connecting cubes or squares of colored paper. The task is to see how many different combinations for a particular number they can make using two parts. (If you wish, you can allow for more than two parts.) Each different combination can be displayed on a small mat, such as a quarter-sheet of construction paper. Here are just a few ideas, each of which is illustrated in Figure 9.8.

- **Use two-color counters such as lima beans spray painted on one side (also available in plastic).**
- **Make bars of connecting cubes. Make each bar with two colors. Keep the colors together.**
- **Color rows of squares on 1-inch grid paper.**
- **Make combinations using two dot strips—strips of poster board about 1 inch wide with stick-on dots. (Make lots of strips with from one to four dots and fewer strips with from five to ten dots.)**
- **Make combinations of "two-column strips." These are cut from tagboard ruled in 1-inch squares. All pieces except the single squares are cut from two columns of the tagboard.**

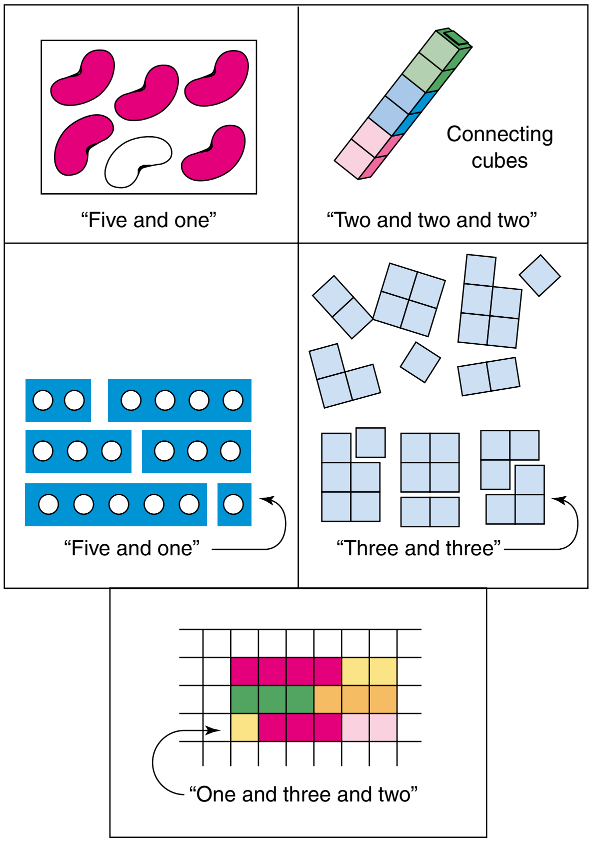


FIGURE 9.8 **Assorted materials for building parts of 6.**

As you observe children working on the "Build It in Parts" activity, ask them to "read" a number sentence to go with each of their designs. Encourage children to read their designs to each other. Two or three children working together with the same materials may have quite a large number of combinations including lots of repeats.

A note about the "two-column cards" mentioned in the activity and shown in Figure 9.8: In her unpublished research, Kuske (2001) makes the case that this two-column model for numbers is more valuable than dot patterns and ten-frames. Her reasoning is that the pieces are "additive." That is, the combination of any two of the pieces is another piece in the set, as can be seen in Figure 9.8. Of course, this is not true of dot patterns or ten-frames. In Kuske's research, sets of ten are given special importance and used as precursors to place value. For example, a 6 and a 7 piece make a 10 and 3 piece. Her research is quite interesting and, if nothing else, reinforces the importance of helping children think about numbers in terms of two parts.

In the "Build It in Parts" activity, the children are focusing on the combinations. To add some interest, vary the activity by adding a design component. Rather than create a two-part illustration for a number, they create an interesting design with an assigned number of elements. For each design, they are then challenged to see and read the design in two parts. Here are some ideas.

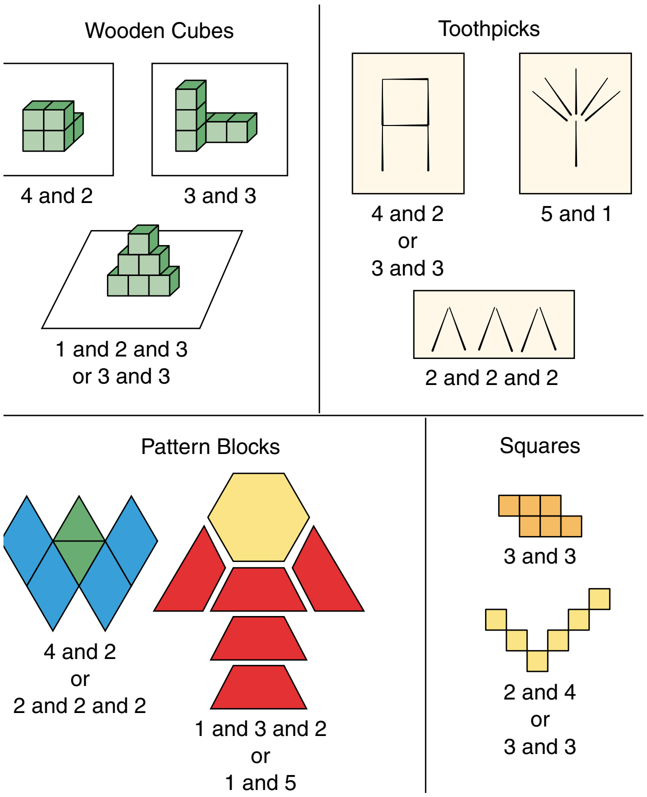


FIGURE 9.9 **Designs for 6.**

- Make arrangements of wooden cubes.
- Make designs with pattern blocks. It is a good idea to use only one or two shapes at a time.
- Make designs with flat toothpicks. These can be dipped in white glue and placed on small squares of construction paper to create a permanent record.
- Make designs with touching squares or triangles. Cut a large supply of small squares or triangles out of construction paper. These can also be pasted down.

It is both fun and useful to challenge children to see their designs in different ways, producing different number combinations. In Figure 9.9, decide how children look at the designs to get the combinations listed under each.

The following activity is strictly symbolic. However, children should use counters if they feel they need to.

ACTIVITY 9.19

Two out of Three

Make lists of three numbers, two of which total the whole that children are focusing on. Here is an example list for the number 5:

2—3—4
5—0—2
1—3—2
3—1—4
2—2—3
4—3—1

With the list on the board, overhead, or worksheet, children can take turns selecting the two numbers that make the whole. As with all problem-solving activities, children should be challenged to justify their answers.

Missing-Part Activities

Missing-part activities require some way for a part to be hidden or unknown. Usually this is done with two children working together or else in a teacher-directed manner with the class. Again, the focus of the activity remains on a single designated quantity as the whole. The next four activities illustrate variations of this important idea.

ACTIVITY 9.20

Covered Parts

A set of counters equal to the target amount is counted out, and the rest are put aside. One child places the counters under a margarine tub or piece of tagboard. The child then pulls some out into view. (This amount could be none, all, or any amount in between.) For example, if 6 is the whole and 4 are showing, the other child says, "Four and *two* is six." If there is hesitation or if the hidden part is unknown, the hidden part is immediately shown (see Figure 9.10).

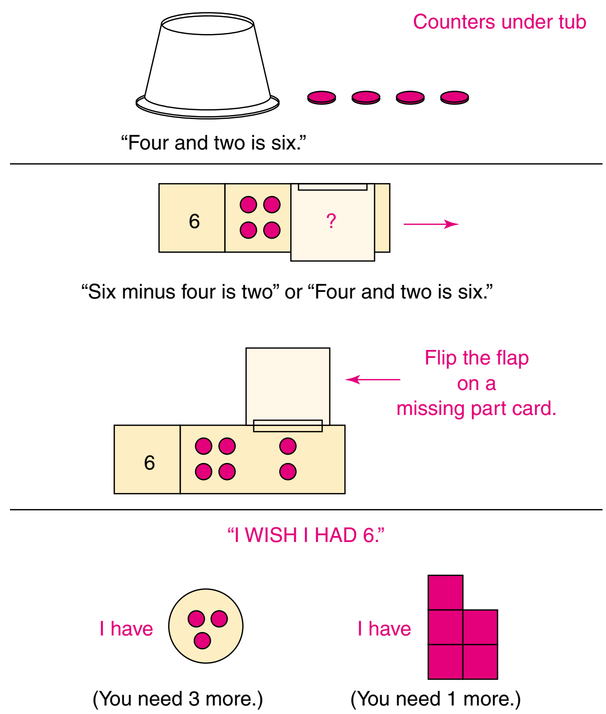


FIGURE 9.10 **Missing-part activities.**

ACTIVITY 9.21

Missing-Part Cards

For each number 4 to 10, make missing-part cards on strips of 3-by-9-inch tagboard. Each card has a numeral for the whole and two dot sets with one set covered by a flap. For the number 8, you need nine cards with the visible part ranging from 0 to 8 dots. Students use the cards as in "Covered Parts," saying, "Four and two is six" for a card showing four dots and hiding two (see Figure 9.10).

ACTIVITY 9.22

I Wish I Had

Hold out a bar of connecting cubes, a dot strip, a two-column strip, or a dot plate showing 6 or less. Say, "I wish I had six." The children respond with the part that is needed to make 6. Counting on can be used to check. The game can focus on a single whole, or the "I wish I had" number can change each time.

The following activity is completely symbolic.

ACTIVITY 9.23

Calculator Parts of 8 Machine

Make a parts of 8 machine by pressing 8 [−] 8 [=]. Now if any number from 0 to 8 is pressed followed by [=], the display shows the other part. (The second part shows as a negative number. Tell students that is how they can tell it is the second part.) Children should try to say the other part before they press [=]. Though this is basically a drill activity, a discussion with any child concerning his or her reasoning returns it to a problem orientation. Machines can be made for any number in the same way.

Technology Note

There are lots of ways you can use computer software to create part-part-whole activities. All that is needed is a program that permits students to create sets of objects on the screen. *Unifix Software* (Hickey, 1996) is especially nice for this purpose. One approach has students create two-part bars to match a single bar that you specify. Here students could make many bars all for the same number. Alternatively, students could put counters in two set loops formed by the computer. As always, the computer should offer something beyond the simple manipulative. In this instance, the advantage is your ability to control the type of feedback provided. Options include showing the total in a bar, the total in a loop, and the total of all blocks on the screen. For example, a student can make a bar of two and a bar of six with a numeral showing on each. When the bars are snapped together, the new bar total of eight appears.

Combining and Breaking Apart Numbers (Tenth Planet, 1998a) is specifically designed for part-part-whole activities. Although the early activities in this package are fairly passive and slow, the "Through" or last section makes the mathematics reasonably problematic. In various animated settings, children attempt to make a number either in two parts or by adding two groups and then removing one or two groups. This is an example of software that requires some teacher guidance to create good problems or else students will not stay engaged. The printed support material includes suggestions for corresponding off-line activities that make this a worthwhile program. ■

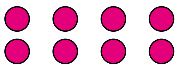
STOP Remember the list you made earlier in the chapter about what children should know about the number 8? Get it out now and see if you would add to it or revise it based on what you have read to this point. Do this before reading on.

Here is a possible list of the kinds of things that children should know about the number 8 (or any number up to about 12) by the end of the first grade. Children should be able to:

- Count to eight (know the number words).
- Count eight objects and know that the last number word tells how many.
- Write the numeral 8.
- Recognize the numeral 8.

The preceding list represents the minimal skills of number. In the following list are the relationships students should have that contribute to number sense:

- More and less by 1 and 2: 8 is one more than 7, one less than 9, two more than 6, and two less than 10.
- Spatial patterns for 8 such as



- Anchors to 5 and 10: 8 is 5 and 3 more and 2 away from 10.
- Part-whole relationships: 8 is 5 and 3, 2 and 6, 7 and 1, and so on.
- Other relationships such as

  Doubles: double 4 is 8.

  Relationships to the real world: 8 is one more than the days of the week, my brother is 8 years old, my reading book is 8 inches wide.

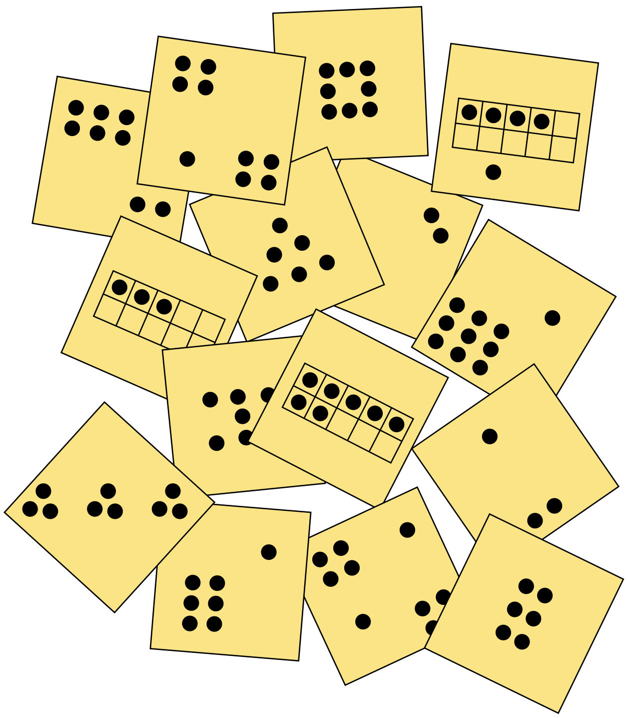


FIGURE 9.11 **Dot cards can be made using the Blackline Masters.**

Dot Card Activities

Many good number development activities involve more than one of the relationships discussed so far. As children learn about ten-frames, patterned sets, and other relationships, the dot cards in the Blackline Masters provide a wealth of activities (see Figure 9.11). The cards contain dot patterns, patterns that require counting, combinations of two and three simple patterns, and ten-frames with "standard" as well as unusual placements of dots. When children use these cards for almost any activity that involves number concepts, the cards make them think about numbers in many different ways. The dot cards add another dimension to many of the activities already described and can be used effectively in the following activities.

ACTIVITY 9.24

Double War

The game of "Double War" (Kamii, 1985) is played like war, but on each play, both players turn up two cards instead of one. The winner is the one with the larger total number. Children playing the game can use many different number relationships to determine the winner without actually finding the total number of dots.

ACTIVITY 9.25

Dot-Card Trains

Make a long row of dot cards from 0 up to 9, then go back again to 1, then up, and so on. Alternatively, begin with 0 or 1 and make a two-more/two-less train.

ACTIVITY 9.26

Difference War

Besides dealing out the cards to the two players as in regular "War," prepare a pile of about 50 counters. On each play, the players turn over their cards as usual. The player with the greater number of dots wins as many counters from the pile as the difference between the two cards. The players keep their cards. The game is over when the counter pile runs out. The player with the most counters wins the game.

ACTIVITY 9.27

Number Sandwiches

Select a number between 5 and 12, and find combinations of two cards that total that number. With the two cards students make a "sandwich" with the dot side out. When they have found at least ten sandwiches, the next challenge is to name the number on the other "slice" of the sandwich. The sandwich is turned over to confirm. The same pairs can then be used again to name the hidden part.

Assessment Notes

The four types of number relationships (spatial representations, one and two more or less than, 5 and 10 anchors, and part-whole relationships) provide an excellent reference for assessing where your students are relative to number concepts. If you have station activities for these relationships, careful observations alone will tell a lot about students' number concepts. For a more careful assessment, each relationship can be assessed separately in a one-on-one setting, taking only a few minutes.

With a set of dot plates and a set of ten-frame cards you can quickly check which dot patterns children recognize without counting and whether they recognize quantities on ten-frames. To check the one and two more or less than relationships, simply write a few numbers on a sheet of paper. Point to a number and have the child tell you the number that is "*two less than* this number," varying the specific request with different numbers. It is not necessary to check every possibility.

For part-whole relationships, use a missing-part assessment similar to Activity 9.20 ("Covered Parts") on p. 125. Begin

Investigations in Number, Data, and Space

Grade K, *How Many in All?*

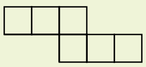
Investigation 2: Six Tiles

Context

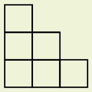
The activities in this investigation take place over one to two weeks near the end of the year but could be done earlier. The entire investigation focuses on the number 6. The authors explain that 6 is an amount most kindergarten children can count, even early in the year. Children need more than one hand to show 6. It provides enough two-part combinations to be interesting, and it is an amount that can be visually recalled and manipulated. Furthermore, most kindergarten children will turn 6 years old during the year.

Task Descriptions

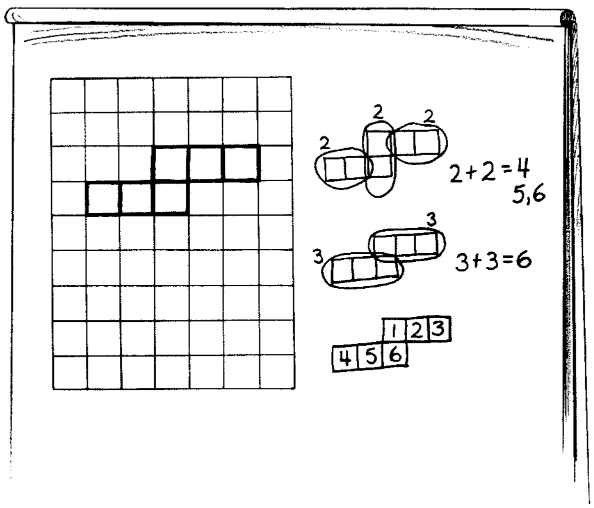
In "Cover Up," the teacher draws an arrangement of six squares on a grid so that all can see. For example:



or



Students are told to look at the drawings and discuss different ways they could remember what they see. Then the picture is covered up and students try to describe the drawing. The discussion is focused on ideas that get at the parts of the picture, not just that there are six squares. "Cover Up" is repeated with several different drawings. "Six Tiles in All" is a follow-up activity in which students paste down six squares on grid paper in as many ways as they can find. Squares have to touch on a full side. The different arrangements are shared and the teacher helps students see number combinations for 6 as in the teacher



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notes shown here. This investigation provides just one opportunity to naturally introduce the symbolism for addition as children say the combinations they see.

The authors point out that it is tempting to think that the activities in this unit could profitably be repeated for other numbers as well. Their experience, however, is that students get bored doing the same thing over and over. These ideas are important for other numbers, but students need different contexts and materials to maintain interest.

with a number you believe the child has "mastered," say, 5. Have the child count out that many counters into your open hand. Close your hand around the counters and confirm that she knows how many are hidden there. Then remove some and show them in the palm of your other hand. (See Figure 9.12.) Ask the child, "How many are hidden?" Repeat with different amounts removed, although it is only necessary to check three or four missing parts for each number. If the child responds quickly and correctly and is clearly not counting in any way, call that a "mastered number." If a number is mastered, repeat the entire process with the next higher number. Continue until the child begins to stumble. In early first grade you will find a range of mastered numbers from 4 to 7 or 8. By spring of the first grade, most children should have mastered numbers to 10. ■

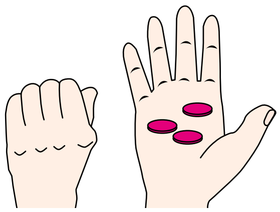


FIGURE 9.12 A missing-part number assessment. Eight in all. "How many are hidden?"

The *Investigations in Number, Data, and Space* curriculum for kindergarten integrates number development throughout most of the year. Four of seven units focus on the development of number, including the unit on data and measurement. The activity described in this excerpt provides some flavor of the *Investigations* program's early approach to number.

Relationships for Numbers 10 to 20

Even though kindergarten, first-, and second-grade children daily experience numbers up to 20 and beyond, it should not be assumed that they will automatically extend the set of relationships they have developed on smaller numbers to the numbers beyond 10. And yet these numbers play a big part in many simple counting activities, in basic facts, and in much of what we do with mental computation. Relationships with these numbers are just as important as relationships involving the numbers through 10.

A Pre–Place-Value Relationship with 10

A set of ten should play a major role in children's initial understanding of numbers between 10 and 20. When children see a set of six with a set of ten, they should know without counting that the total is 16. However, the numbers between 10 and 20 are not an appropriate place to discuss place-value concepts. That is, prior to a much more complete development of place-value concepts (appropriate for second grade and beyond), children should not be asked to explain the 1 in 16 as representing "one ten."

STOP

Say to yourself, "One ten." Now think about that from the perspective of a child just learning to count to 20! What could *one* ten possibly mean when *ten* tells me how many fingers I have and is the number that comes after nine? How can it be one?

The concept of a single ten is just too strange for a kindergarten or early first-grade child to grasp. Some would say that it is not appropriate for grade 1 at all (Kamii, 1985). The inappropriateness of discussing "one ten and six ones" (what's a one?) does not mean that a set of ten should not figure prominently in the discussion of the teen numbers. The following activity illustrates this idea.

ACTIVITY 9.28

Ten and Some More

Use a simple two-part mat, and have children count out ten counters onto one side. Next have them put five counters on the other side. Together count all of the counters by ones. Chorus the combination: "Ten and five is fifteen." Turn the mat around: "Five and ten is fifteen." Repeat with other numbers in a random order but without changing the 10 side of the mat.

Activity 9.28 is designed to teach new number names and, thus, requires a certain amount of directed teaching. Following this activity, explore numbers to 20 in a more open-ended manner. Provide each child with two ten-frames drawn one under the other on a construction paper mat or use the Blackline Master provided. In random order, have children show numbers to 20 on their mats. There is no preferred way to do this as long as there are the correct number of counters. What is interesting is to discuss how the counters can be placed on the mat so that it is easy to see how many are there. Have children share their ideas. Not every child will use a full set of ten, but as this idea becomes more popular, the notion that ten and some more is a teen amount will soon be developed. Do not forget to include numbers less than ten as well. As you listen to your children, you may want to begin challenging them to find ways to show 26 counters or even more.

Extending More and Less Relationships

The relationships of one more than, two more than, one less than, and two less than are important for all numbers. However, these ideas are built on or connected to the same concepts for numbers less than 10. The fact that 17 is one less than 18 is connected to the idea that 7 is one less than 8. Children may need help in making this connection.

ACTIVITY 9.29

More and Less Extended

On the overhead, show 7 counters, and ask what is two more, or one less, and so on. Now add a filled ten-frame to the display (or 10 in any pattern), and repeat the questions. Pair up questions by covering and uncovering the ten-frame as illustrated in Figure 9.13.

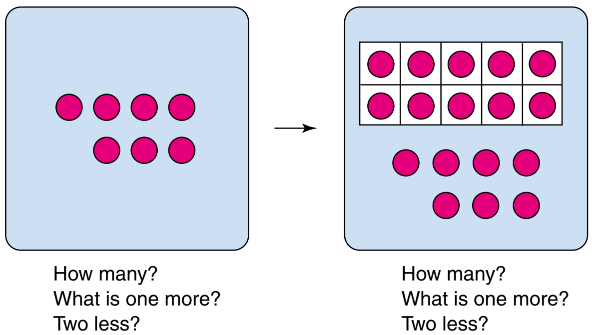


FIGURE 9.13 Extending relationships to the teens.

Double and Near-Double Relationships

The use of doubles (double 6 is 12) and near-doubles (13 is double 6 and 1 more) is generally considered a strategy for memorizing basic addition facts. There is no reason why children should not begin to develop these relationships long before they are concerned with memorizing basic facts. Doubles and near-doubles are simply special cases of the general part-part-whole construct.

Relate the doubles to special images. Thornton (1982) helped first graders connect doubles to these visual ideas:

Double 3 is the bug double: three legs on each side.

Double 4 is the spider double: four legs on each side.

Double 5 is the hand double: two hands.

Double 6 is the egg carton double: two rows of six eggs.

Double 7 is the two-week double: two weeks on the calendar.

Double 8 is the crayon double: two rows of eight crayons in a box.

Double 9 is the 18-wheeler double: two sides, nine wheels on each side.

Children can draw pictures or make posters that illustrate the doubles for each number. There is no reason that the images need be restricted to those listed here. Any images that are strong ideas for your children will be good for them.

Periodically conduct oral exercises in which students double the number you say. Ask children to explain how they knew a particular double. Many will not use the pictures.

ACTIVITY 9.30

The Double Maker

Make the calculator into a "double maker" by pressing 2 $\boxed{\times}$ $\boxed{=}$. Now a press of any digit followed by $\boxed{=}$ will produce the double of that number. Children can work in pairs or individually to try to beat the calculator.

As a related oral task, say a number, and ask students to tell what double it is. "What is fourteen?" (Double 7!) When students can do this well, use any number up to 20. "What is seventeen?" (Double 8 and 1 more.)

Number Sense and the Real World

Here we examine ways to broaden the early knowledge of numbers in a different way. Relationships of numbers to real-world quantities and measures and the use of numbers in simple estimations can help children develop the flexible, intuitive ideas about numbers that are most desired.

Estimation and Measurment

One of the best ways for children to think of real quantities is to associate numbers with measures of things. In the early grades, measures of length, weight, and time are good places to begin. Just measuring and recording results will not be very effective, however, since there is no reason for children to be interested in or think about the result. To help children think or reflect a bit on what number might tell how long the desk is or how heavy the book is, it would be good if they could first write down or tell you an estimate. To produce an estimate is, however, a very difficult task for young children. They do not understand the concept of "estimate" or "about." For example, suppose that you have cut out of poster board an ample supply of very large footprints, say, about 18 inches long. All are exactly the same size. You would like to ask the class, "About how many footprints will it take to measure across the rug in our reading corner?" The key word here is *about,* and it is one that you will need to spend a lot of time helping children understand. To this end, the request of an estimate can be made in ways that help with the concept of "about" yet not require students to give a specific number.

The following estimation questions can be used with most early estimation activities:

- *More or less than* ________? Will it be more or less than 10 footprints? Will the apple weigh more or less than 20 wooden blocks? Are there more or less than 15 connecting cubes in this long bar?
- *Closer to* ________ *or to* ________? Will it be closer to 5 footprints or closer to 20 footprints? Will the apple weigh closer to 10 blocks or closer to 30 blocks? Does this bar have closer to 10 cubes or closer to 50 cubes?
- *About* ________. Use one of these numbers: 5, 10, 15, 20, 25, 30, 35, 40, ... About how many footprints? About how many blocks will the apple weigh? About how many cubes are in this bar?

Asking for estimates using these formats helps children learn what you mean by "about." Every child can make an estimate without having to pull a number out of the air.

To help with numbers and measures, estimate several things in succession using the same unit. For example, suppose that you are estimating and measuring "around things" using a string. To measure, the string is wrapped around the object and then measured in some unit such as craft sticks. After measuring the distance around Demetria's head, estimate the distance around the wastebasket or around the globe or around George's wrist. Each successive measure helps children with the new estimates. See Chapter 19 for a complete discussion of measurement.

More Connections

Here are some additional activities that can help children connect numbers to real situations.

ACTIVITY 9.31

Add a Unit to Your Number

Write a number on the board. Now suggest some units to go with it, and ask the children what they can think of that fits. For example, suppose the number is 9. "What do you think of when I say 9 *dollars?* 9 *hours?* 9 *cars?* 9 *kids?* 9 *meters?* 9 *o'clock?* 9 *hand spans?* 9 *gallons?*" Spend some time in discussion of each. Let children suggest units as well. Be prepared to explore some of the ideas either immediately or as projects or tasks to share with parents at home.

ACTIVITY 9.32

Is It Reasonable?

Select a number and a unit—for example, 15 feet. Could the teacher be 15 feet tall? Could your living room be 15 feet wide? Can a man jump 15 feet high? Could three children stretch their arms 15 feet? Pick any number, large or small, and a unit with which children are familiar. Then make up a series of these questions.

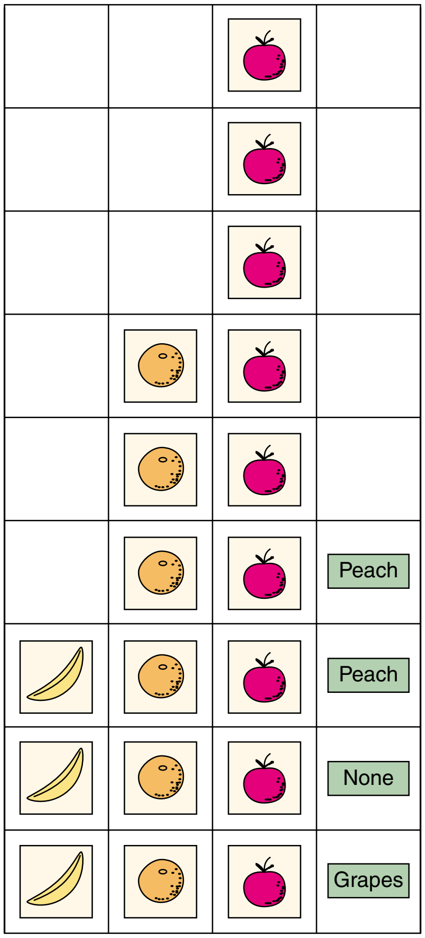
Once children are familiar with Activity 9.32, have them select the number and the unit or things (10 kids, 20 bananas, . . .), and see what kinds of questions children make up. When a difference of opinion develops, capitalize on the opportunity to explore and find out. Resist the temptation to supply your adult-level knowledge. Rather, say, "Well, how can we find out if it is or is not reasonable? Who has an idea about what we could do?"

These activities are problem-based in the truest sense. Not only are there no clear answers, but children can easily begin to pose their own questions and explore number in the part of the environment most interesting to them. Children will not have these real-world connections when you begin, and you may be disappointed in their limited ideas about number. Howden (1989) writes about a first-grade teacher of children from very impoverished backgrounds who told her, "They all have fingers, the school grounds are strewn with lots of pebbles and leaves, and pinto beans are cheap. So we count, sort, compare, and talk about such objects. We've measured and weighed almost everything in this room and almost everything the children can drag in" (p. 6). This teacher's children had produced a wonderfully rich and long list of responses to the question "What comes to your mind when I say twenty-four?" In another school in a professional community where test scores are high, the same question brought almost no response from a class of third graders. It can be a very rewarding effort to help children connect their number ideas to the real world.

Graphs

Graphing activities are another good way to connect children's worlds with number. Chapter 21 discusses ways to make graphs with children in grades K–2. Graphs can be quickly made of almost any data that can be gathered from the students: favorite ice cream, color, sports team, pet; number of sisters and brothers; kids who ride different buses; types of shoes; number of pets; and so on. Graphs can be connected to content in other areas. A unit on sea life might lead to a graph of favorite sea animals.

Once a simple bar graph is made, it is very important to take a few minutes to ask as many number questions as is appropriate for the graph. In the early stages of number development (grades K–1), the use of graphs for number relationships and for connecting numbers to real quantities in the children's environment is a more important reason for building graphs than the graphs themselves. The graphs focus attention on counts of realistic things. Equally important, bar graphs clearly exhibit comparisons between and among numbers that are rarely made when only one number or quantity is considered at a time. See Figure 9.14 for an example of a graph and questions that can be asked. At first, children will have trouble with the questions involving differences, but repeated exposure to these ideas in a bar graph format will



Class graph showing fruit brought for snack. Paper cutouts for bananas, oranges, apples, and cards for "others."

- — Which bar (or refer to what the graph represents) is most, least?
- — Which are more (less) than 7 (or some other number)?
- — Which is one less (more) than this bar?
- — How much more is ______ than ______?
  (Follow this question immediately by reversing the order and asking how much less.)
- — How much less is ______ than ______?
  (Reverse this question after receiving an answer.)
- — How much difference is there between ______ and ______ ?
- — Which two bars together are the same as ______ ?

FIGURE 9.14 Relationships and number sense in a bar graph.

improve their understanding. These comparison concepts add considerably to children's understanding of number.

NCTM Standards

The *Standards* clearly recognizes the value of integrating number development with other areas of the curriculum. "Students' work with numbers should be connected to their work with other mathematics topics. For example, computational fluency . . . can both enable and be enabled by students' investigations of data; a knowledge of patterns supports the development of skip counting and algebraic thinking; and experiences with shape, space, and number help students develop estimation skills related to quantity and size" (p. 79).

Literature Connections

Children's literature abounds with wonderful counting books. Involving children with books in a variety of ways can serve to connect number to reality, make it a personal experience, and provide ample opportunities for problem solving. Be sure to go beyond simply reading a counting book or a number-related book and looking at the pictures. Find a way to extend the book into the children's world. Create problems related to the story. Have children write a similar story. Extend the numbers and see what happens. Create a mural, graphs, or posters. The ideas are as plentiful as the books. Here are a few ideas for making literature connections to number concepts and number sense.

Anno's Counting House

Anno, 1982

In the beautiful style of Anno, this book shows ten children in various parts of a house. As the pages are turned, the house front covers the children, and a few are visible through cutout windows. A second house is on the opposite page. As you move through the book, the children move one at a time to the second house, creating the potential for a 10–0, 9–1, 8–2, . . . , 0–10 pattern of pairs. But as each page shows part of the children through the window, there is an opportunity to discuss how many in the missing part. Have children use counters to model the story as you "read" it the second or third time.

What if the children moved in pairs instead of one at a time? What if there were three houses? What if there were more children? What else could be in the house to count? How many rooms, pictures, windows? What about your house? What about two classrooms or two buses instead of houses?

The Very Hungry Caterpillar

Carle, 1969

This is a predictable-progression counting book about a caterpillar who eats first one thing, then two, and so on. Children can create their own eating stories and illustrate them. What if more than one type of thing were eaten at each stop? What combinations for each number are there? Are seven little things more or less than three very large things? What does all of this stuff weigh? How many things are eaten altogether?

Two Ways to Count to Ten

Dee, 1988

This Liberian folktale is about King Leopard in search of the best animal to marry his daughter. The task devised involves throwing a spear and counting to 10 before the spear lands. Many animals try and fail. Counting by ones proves too lengthy. Finally, the antelope succeeds by counting "2, 4, 6, 8, 10."

The story is a perfect lead-in to skip counting. Can you count to 10 by threes? How else can you count to 10? How many ways can you count to 48? What numbers can you reach if you count by fives? The size of the numbers you investigate is limited only by the children. A hundreds board or counters are useful thinker toys to help with these problems. Be sure to have children write about what they discover in their investigations.

Another fun book to use is *The King's Commissioners* (Friedman, 1994), a hilarious tale that also opens up opportunities to count by different groupings or skip counting. ■

Extensions to Early Mental Mathematics

Teachers in the second and third grades can capitalize on some of the early number relationships and extend them to numbers up to 100. A useful set of materials to help with these relationships is the little ten-frames found in the Blackline Masters. Each child should have a set of 10 tens and a set of frames for each number 1 to 9 with an extra 5.

The following three ideas are illustrated with the little ten-frames in Figure 9.15. First are the relationships of one more than and one less than. If you understand that one more than 6 is 7, then in a similar manner, ten more than 60 is 70 (that is, one more ten). The second idea is really a look ahead to fact strategies. If a child has learned to think about adding on to 8 or 9 by first adding up to 10 and then adding the rest, the extension to similar two-digit numbers is quite simple; see Figure 9.15(b). Finally, the most powerful idea for small numbers is thinking of them in parts. It is a very useful idea (though not one found in textbooks) to take apart larger numbers to begin to develop some flexibility in the same way. Children can begin by thinking of ways to take apart a multiple of 10 such as 80. Once they do it with tens, the challenge can be to think of ways to take apart 80 when one part has a 5 in it, such as 25 or 35.

More will be said about early mental computation in Chapter 13. The point to be made here is that early number relationships have a greater impact on what children know than may be apparent at first. Even teachers in the upper grades may profitably consider the use of ten-frames and part-part-whole activities.

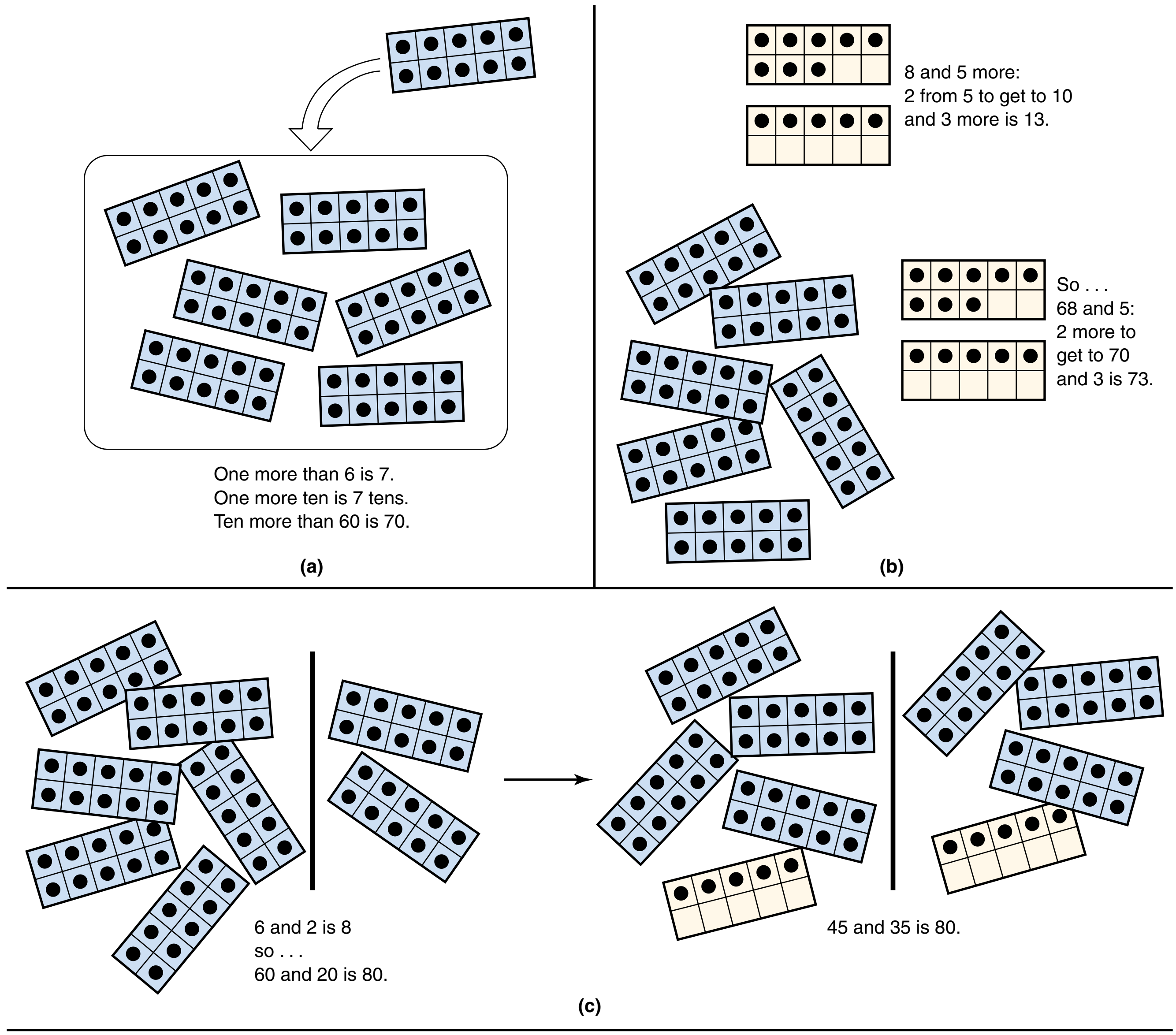


FIGURE 9.15 **Extending early number relationships to mental computation activities.**

Reflections on Chapter 9

Writing to Learn

1. Describe an activity that deals with a basic concept of number that does not require an understanding of counting. Explain the purpose of this activity.
2. What things must a child be able to do in order to count a set accurately?
3. When does a child have the *cardinality principle*? How can you tell if this principle has been acquired?
4. Describe an activity that is a "set-to-numeral match" activity. What ideas must a child have to do these activities meaningfully and correctly?
5. How can "Real Counting On" (Activity 9.8, p. 119) be used as an assessment to determine if children understand counting on or are still in a transitional stage?
6. What are the four types of relationships that have been described for small numbers? Explain briefly what each of these means, and suggest at least one activity for each.
7. Describe a missing-part activity. What should happen if the child trying to give the missing part does not know it?
8. How can a teacher assess each of the four number relationships? Give special attention to part-whole ideas.
9. How can a calculator be used to develop early counting ideas connected with number? How can a calculator be used to help

a child practice number relationships such as part-part-whole or one less than?

10. For numbers between 10 and 20, describe how to develop each of these ideas:
    a. The idea of the teens as a set of 10 and some more
    b. Extension of the one-more/one-less concept to the teens
11. Describe briefly your own idea of what number sense is.
12. What are three ways that children can be helped to connect numbers to real-world ideas?
13. Give two examples of how early number relationships can be used to develop some early mental computation skills.

For Discussion and Exploration

1. Examine a textbook series for grades K–2. Compare the treatment of counting and number concept development with that presented in this chapter. What ideas are stressed? What ideas are missed altogether? If you were teaching in one of these grades, how would you plan your number concept development program? What part would the text play?
2. Many teachers in grades 3 and above find that their children do not possess the number relationships discussed in this chapter but rely heavily on counting. Given the pressures of other content at these grades, how much effort should be made to remediate these number concept deficiencies?

Recommendations for Further Reading

Highly Recommended

Burton, G. (1993). *Number sense and operations: Addenda series, grades K–6.* Reston, VA: National Council of Teachers of Mathematics.
Either this book or the *Number Sense and Operations* addenda series book for your grade level should be a must for developing number concepts. The activities are developed in sufficient detail to give you clear guidance and yet allow considerable flexibility.

Fuson, K. C., Grandau, L., & Sugiyama, P. A. (2001). Achievable numerical understandings for all young children. *Teaching Children Mathematics, 7,* 522–526.
Researchers who have long worked with the number development of young children, provide the reader with a concise overview of number development from ages 3 to 7. This practical reporting of their research is quite useful.

Van de Walle, J. A., & Watkins, K. B. (1993). Early development of number sense. In R. J. Jensen (Ed.), *Research ideas for the classroom: Early childhood mathematics* (pp. 127–150). Old Tappan, NJ: Macmillan.
This chapter includes a discussion of many of the K–2 topics covered in this text as well as a broader discussion of number sense, including mental computation and estimation. It provides a good mix of practical ideas with ample references to the research literature.

Other Suggestions

Baroody, A. J., & Wilkins, J. L. M. (1999). The development of informal counting, number, and arithmetic skills and concepts. In J. V. Copley (Ed.), *Mathematics in the early years* (pp. 48–65). Reston, VA: National Council of Teachers of Mathematics.

Bresser, R., & Holtzzman, C. (1999). *Developing number sense.* Sausalito, CA: Math Solutions Publications.

Clements, D. H. (1999). Subitizing: What is it? Why teach it? *Teaching Children Mathematics, 5,* 400–405.

Dacey, L. S., & Eston, R. (1999). *Growing mathematical ideas in kindergarten.* Sausalito, CA: Math Solutions Publications.

Fosnot, C. T., & Dolk, M. (2001). *Young mathematicians at work: Constructing number sense, addition, and subtraction.* Portsmouth, NH: Heinemann.

Fuson, K. C. (1989). *Children's counting and concepts of number.* New York: Springer-Verlag.

Kamii, C., Kirkland, L., & Lewis, B. (2001). Representation and abstraction in young children's numerical reasoning. In A. A. Cuoco (Ed.), *The roles of representation in school mathematics* (pp. 24–34). Reston, VA: National Council of Teachers of Mathematics.

Kouba, V. L., Zawojewski, J. S., & Strutchens, M. E. (1997). What do students know about numbers and operations? In P. A. Kenney, & E. A. Silver (Eds.), *Results from the sixth mathematics assessment of educational progress* (pp. 87–140). Reston, VA: National Council of Teachers of Mathematics.

McClain, K., & Cobb, P. (1999). Supporting students' ways of reasoning about patterns and partitions. In J. V. Copley (Ed.), *Mathematics in the early years* (pp. 112–118). Reston, VA: National Council of Teachers of Mathematics.

McIntosh, A., Reys, B., & Reys, B. (1997). *Number SENSE: Simple effective number sense experiences* [Grades 1–2]. White Plains, NY: Dale Seymour Publications.

Ritchhart, R. (1994). *Making numbers make sense: A sourcebook for developing numeracy.* Reading, MA: Addison-Wesley.

Weinberg, S. (1996). Going beyond ten black dots. *Teaching Children Mathematics, 2,* 432–435.

Whitin, D. J., Mills, H., & O'Keefe, T. (1994). Exploring subject areas with a counting book. *Teaching Children Mathematics, 1,* 170–174.

Whitin, D. J., & Wilde, S. (1995). *It's the story that counts: More children's books for mathematical learning, K–6.* Portsmouth, NH: Heinemann.

Wicket, M. S. (1997). Serving up number sense and problem solving: *Dinner at the Panda Palace. Teaching Children Mathematics, 3,* 476–480.