
# Electric Charge and Electric Field

In a thundercloud, it is believed that collisions between ice and slush particles give the ice particles a slight positive charge. Although the details of this process are not understood, the resulting charge separation can produce enormous electric fields that result in a lightning bolt.

By the end of this chapter, you will be able to:

1. Sketch the distribution of charges for both conducting and insulating objects in various arrangements.
2. Calculate the number of fundamental units of charge in a particular quantity of charge.
3. Determine both the magnitude and direction of the force one charge exerts on another using Coulomb's law.
4. Determine the net force acting on a charge due to an array of point charges.
5. Relate both the magnitude and direction of the electric field at a point to the force felt by a charge placed at that point.
6. Determine the net electric field at a point due to both an array of point charges and a symmetric charge distribution.
7. Determine the electric flux through a surface.
8. Relate the net electric flux through a closed surface to the amount of charge enclosed by the surface.

When you scuff your shoes across a carpet, you can get zapped by an annoying spark of static electricity. That same spark could, in principle, totally destroy an integrated circuit chip in your computer. Fortunately, most modern electronic devices are designed to prevent such a catastrophe. Lightning, the same phenomenon on a vastly larger scale, can destroy a lot more than computer chips. All these phenomena involve electric charges and the interactions between such charges.

In this chapter, we'll study how electric charges that are at rest in our frame of reference influence each other; we call these **electrostatic interactions.** We'll find that charge has interesting properties. It is *quantized:* The total electric charge in a system must be an integer multiple of the charge of a single electron. Electric charge also obeys a *conservation* law: Charge can be neither created nor destroyed. However, most of the chapter will be devoted to the forces that charges produce on other charges.

These electrostatic forces are governed by Coulomb's law and are mediated by *electric fields.* Electrostatic forces hold atoms, molecules, and our bodies together, but they also are constantly trying to tear apart the nuclei of atoms. We'll explore all these concepts in this chapter.

▲ **Application**
**Run!**

The person in this vacation snapshot, taken at a scenic overlook in Sequoia National Park, was amused to find her hair standing on end. Luckily, she and her companion left the overlook after taking the photo—and before it was hit by lightning. Just before lightning strikes, strong charges build up in the ground and in the clouds overhead. If you're standing on charged ground, the charge will spread onto your body. Because like charges repel, all your hairs tend to get as far from each other as they can. But the key thing is for *you* to get as far from that spot as *you* can!

## 17.1 Electric Charge

The ancient Greeks discovered as early as 600 B.C. that when they rubbed amber with wool, the amber could attract other objects. Today we say that the amber has acquired a net **electric charge,** or has become *charged.* The word *electric* is derived from the Greek word *elektron,* meaning "amber." When you scuff your shoes across a nylon carpet, you become electrically charged, and you can charge a comb by passing it through dry hair.

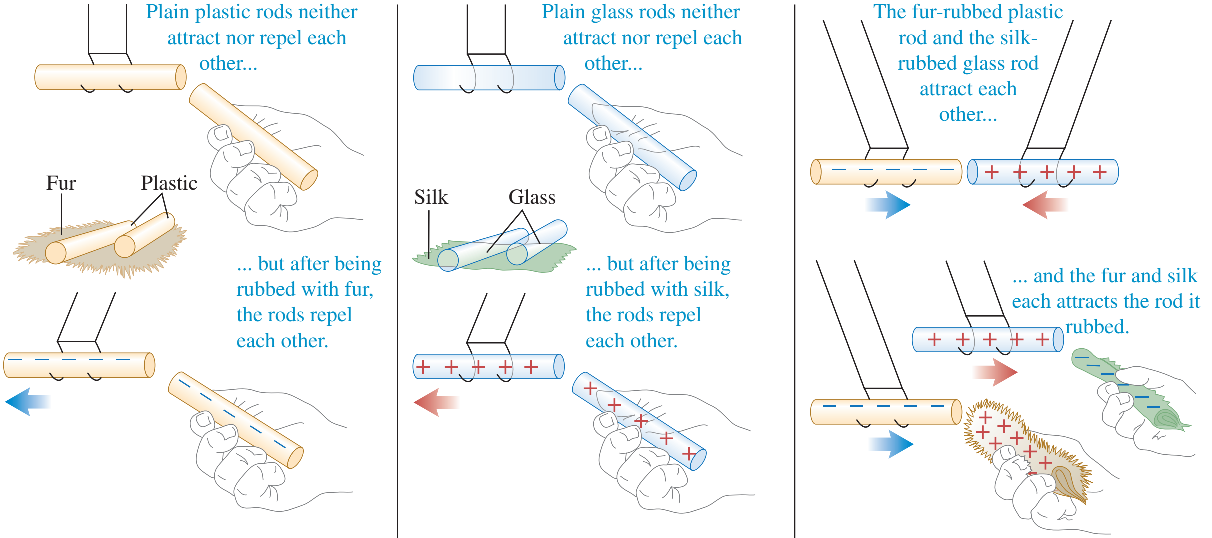
Plastic rods and fur (real or fake) are particularly good for demonstrating electric-charge interactions. In Figure 17.1a, we charge two plastic rods by rubbing them on a piece of fur. We find that the rods repel each other. When we rub glass rods with silk (Figure 17.1b), the glass rods also become charged and repel each other. But a charged plastic rod *attracts* a charged glass rod (Figure 17.1c, top). Furthermore, the plastic rod and the fur attract each other, and the glass rod and the silk attract each other (Figure 17.1c, bottom).

These experiments and many others like them have shown that there are exactly two (no more) kinds of electric charge: the kind on the plastic rod rubbed with fur and the kind on the glass rod rubbed with silk. Benjamin Franklin (1706–1790) suggested calling these two kinds of charge *negative* and *positive,* respectively, and these names are still used.

**Like and unlike charges**
**Two positive charges or two negative charges repel each other; a positive and a negative charge attract each other.**

In Figure 17.1, the plastic rod and the silk have negative charge; the glass rod and the fur have positive charge.

When we rub a plastic rod with fur (or a glass rod with silk), *both* objects acquire net charges, and the net charges of the two objects are always equal in magnitude and opposite in sign. These experiments show that in the charging process we are not *creating* electric




**(a)** Interaction between plastic rods rubbed on fur

**(b)** Interaction between glass rods rubbed on silk

**(c)** Interaction between objects with opposite charges

▲ **FIGURE 17.1** Experiments illustrating the nature of electric charge.

charge, but *transferring* it from one object to another. We now know that the plastic rod acquires extra electrons, which have negative charge. These electrons are taken from the fur, which is left with a deficiency of electrons (that is, fewer electrons than positively charged protons) and thus a net positive charge. The *total* electric charge on *both* objects does not change. This is an example of *conservation of charge;* we'll come back to this important principle later.

**CONCEPTUAL ANALYSIS 17.1**

### The sign of the charge

Three balls made of different materials are rubbed against different types of fabric—silk, polyester, and others. It is found that balls 1 and 2 repel each other and that balls 2 and 3 repel each other. From this result, we can conclude that

A. balls 1 and 3 carry charges of opposite sign.
B. balls 1 and 3 carry charges of the same sign; ball 2 carries a charge of the opposite sign.
C. all three balls carry charges of the same sign.

**SOLUTION** Since balls 1 and 2 repel, they must be of the same sign, either both positive or both negative. Since balls 2 and 3 repel each other, they also must be of the same sign. This means that 1 and 3 both have the same sign as 2, so all three balls have the same sign. The correct answer is C.

## The physical basis of electric charge

When all is said and done, we can't say what electric charge *is;* we can only describe its properties and its behavior. However, we *can* say with certainty that electric charge is one of the fundamental attributes of the particles of which matter is made. The interactions responsible for the structure and properties of atoms and molecules—and, indeed, of all ordinary matter—are primarily *electrical* interactions between electrically charged particles.

The structure of ordinary matter can be described in terms of three particles: the negatively charged **electron,** the positively charged **proton,** and the uncharged **neutron.** The protons and neutrons in an atom make up a small, very dense core called the **nucleus,** with a diameter on the order of $10^{-15}$ m (Figure 17.2). Surrounding the nucleus are the electrons, which orbit the nucleus out to distances on the order of $10^{-10}$ m. If an atom were a few miles across, its nucleus would be the size of a tennis ball.

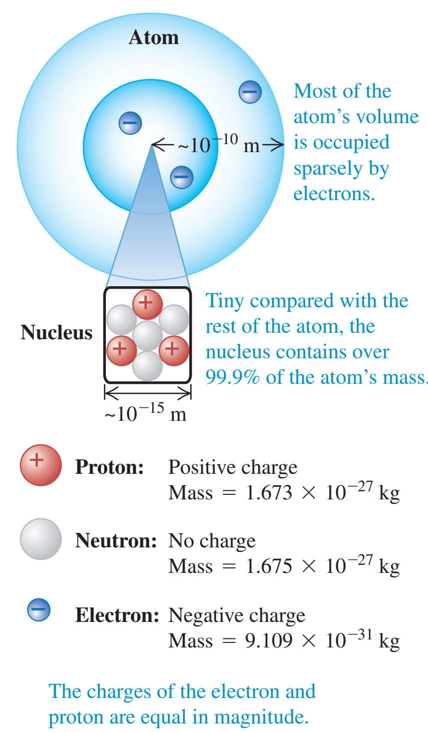
The masses of the individual particles, to the precision that they are currently known, are as follows:

$$\text{Mass of electron} = m_e = 9.10938291(40) \times 10^{-31}\ \text{kg},$$
$$\text{Mass of proton} = m_p = 1.672621777(74) \times 10^{-27}\ \text{kg},$$
$$\text{Mass of neutron} = m_n = 1.674927351(74) \times 10^{-27}\ \text{kg}.$$

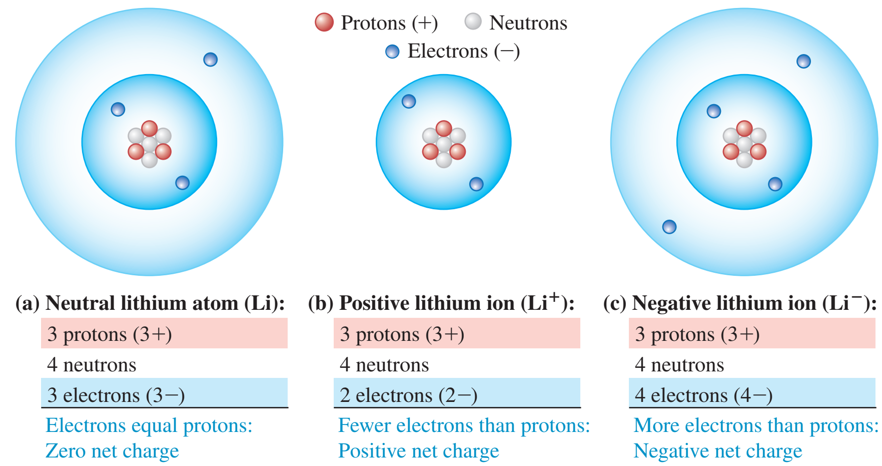
The numbers in parentheses are the uncertainties in the last two digits. Note that the masses of the proton and neutron are nearly equal (within about 0.1%) and that the mass of the proton is roughly 2000 times that of the electron. Over 99.9% of the mass of any atom is concentrated in its nucleus.

The negative charge of the electron has (within experimental error) *exactly* the same magnitude as the positive charge of the proton. In a neutral atom, the number of electrons equals the number of protons in the nucleus, and the net electric charge (the algebraic sum of all the charges) is exactly zero (Figure 17.3a). The number of protons or electrons in neutral atoms of any element is called the **atomic number** of the element.

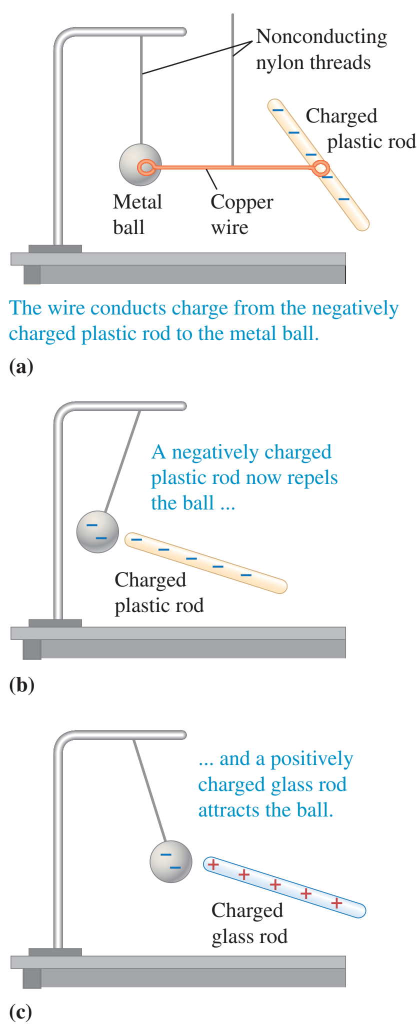
When the number of protons in an object equals the number of electrons in the object, the total charge is zero, and the object as a whole is electrically neutral. To give a neutral object an excess negative charge, we may either *add negative* charges to it or *remove positive* charges from it. Similarly, we can give an excess positive charge to a neutral body by either *adding positive* charge or *removing negative* charge. When we speak of the charge on an object, we always mean its *net* charge.




▲ **FIGURE 17.2** Schematic depiction of the structure and components of an atom.




▲ **FIGURE 17.3** The neutral lithium (Li) atom and positive and negative lithium ions.




▲ **FIGURE 17.4** Charging by conduction. A copper wire is a good conductor. (a) The wire conducts charge between the plastic rod and the metal ball, giving the ball a negative charge. The charged ball is then (b) repelled by a like charge and (c) attracted by an unlike charge.

An **ion** is an atom that has lost or gained one or more electrons. If one or more electrons are removed, the remaining positively charged structure is called a **positive ion** (Figure 17.3b). A **negative ion** is an atom that has *gained* one or more electrons (Figure 17.3c). This gaining or losing of electrons is called **ionization.**

Ordinarily, when an ion is formed, the structure of the nucleus is unchanged. In a solid object such as a carpet or a copper wire, the nuclei of the atoms are not free to move about, so a net charge is due to an excess or deficit of electrons. However, in a liquid or a gas, a net electric charge may be due to movements of ions. Thus, a positively charged region in a fluid could represent an excess of positive ions, a deficit of negative ions, or both.

## 17.2 Conductors and Insulators

Some materials permit electric charge to move from one region of the material to another; others do not. For example, Figure 17.4 shows a copper wire supported by a nylon thread. Suppose you touch one end of the wire to a charged plastic rod and touch the other end to a metal ball that is initially uncharged. When you remove the copper wire and bring another charged object near the ball, the ball is attracted or repelled, showing that it has become electrically charged. Electric charge has been transferred through the copper wire between the ball and the surface of the plastic rod.

The wire is called a **conductor** of electricity. If you repeat the experiment, but this time using a rubber band or nylon thread in place of the wire, you find that *no* charge is transferred to the ball. These materials are called **insulators.** Conductors permit charge to move through them; insulators do not. Carpet fibers on a dry day are good insulators and allow charge to build up on us as we walk across the carpet. Coating the fibers with an antistatic layer that does not easily transfer electrons to or from our shoes is one solution to the charge-buildup problem; another is to wind some of the fibers around conducting cores.

Most of the materials we call *metals* are good conductors, and most *nonmetals* are insulators. Within a solid metal such as copper, one or more outer electrons in each atom become detached and can move freely throughout the material, just as the molecules of a gas can move through the spaces between the grains in a bucket of sand. The other electrons remain bound to the positively charged nuclei, which themselves are bound in fixed positions within the material. In an insulator, there are no, or at most very few, free electrons, and electric charge cannot move freely through the material.

Some materials called *semiconductors* are intermediate in their properties between good conductors and good insulators. Unlike copper, which is always a good conductor, no matter what you do to it, or rubber, which is always a bad conductor, no matter what you do to it, a semiconductor such as silicon can be engineered to have a controllable conductivity.

This is the basis of the silicon-based transistor, which is the fundamental building block of the modern computer.

Finally, we note that, in a liquid or gas, charge can move in the form of positive or negative ions. Ionic solutions are usually good conductors. For example, when ordinary table salt (NaCl) dissolves in water, each sodium (Na) atom loses an electron to become a positively charged sodium ion $(Na^+)$, and each chlorine $(Cl)$ atom gains an electron to become a negatively charged chloride ion $(Cl^-)$. These charged particles can move freely in the solution and thus conduct charge from one region of the fluid to another, providing a mechanism for conductivity. Ionic solutions are the dominant conductivity mechanism in many biological processes.

**Application**
**Good conductor, bad conductor.**

Salt water is salty because it contains an abundance of dissolved ions. These ions are charged and can move freely, so salt water is an excellent conductor of electricity. Ordinary tap water contains enough ions to conduct electricity reasonably well—which is why you should never, ever, use an electrical device in a bathtub. However, absolutely pure distilled water is an insulator because it consists of only neutral water molecules.

## Induction

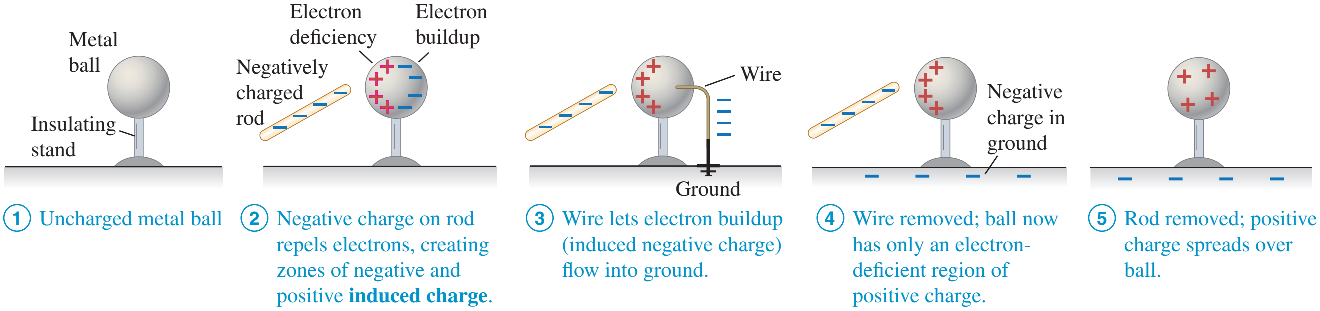
When we charge a metal ball by touching it with an electrically charged plastic rod, some of the excess electrons on the rod move from it to the ball, leaving the rod with a smaller negative charge. In another technique, called charging by **induction,** the plastic rod can give another object a charge of *opposite* sign without losing any of its own charge.

Figure 17.5 shows an example of charging by induction. A metal sphere is supported on an insulating stand (step 1). When you bring a negatively charged rod near the sphere, without actually touching it (step 2), the free electrons on the surface of the sphere are repelled by the excess electrons on the rod, and they shift toward the right, away from the rod. They cannot escape from the sphere because the supporting stand and the surrounding air are insulators. As a result, negative charge accumulates on the right side of the surface of the sphere and positive charge (due to the positive nuclei that the electrons left behind) accumulates on the left side. These excess charges are called **induced charges.**

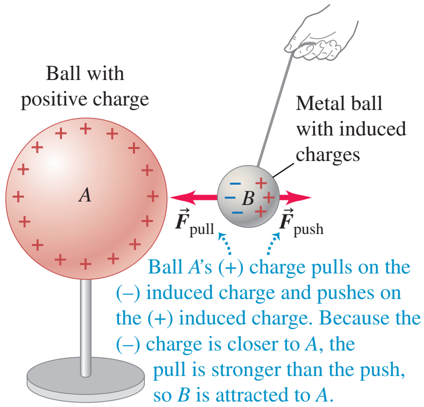
Not all of the free electrons move to the right side of the surface of the sphere. As soon as any induced charge develops, it exerts forces toward the *left* on the other free electrons. These electrons are repelled from the negative induced charge on the right and attracted toward the positive induced charge on the left. The system reaches an equilibrium state in which the force toward the right on an electron, due to the charged rod, is just balanced by the force toward the left, due to the induced charge. If we remove the charged rod, the free electrons shift back to the left, and the original neutral condition is restored.

What happens if, while the plastic rod is nearby, you touch one end of a conducting wire to the right surface of the sphere and the other end to the earth (step 3 in Figure 17.5)? The earth is a conductor, and it is so large that it can act as a practically infinite source of extra electrons or sink of unwanted electrons. Some of the negative charge flows through the wire to the earth. Now suppose you disconnect the wire (step 4) and then remove the rod (step 5); a net positive charge is left on the sphere. The charge on the negatively charged rod has not changed during this process. The earth acquires a negative charge that is equal in magnitude to the induced positive charge remaining on the sphere.

Charging by induction would work just as well if the mobile charges in the sphere were positive charges instead of (negatively charged) electrons or even if both positive and negative mobile charges were present (as would be the case if we replaced the sphere with a flask of salt water). In this book, we'll talk mostly about metallic conductors, in which the mobile




▲ **FIGURE 17.5** Charging a metal ball by induction.




▲ **FIGURE 17.7** The charge on ball $A$ induces charges in ball $B$, resulting in a net attractive force between the balls.

▲ **FIGURE 17.6** A charged plastic comb picks up *uncharged* bits of paper.

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**PhET:** Balloons and Static Electricity

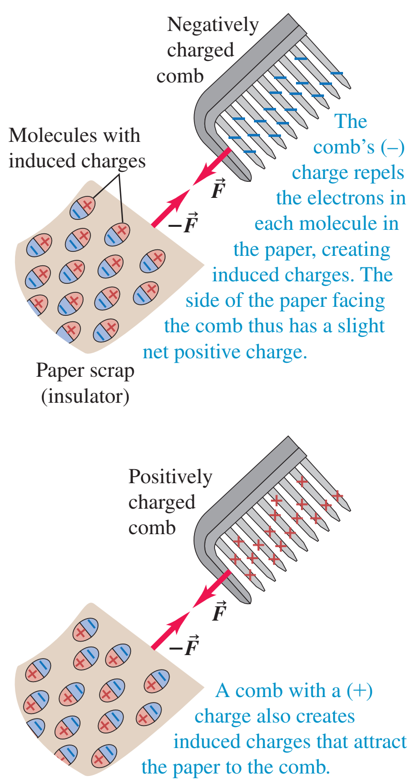
charges are negative electrons. However, even in a metal, we can describe conduction *as though* the moving charges were positive. In terms of transfer of charge in a conductor, a movement of electrons to the left is exactly equivalent to a movement of imaginary positive particles to the right. In fact, when we study electric currents, we will find that, for historical reasons, currents in wires are described as though the moving charges were positive.

When excess charge is placed on a solid conductor and is at rest (i.e., an electrostatic situation), the excess charge rests entirely on the surface of the conductor. If there were excess charge in the interior of the conductor, there would be electric forces among the excess charges that would cause them to move, and the situation couldn't be electrostatic.

## Polarization

A charged object can exert forces even on objects that are *not* charged themselves. If you rub a balloon on a rug and then hold the balloon against the ceiling, it sticks, even though the ceiling has no net electric charge. After you electrify a comb by running it through your hair, you can pick up uncharged bits of paper on the comb (Figure 17.6). How is this possible?

The interaction between the balloon and the ceiling or between the comb and the paper is an induced-charge effect. In step 2 of Figure 17.5, the plastic rod exerts a net attractive force on the sphere, even though the total charge on the sphere is zero, because the positive charges are closer to the rod than the negative charges are. Figure 17.7 shows this effect more clearly. The large ball $A$ has a positive charge; the conducting metal ball $B$ is uncharged. When we bring $B$ close to $A$, the positive charge on $A$ pulls on the electrons in $B$, setting up induced charges. Because the negative induced charge on the surface of $B$ is closer to $A$ than the positive induced charge is, $A$ exerts a net attraction on $B$. (We'll study the dependence of electric forces on distance in Section 17.4.) Even in an insulator, the electric charges can shift back and forth a little when there is charge nearby. Figure 17.8 shows how a static charge enables a charged plastic comb to pick up uncharged bits of paper. Although the electrons in the paper are bound to their molecules and cannot move freely through the paper, they can still shift slightly to produce a net charge on one side and the opposite charge on the other. Thus, the comb causes each molecule in the paper to develop induced charges (an effect called **polarization**). The net result is that the scrap of paper shows a slight induced charge—enough to enable the comb to pick it up.




▲ **FIGURE 17.8** A charged comb picks up uncharged paper by polarizing the paper's molecules.

**Video Tutor Demo**

## 17.3 Conservation and Quantization of Charge

As we've discussed, an electrically neutral object is an object that has equal numbers of electrons and protons. The object can be given a charge by adding or removing either positive or negative charges. Implicit in this discussion are two very important principles. First is the principle of **conservation of charge:**

**Conservation of charge**
The algebraic sum of all the electric charges in any closed system is constant. Charge can be neither created nor destroyed; it can only move from one place or object to another.

Conservation of charge is believed to be a *universal* conservation law; there has never been any experimental evidence for a violation of this principle. Even in high-energy interactions in which subatomic particles are created and destroyed, the net charge of all the particles is exactly constant.

Second, the magnitude of the charge of the electron or proton is a natural unit of charge. Every amount of observable electric charge is always an integer multiple of this basic unit. Hence we say that charge is *quantized.* A more familiar example of quantization is money. When you pay cash for an item in a store, you have to do it in 1-cent increments. If grapefruits are selling three for a dollar, you can't buy one for $33\frac{1}{3}$ cents; you have to pay 34 cents. Cash can't be divided into smaller amounts than 1 cent, and electric charge can't be divided into smaller amounts than the charge of one electron or proton.

**QUANTITATIVE ANALYSIS 17.1**

### Determine the charge

Three identical metal balls $A$, $B$, and $C$ are mounted on insulating rods. Ball $A$ has a charge $+q$, and balls $B$ and $C$ are initially uncharged ($q$ is the usual symbol for electric charge). Ball $A$ is touched first to ball $B$ and then separately to ball $C$. At the end of this experiment, the charge on ball $A$ is

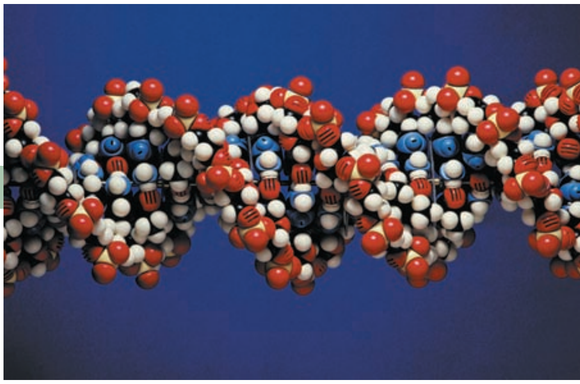
A. $+q/2$. B. $+q/3$. C. $+q/4$.

**SOLUTION** When identical metal objects come in contact, any net charge they carry is shared equally between them. Thus, when $A$ touches $B$, each ends up with a charge $+q/2$. When $A$ then touches $C$, this charge is shared equally, leaving $A$ and $C$ each with a charge of $+q/4$. The correct answer is C.

The forces that hold atoms and molecules together are fundamentally electrical in nature. The attraction between electrons and protons holds the electrons in atoms, holds atoms together to form polyatomic molecules, holds molecules together to form solids or liquids, and accounts for phenomena such as surface tension and the stickiness of glue. Within the atom, the electrons repel each other, but they are held in the atom by the attractive force of the protons in the nucleus. But what keeps the positively charged protons together in the tiny nucleus despite their mutual repulsion? They are held by another, even stronger interaction called the *nuclear force.* (We will learn about the nuclear force in Chapter 30.)

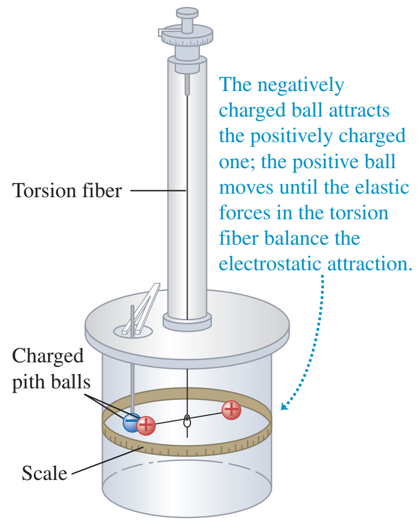
## 17.4 Coulomb's Law

Charles Augustin de Coulomb (1736–1806) studied the forces between charged particles in detail in 1784 using a torsion balance. The torsion balance, which is depicted in Figure 17.9a, consisted of a small rod that was suspended from its midpoint by a fine wire. On each end of the rod was a charged sphere. When Coulomb brought a third charged sphere near one of the ends of the rod, it caused the rod to rotate slightly about its center of mass. By measuring the direction and magnitude of the angular deflection, Coulomb was able to deduce some of the basic properties of the electric force between charges. This very sensitive technique would

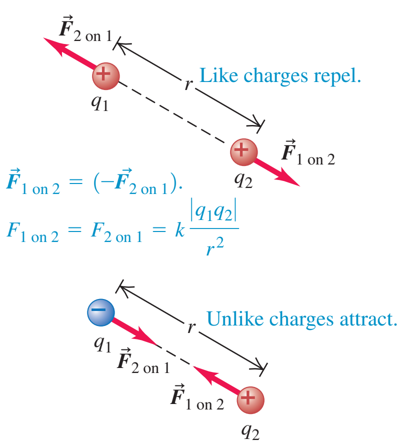


► BIO Application **Static cling.**

The genetic code is carried by the "double helix" of DNA, which consists of two DNA strands wound around each other. The two strands stick together by what is essentially static cling. Along each strand, specific molecular groups form dipoles, with a positive or negative end projecting outward. The positive charges on one strand interact precisely with the negative charges on the other, "zipping" the two strands together. Crucially, these interactions are strong enough to keep the strands from coming apart on their own, but weak enough that the cellular machinery can "unzip" the strands for copying.




**(a)** A torsion balance of the type used by Coulomb to measure the electric force




**(b)** Interaction of like and unlike charges

▲ **FIGURE 17.9** Schematic depiction of the apparatus Coulomb used to determine the forces between charged objects that can be treated as point charges.

be used 13 years later by Cavendish to study the (much weaker) gravitational force between lead spheres, as we discussed in Section 6.3. Coulomb's experiments led to the very important discovery that the electric force between two *point charges* (charged bodies that are very small in comparison with the distance $r$ between them) is proportional to the inverse square of the distance between the charges, $1/r^2$.

The force also depends on the quantity of charge on each object, which we'll denote by $q$ or $Q$. To explore this dependence, Coulomb divided a charge into two equal parts by placing a small charged spherical conductor in contact with an identical but uncharged sphere; by symmetry, the charge is shared equally between the two spheres. (Note the essential role of the principle of conservation of charge in this procedure.) Thus, Coulomb could obtain one-half, one-quarter, and so on, of any initial charge. He found that the forces that two point charges $q_1$ and $q_2$ exert on each other are proportional to each charge and therefore are proportional to the *product* $q_1 q_2$ of the two charges.

**Coulomb's law**

The magnitude $F$ of the force that each of two point charges $q_1$ and $q_2$ a distance $r$ apart exerts on the other (Figure 17.9b) is directly proportional to the product of the charges and inversely proportional to the square of the distance between them. The relationship is expressed symbolically as

$$F = k\frac{|q_1 q_2|}{r^2}. \tag{17.1}$$

This relationship is called **Coulomb's law.**
Units: $q_1$ and $q_2$ are in coulombs (C); $F$ is in newtons (N).
Notes:

- $k$ is a fundamental constant of nature: $k = 8.987551789 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$.
- $F$ represents only the magnitude of the force; the direction is determined using the fact that like charges repel and unlike charges attract.
- $r$ is the distance between the two charges.

The SI unit of electric charge is called one **coulomb** (1 C). For numerical calculations in problems, we'll often use the approximate value

$$k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2,$$

which is in error by about 0.03%.

The forces that two charges exert on each other always act along the line joining the charges. The two forces are always equal in magnitude and opposite in direction, even when the charges are not equal. *The forces obey Newton's third law.*

As we've seen, $q_1$ and $q_2$ can be either positive or negative quantities. When the charges have the same sign (both positive or both negative), the forces are repulsive; when they are unlike, the forces are attractive. We need the absolute value bars in Equation 17.1 because $F$ is the magnitude of a vector quantity. By definition, $F$ is always positive, but the product $q_1 q_2$ is negative whenever the two charges have opposite signs.

The proportionality of the electric force to $1/r^2$ has been verified with great precision. There is no experimental evidence that the exponent is anything different from precisely 2. The form of Equation 17.1 is the same as that of the law of gravitation, but electrical and gravitational interactions are two distinct classes of phenomena. The electrical interaction depends on electric charges and can be either attractive or repulsive; the gravitational interaction depends on mass and is always attractive (because there is no such thing as negative mass).

Strictly speaking, Coulomb's law, as we have stated it, should be used only for point charges *in vacuum.* If matter is present in the space between the charges, the net force acting on each charge is altered because charges are induced in the molecules of the intervening material. We'll describe this effect later. As a practical matter, though, we can use

**▶ Application Great balls of fire?**

Before the invention of the cyclotron, which uses both electric and magnetic fields to accelerate subatomic particles, physicists used electric-field generators in atom-smashing experiments. These generators, like the Van de Graaff generators shown here, can accumulate either positive or negative charges on the surface of a metal sphere, thus generating immense electric fields. Charged particles in such an electric field are acted upon by a large electric force, which can be used to accelerate the particles to very high velocities.

Coulomb's law unaltered for point charges in air; at normal atmospheric pressure, the presence of air changes the electric force from its vacuum value by only about 1 part in 2000.

In SI units, the constant $k$ in Equation 17.1 is often written as

$$k = \frac{1}{4\pi\epsilon_0},$$

where $\epsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2/(N \cdot m^2)}$ is another constant. This alternative form may appear to complicate matters, but it actually simplifies some of the formulas that we'll encounter later. When we study electromagnetic radiation (in Chapter 23), we'll show that the numerical value of $\epsilon_0$ is closely related to the speed of light.

The most fundamental unit of charge is the magnitude of the charge of an electron or a proton, denoted by $e$:

$$e = 1.60217653 \times 10^{-19}\ \mathrm{C}.$$

The electron has a charge of $-e$ and the proton has a charge of $+e$. One coulomb represents the total charge carried by about $6 \times 10^{18}$ protons, or the negative of the total charge of about $6 \times 10^{18}$ electrons. For comparison, the population of the earth is about $7 \times 10^9$ persons, and a cube of copper 1 cm on a side contains about $2.4 \times 10^{24}$ electrons.

In electrostatics problems, charges as large as 1 coulomb are very unusual. Two charges with magnitude 1 C, at a distance 1 m apart, would exert forces of magnitude $9 \times 10^9$ N (about a million tons) on each other! A more typical range of magnitudes is $10^{-9}$ to $10^{-6}$ C. The *microcoulomb* ($1\ \mu\mathrm{C} = 10^{-6}\ \mathrm{C}$) and the *nanocoulomb* ($1\ \mathrm{nC} = 10^{-9}\ \mathrm{C}$) are often used as practical units of charge. The total charge of all the electrons in a penny is about $1.4 \times 10^5$ C. This number shows that we can't disturb electrical neutrality very much without using enormous forces.

**CONCEPTUAL ANALYSIS 17.3**

### Charged spheres in motion

Two small identical balls $A$ and $B$ are held a distance $r$ apart on a frictionless surface; $r$ is large compared with the size of the balls. Ball $A$ has a net charge $+q$; ball $B$ has a net charge $+4q$. When both balls are released at the same instant, which of the statements about the acceleration of the two balls are correct? (There may be more than one correct choice.)

A. Both balls accelerate away from each other.
B. The acceleration of ball $B$ is four times larger than the acceleration of ball $A$.
C. The acceleration of both balls is constant as they move away from each other.
D. The acceleration of both balls decreases as they move away from each other.

**SOLUTION** Coulomb's law states that the magnitude of the force between two charged objects that can be treated as particles is $F = (k|q_1q_2|)/r^2$. Is this force somehow divided between the two objects? Does the object with the larger charge exert a stronger force? Should the force on each object be calculated separately? No; Newton's third law gives the answer. Whenever two objects interact, the forces that the two objects exert on each other are equal in magnitude (and opposite in direction). Since the balls experience the same magnitude of force and have the same mass, by Newton's second law they have the same magnitude of acceleration at any instant. In addition, the two balls will repel each other because they are both positively charged. As they move apart and $r$ increases, the magnitude of acceleration decreases. Therefore, both A and D are correct answers.

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## Superposition

When two charges exert forces simultaneously on a third charge, the total force acting on that charge is the *vector sum* of the forces that the two charges would exert individually. This important property, called the **principle of superposition,** holds for any number of charges. Coulomb's law, as we have stated it, describes only the interaction between two *point* charges, but by using the superposition principle, we can apply it to *any* collection of charges. Several of the examples that follow illustrate the superposition principle.

### PROBLEM-SOLVING STRATEGY 17.1 Coulomb's law

**SET UP**

1. As always, consistent units are essential. With the value of $k$ given earlier, distances *must* be in meters, charges in coulombs, and forces in newtons. If you are given distances in centimeters, inches, or furlongs, don't forget to convert! When a charge is given in microcoulombs, remember that $1\ \mu C = 10^{-6}\ C$.

**SOLVE**

2. When the forces acting on a charge are caused by two or more other charges, the total force on the charge is the *vector sum* of the individual forces. If you're not sure you remember how to do vector addition, you may want to review Sections 1.7 and 1.8. It's often useful to use components in an $x$-$y$ coordinate system. As always, it's essential to distinguish among vectors, their magnitudes, and their components (using correct notation!) and to treat vectors properly as such.
3. Some situations involve a continuous distribution of charge along a line or over a surface. In this book, we'll consider only situations for which the vector sum described in step 2 can be evaluated by using vector addition and symmetry considerations. In other cases, methods of integral calculus would be needed.

**REFLECT**

4. Try to think of particular cases where you can guess what the result should be, and compare your intuitive expectations with the results of your calculations.

### EXAMPLE 17.1 Charge imbalance

In this example we will use the quantization of charge to determine the number of excess electrons in an object. **(a)** A large plastic block has a net charge of $-1.0\ \mu C = -1.0 \times 10^{-6}\ C$. How many more electrons than protons are in the block? **(b)** When rubbed with a silk cloth, a glass rod acquires a net positive charge of 1.0 nC. If the rod contains 1.0 mole of molecules, what fraction of the molecules have been stripped of an electron? Assume that at most one electron is removed from any molecule.

Video Tutor Solution

**SOLUTION**

**SET UP AND SOLVE Part (a):** We want to find the number of electrons $N_e$ needed for a net charge of $-1.0 \times 10^{-6}\ C$ on the object. Each electron has charge $-e$. We divide the total charge by $-e$:

$$N_e = \frac{-1.0 \times 10^{-6}\ C}{-1.60 \times 10^{-19}\ C} = 6.2 \times 10^{12}\ \text{electrons}.$$

**Part (b):** First we find the number $N_{ion}$ of positive ions needed for a total charge of 1.0 nC if each ion has charge $+e$. The number of molecules in a mole is Avogadro's number, $6.02 \times 10^{23}$, so the rod contains $6.02 \times 10^{23}$ molecules. As in part (a), we divide the total charge on the rod by the charge of one ion. Remember that $1\ nC = 10^{-9}\ C$. Thus,

$$N_{ion} = \frac{1.0 \times 10^{-9}\ C}{1.6 \times 10^{-19}\ C} = 6.25 \times 10^{9}.$$

The fraction of all the molecules that are ionized is

$$\frac{N_{ion}}{6.02 \times 10^{23}} = \frac{6.25 \times 10^{9}}{6.02 \times 10^{23}} = 1.0 \times 10^{-14}.$$

**REFLECT** A charge imbalance of about $10^{-14}$ is typical for charged objects. Common objects contain a huge amount of charge, but they have very nearly equal amounts of positive and negative charges.

**Practice Problem:** A tiny object contains $5.26 \times 10^{12}$ protons and $4.82 \times 10^{12}$ electrons. What is the net charge on the object? *Answer:* $7.0 \times 10^{-8}\ C$.

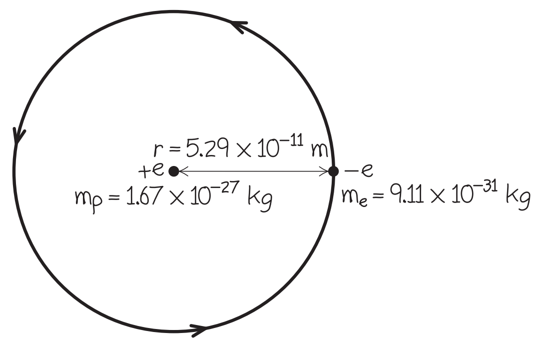
## EXAMPLE 17.2 Gravity in the hydrogen atom

Video Tutor Solution

Now let's compare the strength of the electrostatic force between the electron and proton in a hydrogen atom with the corresponding gravitational force between the two. Remember that a hydrogen atom consists of a single electron in orbit around a proton. In an early, simple model of the hydrogen atom called the *Bohr model,* the electron is pictured as moving around the proton in a circular orbit with radius $r = 5.29 \times 10^{-11}$ m. (In Chapter 29, we'll study the Bohr model and also more sophisticated models of atomic structure.) What is the ratio of the magnitude of the electric force between the electron and proton to the magnitude of the gravitational attraction between them? The electron has mass $m_e = 9.11 \times 10^{-31}$ kg, and the proton has mass $m_p = 1.67 \times 10^{-27}$ kg.

**SOLUTION**

**SET UP** Figure 17.10 shows our sketch. The distance between the proton and electron is the radius $r$. Each particle has charge of magnitude $e$. The electric force is given by Coulomb's law and the gravitational force by Newton's law of gravitation.




▲ **FIGURE 17.10** Our sketch for this problem.

**SOLVE** Coulomb's law gives the magnitude $F_e$ of the electric force between the electron and proton as

$$F_e = k\frac{|q_1 q_2|}{r^2} = k\frac{e^2}{r^2},$$

where $k = 8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2$. The gravitational force $\vec{F}_g$ has magnitude $F_g$:

$$F_g = G\frac{m_1 m_2}{r^2} = G\frac{m_e m_p}{r^2},$$

where $G = 6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$. The ratio of the two forces is

$$\frac{F_e}{F_g} = \left(\frac{ke^2}{r^2}\right)\left(\frac{r^2}{Gm_e m_p}\right) = \frac{ke^2}{Gm_e m_p}$$

$$= \left(\frac{8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2}{6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2}\right) \times \frac{(1.60 \times 10^{-19}\ \text{C})^2}{(9.11 \times 10^{-31}\ \text{kg})\,(1.67 \times 10^{-27}\ \text{kg})},$$

$$\frac{F_e}{F_g} = 2.27 \times 10^{39}.$$

**REFLECT** In our expression for the ratio, all the units cancel and the ratio is dimensionless. The astonishingly large value of $F_e/F_g$—about $10^{39}$—shows that, in atomic structure, the gravitational force is negligible compared with the electrostatic force. The reason gravitational forces dominate in our daily experience is that positive and negative electric charges are always nearly equal in number and thus cancel nearly completely. Since there is no "negative" gravitation, gravitational forces always add. Note also that because both $F_e$ and $F_g$ are proportional to $1/r^2$, the ratio $F_e/F_g$ does not depend on the distance between the two particles.

**Practice Problem:** A hydrogen atom is at the earth's surface. The electron and proton in the atom are separated by a distance of $5.29 \times 10^{-11}$ m. What is the ratio of the magnitude of the electric force exerted by the proton on the electron to the weight of the electron? *Answer:* $9.2 \times 10^{21}$.

For all fundamental particles, the gravitational attraction is always much, much weaker than the electrical interaction. But suppose that the electric force were a million $(10^6)$ times weaker than it really is. In that case, the ratio of electric to gravitational forces between an electron and a proton would be about $10^{39} \times 10^{-6} = 10^{33}$ and the universe would be a very different place. Materials would be a million times weaker than we are used to, because they are held together by electrostatic forces. Insects would need to have much thicker legs to support the same mass. In fact, animals couldn't get much larger than an insect unless they were made of steel, and even a hypothetical animal made of steel could be only a few centimeters in size before collapsing under its own weight. More significantly, if the electric force were a million times weaker, the lifetime of a typical star would decrease from 10 billion years down to 10 thousand years! This is hardly enough time for *any* living organisms—much less such complicated ones as insects or humans—to evolve.

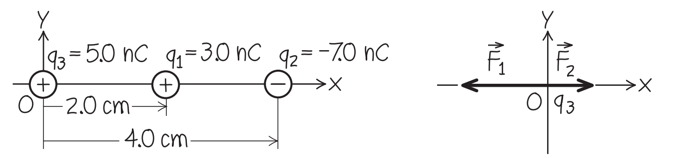
## EXAMPLE 17.3 Adding forces

Now let's apply Coulomb's law and the superposition principle to calculate the force on a point charge due to the presence of other nearby charges. Two point charges are located on the positive $x$ axis of a coordinate system. Charge $q_1 = 3.0\ \text{nC}$ is 2.0 cm from the origin, and charge $q_2 = -7.0\ \text{nC}$ is 4.0 cm from the origin. What is the total force (magnitude and direction) exerted by these two charges on a third point charge $q_3 = 5.0\ \text{nC}$ located at the origin?

Video Tutor Solution

### SOLUTION

**SET UP** We sketch the situation and draw a free-body diagram for charge $q_3$, using $\vec{F}_1$ to denote the force exerted by $q_1$ on $q_3$ and $\vec{F}_2$ for the force exerted by $q_2$ on $q_3$ (Figure 17.11). The directions of these forces are determined by the rule that like charges repel and unlike charges attract, so $\vec{F}_1$ points in the $-x$ direction and $\vec{F}_2$ points in the $+x$ direction. We don't yet know their relative magnitudes, so we draw them to arbitrary length.




**(a)** Our diagram of the situation **(b)** Free-body diagram for $q_3$

▲ **FIGURE 17.11** Our sketches for this problem.

**SOLVE** We use Coulomb's law to find the magnitudes of the forces $\vec{F}_1$ and $\vec{F}_2$; then we add these two forces (as vectors) to find the resultant force on $q_3$:

$$\vec{F}_{\text{total}} = \vec{F}_1 + \vec{F}_2,$$
$$\text{so}\quad F_{\text{total},x} = F_{1x} + F_{2x}$$
$$\text{and}\ F_{\text{total},y} = F_{1y} + F_{2y}.$$

$$F_1 = k\frac{|q_1 q_3|}{r_{12}{}^2}$$
$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.0 \times 10^{-9}\ \text{C})(5.0 \times 10^{-9}\ \text{C})}{(0.020\ \text{m})^2}$$
$$= 3.37 \times 10^{-4}\ \text{N},$$

$$F_2 = k\frac{|q_2 q_3|}{r_{23}{}^2}$$
$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(7.0 \times 10^{-9}\ \text{C})(5.0 \times 10^{-9}\ \text{C})}{(0.040\ \text{m})^2}$$
$$= 1.97 \times 10^{-4}\ \text{N}.$$

Both $F_1$ and $F_2$ are positive because they are the magnitudes of vector quantities. Since $\vec{F}_1$ points in the $-x$ direction, $F_{1x} = -F_1 = -3.37 \times 10^{-4}\ \text{N}$. Since $\vec{F}_2$ points in the $+x$ direction, $F_{2x} = +F_2 = +1.97 \times 10^{-4}\ \text{N}$. Adding $x$ components, we find that $F_{\text{total},x} -3.37 \times 10^{-4}\ \text{N} + 1.97 \times 10^{-4}\ \text{N} = -1.40 \times 10^{-4}\ \text{N}$. There are no $y$ components. Since $F_{\text{total},x} = -1.40 \times 10^{-4}\ \text{N}$ and $F_{\text{total},y} = 0$, $\vec{F}_{\text{total}}$ has magnitude $1.40 \times 10^{-4}\ \text{N}$ and is in the $-x$ direction.

**REFLECT** Because the distance term $r$ in Coulomb's law is squared, $F_1$ is greater than $F_2$ even though $|q_2|$ is greater than $|q_1|$.

**Practice Problem:** In the same example, what is the total force (magnitude and direction) exerted on $q_1$ by $q_2$ and $q_3$? *Answer:* $8.1 \times 10^{-4}\ \text{N}$, in the $+x$ direction.

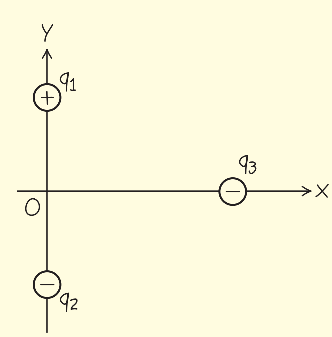
## CONCEPTUAL ANALYSIS 17.4

### Charges and the net force

Three point charges $q_1$, $q_2$, and $q_3$ are placed at the corners of an equilateral triangle, as shown in Figure 17.12. $q_1$ has a charge of $+Q$. $q_2$ and $q_3$ both have a charge of $-Q$. In which direction does the net force point on $q_3$?

A. In the $+x$ direction
B. In the $-x$ direction
C. In the $+y$ direction
D. In the $-y$ direction

**SOLUTION** The magnitude of all three charges is the same, so the magnitude of the force exerted on $q_3$ by $q_1$ is the same as the magnitude of the force exerted on $q_3$ by $q_2$. $q_3$ is attracted to $q_1$ (so $\vec{F}_1$ points upward and to the left) but is repelled by $q_2$ (so $\vec{F}_2$ points upward and to the right). The $x$ components of both forces cancel out and the $y$ components add together, so the net force points straight up. Therefore, the correct answer is C.




▲ **FIGURE 17.12**

**EXAMPLE 17.4** **Vector addition of forces**

Video Tutor Solution

This example is similar to the preceding one, but now we will consider a two-dimensional arrangement of charges. A point charge $q_1 = 2.0\ \mu\text{C}$ is located on the positive $y$ axis at $y = 0.30$ m, and an identical charge $q_2$ is at the origin. Find the magnitude and direction of the total force that these two charges exert on a third charge $q_3 = 4.0\ \mu\text{C}$ that is on the positive $x$ axis at $x = 0.40$ m.

**SOLUTION**

**SET UP** As in the preceding example, we sketch the situation and draw a free-body diagram for $q_3$ (Figure 17.13), using $\vec{F}_1$ and $\vec{F}_2$ for the forces exerted on $q_3$ by $q_1$ and $q_2$, respectively. The directions of $\vec{F}_1$ and $\vec{F}_2$ are determined by the fact that like charges repel.

**SOLVE** As in Example 17.3, the net force acting on $q_3$ is the vector sum of $\vec{F}_1$ and $\vec{F}_2$. We use Coulomb's law to find the magnitudes $F_1$ and $F_2$ of the forces:

$$F_1 = k\frac{|q_1 q_3|}{{r_{13}}^2}$$
$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(2.0 \times 10^{-6}\ \text{C})(4.0 \times 10^{-6}\ \text{C})}{(0.50\ \text{m})^2}$$
$$= 0.288\ \text{N},$$

$$F_2 = k\frac{|q_2 q_3|}{{r_{23}}^2}$$
$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(2.0 \times 10^{-6}\ \text{C})(4.0 \times 10^{-6}\ \text{C})}{(0.40\ \text{m})^2}$$
$$= 0.450\ \text{N}.$$

We now calculate the $x$ and $y$ components of $\vec{F}_1$ and add them to the $x$ and $y$ components of $\vec{F}_2$, respectively, to obtain the components of the total force $\vec{F}_{\text{total}}$ on $q_3$. From Figure 17.13a, $\sin\theta = (0.30\ \text{m})/(0.50\ \text{m}) = 0.60$ and $\cos\theta = (0.40\ \text{m})/(0.50\ \text{m}) = 0.80$. Since the $y$ component of $\vec{F}_2$ is zero and its $x$ component is positive, the $x$ component of $\vec{F}_2$ is $F_{2x} = F_2 = 0.450$ N. The total $x$ and $y$ components are

$$\begin{aligned} F_{\text{total},x} &= F_{1x} + F_{2x} \\ &= (0.288\ \text{N})\cos\theta + 0.450\ \text{N} \\ &= (0.288\ \text{N})(0.80) + 0.450\ \text{N} = 0.680\ \text{N}, \\ F_{\text{total},y} &= F_{1y} + F_{2y} = -(0.288\ \text{N})\sin\theta + 0 \\ &= -(0.288\ \text{N})(0.60) = -0.173\ \text{N}. \end{aligned}$$

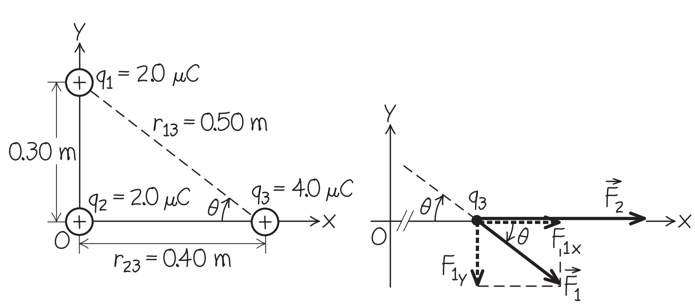
These components combine to form $\vec{F}_{\text{total}}$, as shown in Figure 17.14:

$$\begin{aligned} F_{\text{total}} &= \sqrt{{F_{\text{total},x}}^2 + {F_{\text{total},y}}^2} \\ &= \sqrt{(0.680\ \text{N})^2 + (-0.173\ \text{N})^2} = 0.70\ \text{N}, \\ \tan\phi &= \frac{F_{\text{total},y}}{F_{\text{total},x}} = \frac{-0.173\ \text{N}}{0.680\ N} \text{ and } \phi = -14°. \end{aligned}$$

The resultant force has magnitude 0.70 N and is directed at 14° below the $+x$ axis.

**REFLECT** The forces exerted by $q_1$ and $q_2$ both have components in the $+x$ direction, so these components add. The force exerted by $q_1$ also has a component in the $-y$ direction, so the net force is in the fourth quadrant. Even though $q_1$ and $q_2$ are identical, the force exerted by $q_2$ is larger than the force exerted by $q_1$ because $q_3$ is closer to $q_2$.

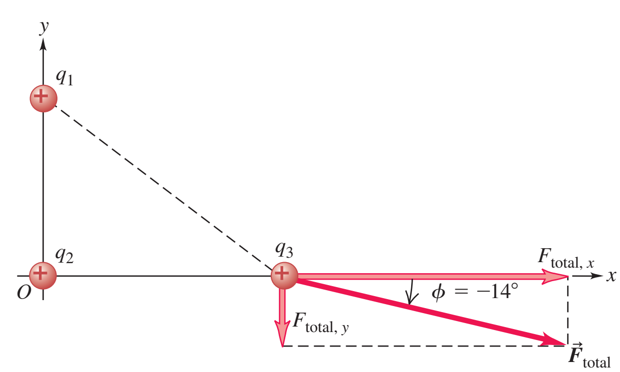
**Practice Problem:** In the same example, what is the net force on $q_3$ if $q_1 = 2.0\ \mu\text{C}$, as in the example, but $q_2 = -2.0\ \mu\text{C}$? *Answer:* 0.28 N, 38° below the $-x$ axis.




**(a)** Our sketch of the situation

**(b)** Free-body diagram for $q_3$

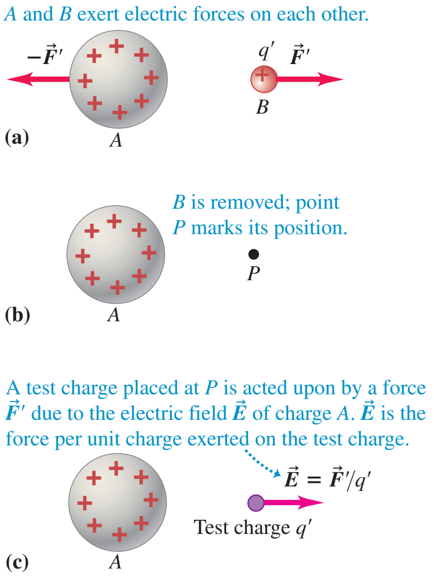
▲ **FIGURE 17.13** Our sketches for this problem.



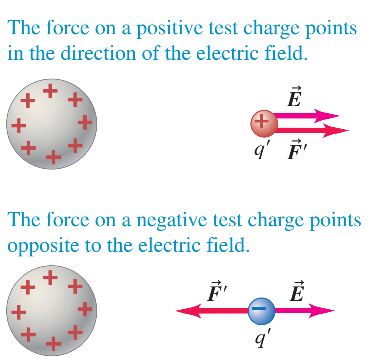

▲ **FIGURE 17.14** The total force on $q_3$.

## 17.5 Electric Field and Electric Forces

When two electrically charged particles in empty space interact, how does each one "know" that the other is there? What goes on in the space between them to transmit the effect of each one to the other? We can begin to answer these questions, and at the same time reformulate Coulomb's law in a very useful way, by using the concept of *electric field.* To introduce this concept, let's look at the mutual repulsion of two positively charged objects $A$ and $B$




▲ **FIGURE 17.15** A charged object creates an electric field in the space around it.




▲ **FIGURE 17.16** The direction of the electric force on positive and negative test charges relative to the direction of the electric field.

(Figure 17.15a). Suppose $B$ is a point charge $q'$, and let $\vec{F}'$ be the force on $B$, as shown in the figure. One way to think about this force is as an "action-at-a-distance" force—that is, as a force that acts across empty space without needing any matter (such as a pushrod or a rope) to transmit it through the intervening space.

Now think of object $A$ as having the effect of somehow modifying the properties of the space around it. We remove object $B$ and label its former position as point $P$ (Figure 17.15b). We say that the charged object $A$ produces or causes an **electric field** at point $P$ (and at all other points in the neighborhood). Then, when point charge $B$ is placed at point $P$ and is acted upon by the force $\vec{F}'$, we take the point of view that the force is exerted on $B$ *by the electric field* at $P$. Because $B$ would be acted upon by a force at *any* point in the neighborhood of $A$, the electric field exists at all points in the region around $A$. (We could also say that point charge $B$ sets up an electric field, which in turn exerts a force on object $A$.)

To find out experimentally whether there is an electric field at a particular point, we place a charged object, which we call a **test charge,** at the point (Figure 17.15c). If we find that the test charge experiences a nonzero electric force, then there is an electric field at that point.

Force is a vector quantity, so electric field is also a vector quantity. (Note the use of boldface letters with arrows on top of them in the discussion that follows.) To define the *electric field* $\vec{E}$ at any point, we place a test charge $q'$ at the point and measure the electric force $\vec{F}'$ on it (Figure 17.15c). We define $\vec{E}$ at this point to be equal to $\vec{F}'$ divided by $q'$:

**Definition of electric field**

When a charged particle with charge $q'$ at a point $P$ is acted upon by an electric force $\vec{F}'$, the electric field $\vec{E}$ at that point is defined as

$$\vec{E} = \frac{\vec{F}'}{q'}. \tag{17.2}$$

The test charge $q'$ can be either positive or negative. If it is positive, the directions of $\vec{E}$ and $\vec{F}'$ are the same; if it is *negative,* they are opposite (Figure 17.16).
Units: newton per coulomb (N/C)

The force acting on the test charge $q'$ varies from point to point, so the electric field is also different at different points. Be sure you understand that $\vec{E}$ is not a single vector quantity but an infinite set of vector quantities, one associated with each point in space. We call this situation a **vector field**—a vector quantity associated with every point in a region of space, different at different points. In general, each component of $\vec{E}$ at any point depends on (i.e., is a function of) all the coordinates of the point.

If an electric field exists within a *conductor,* the field exerts a force on every charge in the conductor, causing the free charges to move. By definition, an *electrostatic* situation is a situation in which the charges *do not* move. We conclude that **in an electrostatic situation, the electric field at every point within the material of a conductor must be zero.** (In Section 17.9, we'll consider the special case of a conductor that has a central cavity.)

In general, the magnitude and direction of an electric field can vary from point to point. If, in a particular situation, the magnitude and direction of the field are *constant* throughout a certain region, we say that the field is *uniform* in that region.

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**PhET:** Charges and Fields
**PhET:** Electric Field of Dreams

**CONCEPTUAL ANALYSIS 17.5**

### A moving electron

A vacuum chamber contains a uniform electric field directed downward. If an electron is shot horizontally into this region, its acceleration is

A. downward and constant.
B. upward and constant.
C. upward and changing.
D. downward and changing.

**SOLUTION** The electron has a negative charge, so it is acted upon by a force directed *opposite* to the electric field—that is, an upward force giving an upward acceleration. We're told that the electric field is *uniform,* meaning that its magnitude and direction are constant. Therefore, the force exerted on the electron by the electric field is constant ($\vec{F}' = \vec{E}/q'$). Because the force is constant, so is the acceleration (by Newton's second law). The correct answer is B.

## EXAMPLE 17.5 Accelerating an electron

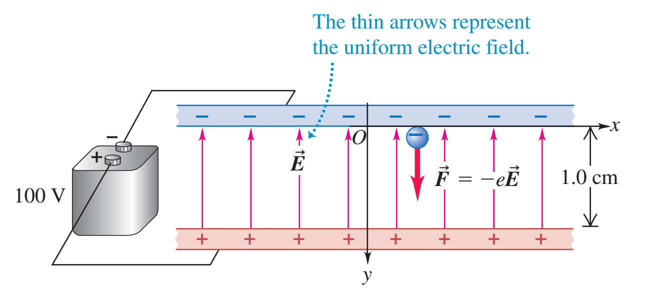
In this example we will analyze the motion of an electron that is released in an electric field. The terminals of a 100 V battery are connected to two large, parallel, horizontal plates 1.0 cm apart. The resulting charges on the plates produce an electric field $\vec{E}$ in the region between the plates that is very nearly uniform and has magnitude $E = 1.0 \times 10^4$ N/C. Suppose the lower plate has positive charge, so that the electric field is vertically upward, as shown in Figure 17.17. (The thin pink arrows represent the electric field.) If an electron is released from rest at the upper plate, what is its speed just before it reaches the lower plate? How much time is required for it to reach the lower plate? The mass of an electron is $m_e = 9.11 \times 10^{-31}$ kg.

Video Tutor Solution

**SOLUTION**

**SET UP** We place the origin of coordinates at the upper plate and take the $+y$ direction to be downward, toward the lower plate. The electron has negative charge, $q = -e$, so the direction of the force on the electron is downward, opposite to the electric field. The field is uniform, so the force on the electron is constant. Thus the electron has constant acceleration, and we can use the constant-acceleration equation $v_y{}^2 = v_{0y}{}^2 + 2a_y y$. The electron's initial velocity $v_{0y}$ is zero, so

$$v_y{}^2 = 2a_y y.$$




▲ **FIGURE 17.17**

**SOLVE** The force on the electron has only a $y$ component, which is positive, and we can solve Equation 17.2 to find this component:

$$F_y = |q|E = (1.60 \times 10^{-19}\,\text{C})(1.0 \times 10^4\,\text{N/C})$$
$$= 1.60 \times 10^{-15}\,\text{N}.$$

Newton's second law then gives the electron's acceleration:

$$a_y = \frac{F_y}{m_e} = \frac{1.60 \times 10^{-15}\,\text{N}}{9.11 \times 10^{-31}\,\text{kg}} = +1.76 \times 10^{15}\,\text{m/s}^2.$$

We want to find $v_y$ when $y = 0.010$ m. The equation for $v_y$ gives

$$v_y = \sqrt{2a_y y} = \sqrt{2(1.76 \times 10^{15}\,\text{m/s}^2)(0.010\,\text{m})}$$
$$= 5.9 \times 10^6\,\text{m/s}.$$

Finally, $v_y = v_{0y} + a_y t$ gives the total travel time $t$:

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{5.9 \times 10^6\,\text{m/s} - 0}{1.76 \times 10^{15}\,\text{m/s}^2} = 3.4 \times 10^{-9}\,\text{s}.$$

**REFLECT** The acceleration produced by the electric field is enormous; to give a 1000 kg car this acceleration, we would need a force of about $2 \times 10^{18}$ N, or about $2 \times 10^{14}$ tons. The effect of gravity is negligible. Note again that negative charges gain speed when they move in a direction opposite to the direction of the electric field.

**Practice Problem:** In this example, suppose a proton ($m_p = 1.67 \times 10^{-27}$ kg) is released from rest at the positive plate. What is its speed just before it reaches the negative plate? *Answer:* $1.38 \times 10^5$ m/s.

## 17.6 Calculating Electric Fields

In this section, we'll discuss several situations in which electric fields produced by specific charge distributions can be determined with fairly simple calculations. The key to these calculations is the principle of superposition, which we mentioned in Section 17.4. Restated in terms of electric fields, the principle is as follows:

▲ **BIO Application**
**Sensitive snout.**

As a rule, mammals cannot sense external electric fields, but the platypus is an exception. It feeds on small underwater creatures, which it finds by nosing around the bottom of streams and ponds. It hunts with its eyes shut, and usually at night, so it cannot see its prey. Instead, its rubbery bill detects the tiny electric fields created by the nerves and muscles of the prey. (The bill is also highly sensitive to touch.) Because water is a good conductor but air is not, the ability to sense electric fields is found almost exclusively among water-dwelling creatures (mainly fish).

**Principle of superposition**

The total electric field at any point due to two or more charges is the vector sum of the fields that would be produced at that point by the individual charges.

To find the field caused by several charges or an extended distribution of charge, we imagine that the source is made up of many point charges. We call the location of one of these points a **source point** (denoted by $S$, possibly with a subscript), and the point where we want to find the field is called the **field point** (denoted by $P$). We calculate the fields $\vec{E}_1, \vec{E}_2, \vec{E}_3, \ldots$ at point $P$ caused by the individual point charges $q_1, q_2, q_3, \ldots$ located at points $S_1, S_2, S_3, \ldots$ and take their vector sum (using the superposition principle) to find the total field $\vec{E}_{\text{total}}$ at point $P$; that is,

$$\vec{E}_{\text{total}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots.$$

## Electric field due to a point charge

If the source distribution is a single point charge $q$, it is easy to find the electric field that it produces. We call the location of the charge the source point $S$, and we call the point $P$ where we are determining the field the field point. If we place a small test charge $q'$ at the field point $P$, at a distance $r$ from the source point, the magnitude of the force $\vec{F}'$ is given by Coulomb's law, Equation 17.1:

$$F' = k\frac{|qq'|}{r^2}.$$

From Equation 17.2, we find the magnitude $E$ of the electric field at $P$:

**Electric field due to a point charge**

The magnitude $E$ of the electric field $\vec{E}$ at point $P$ due to a point charge $q$ at point $S$, a distance $r$ from $P$, is given by

$$E = k\frac{|q|}{r^2}. \tag{17.3}$$

By definition, the electric field produced by a positive point charge always points *away from* it, but the electric field produced by a negative point charge points *toward* it.

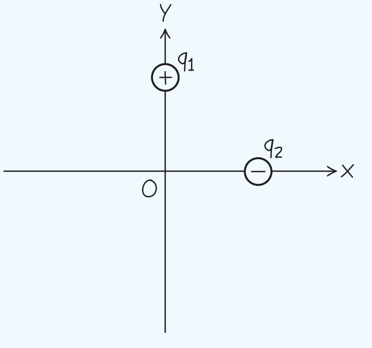
**QUANTITATIVE ANALYSIS 17.6**

### Direction of the net field

Figure 17.18 shows two charges in the $x$-$y$ plane. Charge $q_1$ sits on the positive $y$ axis and has a positive charge. Charge $q_2$ sits on the positive $x$ axis and is negatively charged. The electric field at the origin points toward the

A. first quadrant. B. second quadrant.
C. third quadrant. D. fourth quadrant.

**SOLUTION** $\vec{E} = \vec{F}/q'$ (Equation 17.2) tells us that the electric field vector points in the same direction as the force vector if we use a positive test charge to test the field at the point we are interested in. So the easiest way to determine the direction of the electric field at the origin is to think about the direction of the net force acting on a positive charge placed at the origin. A positive test charge would be repelled by $q_1$ and attracted to $q_2$, so the net force acting on it would point downward and to the right. Therefore, the correct answer is D.




▲ **FIGURE 17.18**

## Spherical charge distributions

In applications of electrostatics, we often encounter charge distributions that have spherical symmetry. Familiar examples include electric charge distributed uniformly over the surface of a conducting sphere and charge distributed uniformly throughout the volume of an insulating sphere. It turns out that the electric field produced by *any* spherically symmetric charge distribution, at all points outside this distribution, is the same as though all the charge were concentrated at a point at the center of the sphere. In field calculations, the field outside any spherical charge distribution can be obtained by replacing the distribution with a single point charge at the center of the sphere and equal to the total charge of the sphere.

### EXAMPLE 17.6 Electric field in a hydrogen atom

Let's now calculate the electric field of two simple spherical charge distributions. **(a)** In the Bohr model of the hydrogen atom (described in Example 17.2), when the atom is in its lowest-energy state, the distance from the proton to the electron is $5.29 \times 10^{-11}$ m. Find the electric field due to the proton at this distance. **(b)** A device called a Van de Graaff generator (shown on page 533) can build up a large static charge on a metal sphere. Suppose the sphere of a Van de Graaff generator has a radius of 0.50 m and a net charge of 1.0 $\mu$C. What is the magnitude of the electric field 1.0 m from the center of the sphere? Compare this electric field with the field calculated in part (a).

Video Tutor Solution

**SOLUTION**

**SET UP** Figure 17.19 shows our diagrams for these cases.

**SOLVE Part (a):** We are asked to calculate the electric-field magnitude $E$ at a distance of $5.29 \times 10^{-11}$ m from a point charge (the proton). We use Equation 17.3; a proton has charge $q = +e = 1.60 \times 10^{-19}$ C, so

$$E = k\frac{|q|}{r^2}$$

$$= (8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)\frac{(1.60 \times 10^{-19}\ \text{C})}{(5.29 \times 10^{-11}\ \text{m})^2}$$

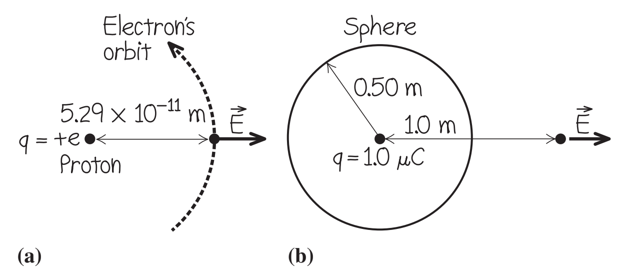
$$= 5.14 \times 10^{11}\ \text{N/C}.$$

**Part (b):** To calculate the field of the Van de Graaff sphere, we use the principle discussed above: A uniform spherical distribution of charge creates the same field as an equal point charge located at the center of the sphere. Thus, we can again use Equation 17.3:

$$E = k\frac{|q|}{r^2} = (8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)\frac{(1.0 \times 10^{-6}\ \text{C})}{(1.0\ \text{m})^2}$$

$$= 9.0 \times 10^3\ \text{N/C}.$$

The electric field in part (a) is larger than that in part (b) by a factor of $5.7 \times 10^7$.



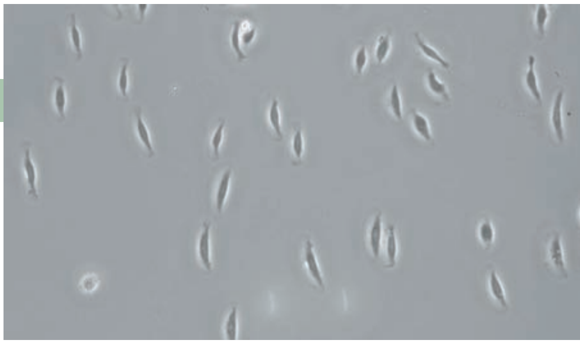

▲ **FIGURE 17.19** Our sketches for this problem.

**REFLECT** The electric field in an atom is extremely large compared with the electric fields of macroscopic objects that have easily obtainable electric charges.

**Practice Problem:** At what distance from a proton does the electric field of the proton have magnitude $9.0 \times 10^3$ N/C? How does this distance compare with the Bohr orbit radius ($r = 5.29 \times 10^{-11}$ m) of the electron in the lowest-energy state of the hydrogen atom? *Answers:* $4.0 \times 10^{-7}$ m, $7.6 \times 10^3$ times larger.

**▶ BIO Application They got their electrical marching orders.**

Many cells, including nerve cells and skin cells, are remarkably sensitive to electric fields. The photograph shows cultured skin cells of the zebrafish (an important experimental animal for biology and medicine). These cells are highly mobile in culture, moving at speeds of 10 $\mu$m/min. Left to their own devices, these cells move at random, independently of one another; however, when exposed to a modest electric field of 100 N/C, they align their long axes perpendicular to the field lines and move in the direction of the field. These cells respond to fields as small as 7 N/C, which is well within the range of electric fields that have been measured near skin wounds in vertebrates. It may be that the wound-healing response is controlled in part by natural electric fields.



**PROBLEM-SOLVING STRATEGY 17.2** **Electric-field calculations**

**SET UP**

1. Be sure to use a consistent set of units. Distances must be in meters, charges in coulombs. If you are given cm or nC, don't forget to convert.
2. Usually, you will use components to compute vector sums. As we suggested for problems involving Coulomb's law, it may be helpful to review Sections 1.7 and 1.8. Use proper vector notation; distinguish carefully among scalars, vectors, and components of vectors. Indicate your coordinate axes clearly on your diagram, and be certain that the components are consistent with your choice of axes.

**SOLVE**

3. In working out directions of $\vec{E}$ vectors, be careful to distinguish between the *source point S* and the *field point P*. The field produced by a positive point charge always points in the direction from source point to field point; the opposite is true for a negative point charge.

**REFLECT**

4. If your result is a symbolic expression, check to see whether it depends on the variables in the way you expect. If it is numeric, estimate what you expect the result to be and check for consistency with the result of your calculations.

## EXAMPLE 17.7 Electric field of an electric dipole

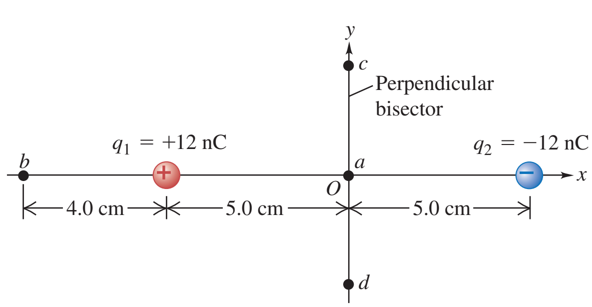
One of the most important charge distributions in physics and chemistry is the **electric dipole.** The dipole consists of equal and opposite charges that are separated by a small distance. Although the dipole itself is neutral, there is an electric field because the two charges are not at exactly that same position. Suppose point charges $q_1$ and $q_2$ of $+12$ nC and $-12$ nC, respectively, are placed 10.0 cm apart (Figure 17.20). Compute the resultant electric field (magnitude and direction) at **(a)** point $a$, midway between the charges, and **(b)** point $b$, 4.0 cm to the left of $q_1$. **(c)** What is the direction of the resultant electric field produced by these two charges at points along the perpendicular bisector of the line connecting the charges? Consider points both above and below the line connecting the charges.

Video Tutor Solution

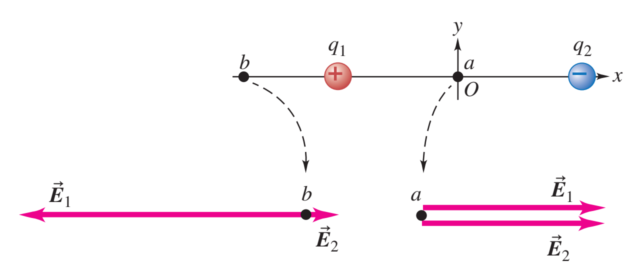
**SOLUTION**

**SET UP** We use a coordinate system with the origin midway between the two charges and with the $+x$ axis toward $q_2$, as shown in Figure 17.20. The perpendicular bisector then lies along the $y$ axis. We use $\vec{E}_1$ and $\vec{E}_2$ to denote the electric fields due to $q_1$ and $q_2$, respectively; the resultant electric field is the vector sum of these fields. The point charges are the source points, and points $a$, $b$, $c$, and $d$ are the field points.

**SOLVE** For a point charge, the magnitude of the electric field is given by $E = k\dfrac{|q|}{r^2}$.




▲ **FIGURE 17.20**




▲ **FIGURE 17.21** The electric fields due to the two charges at points $a$ and $b$.

**Part (a):** The electric fields at point $a$ are shown in Figure 17.21. $\vec{E}_1$ points away from $q_1$ (because $q_1$ is positive), and $\vec{E}_2$ points toward $q_2$ (because $q_2$ is negative). Thus,

$$E_1 = E_2 = k\frac{|q_1|}{r_1^{\,2}} = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{(12 \times 10^{-9}\ \mathrm{C})}{(0.050\ \mathrm{m})^2}$$
$$= 4.32 \times 10^4\ \mathrm{N/C}.$$

Since $\vec{E}_1$ and $\vec{E}_2$ point in the same direction, $E_{\text{total}} = E_1 + E_2 = 8.6 \times 10^4$ N/C and $E_{\text{total}}$ points in the $+x$ direction, from the positive charge toward the negative charge.

CONTINUED

**Part (b):** The electric fields at point $b$ are also shown in Figure 17.21. Again, $\vec{E}_1$ points away from $q_1$ and $\vec{E}_2$ points toward $q_2$. Hence,

$$E_1 = k\frac{|q_1|}{r_1^2} = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{(12 \times 10^{-9}\ \mathrm{C})}{(0.040\ \mathrm{m})^2}$$
$$= 6.74 \times 10^4\ \mathrm{N/C},$$

$$E_2 = k\frac{|q_2|}{r_2^2} = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{(12 \times 10^{-9}\ \mathrm{C})}{(0.140\ \mathrm{m})^2}$$
$$= 5.50 \times 10^3\ \mathrm{N/C}.$$

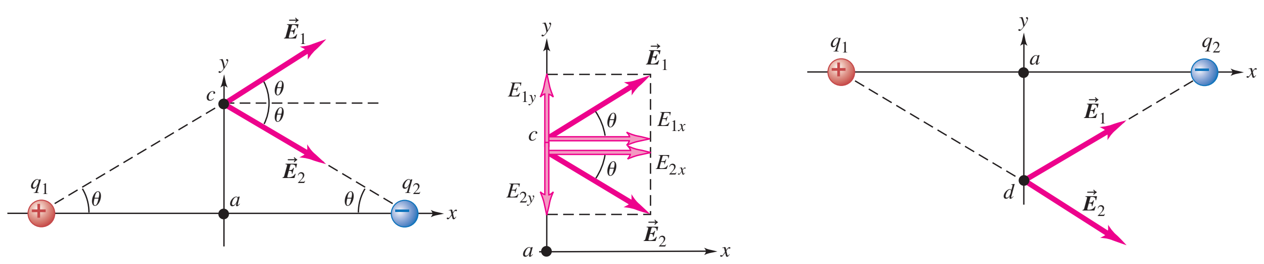
$E_1$ is larger than $E_2$ because point $b$ is closer to $q_1$ than to $q_2$. Since $\vec{E}_1$ and $\vec{E}_2$ point in opposite directions, $E_{\text{total}} = E_1 - E_2 = 6.2 \times 10^4\ \mathrm{N/C}$. $\vec{E}_{\text{total}}$ points to the left, in the direction of the stronger field.

**Part (c):** At point $c$ in Figure 17.20, the two electric fields are directed as shown in Figure 17.22a. In Figure 17.22b, each electric field is replaced by its $x$ and $y$ components. Point $c$ is equidistant from the two charges, and $|q_1| = |q_2|$, so $E_1 = E_2$. The $y$ components of $\vec{E}_1$ and $\vec{E}_2$ are equal in magnitude and opposite in direction, and their sum is zero. The $x$ components are equal in magnitude and are both in the $+x$ direction, so the resultant field is in the $+x$ direction.

At point $d$ in Figure 17.20, the two electric fields are directed as shown in Figure 17.22c. The resultant field is again in the $+x$ direction. At all points along the perpendicular bisector of the line connecting the two charges, the resultant field is in the $+x$ direction, parallel to the direction from the positive charge toward the negative charge.

**REFLECT** Our general result in part (c) is consistent with the direction of the electric field calculated at point $a$. The resultant electric field has the same direction at every point along the perpendicular bisector, but the magnitude decreases at points farther from the charges.

**Practice Problem:** Repeat the calculations of this example, using the same value of $q_1$ but with $q_2 = +12$ nC (so that both charges are positive). *Answers:* (a) 0; (b) $7.3 \times 10^4$ N/C, in the $-x$ direction; (c) along the $y$ axis and away from the charges.




(a) The electric fields at point $c$

(b) The electric fields at point $c$ and their components

(c) The electric fields at point $d$

▲ **FIGURE 17.22**

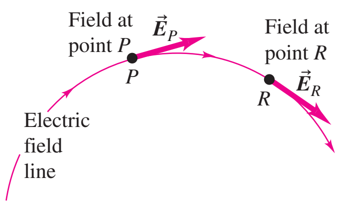
## 17.7 Electric Field Lines

The concept of an electric field may seem rather abstract; you can't see or feel one (although some animals can). It's often useful to draw a diagram that helps you to visualize electric fields at various points in space. A central element in such a diagram is the concept of **electric field lines.** An electric field line is an imaginary line drawn through a region of space so that, at every point, it is tangent to the direction of the electric field vector at that point. The basic idea is shown in Figure 17.23. Michael Faraday (1791–1867) first introduced the concept of field lines. He called them "lines of force," but the term "field lines" is preferable.

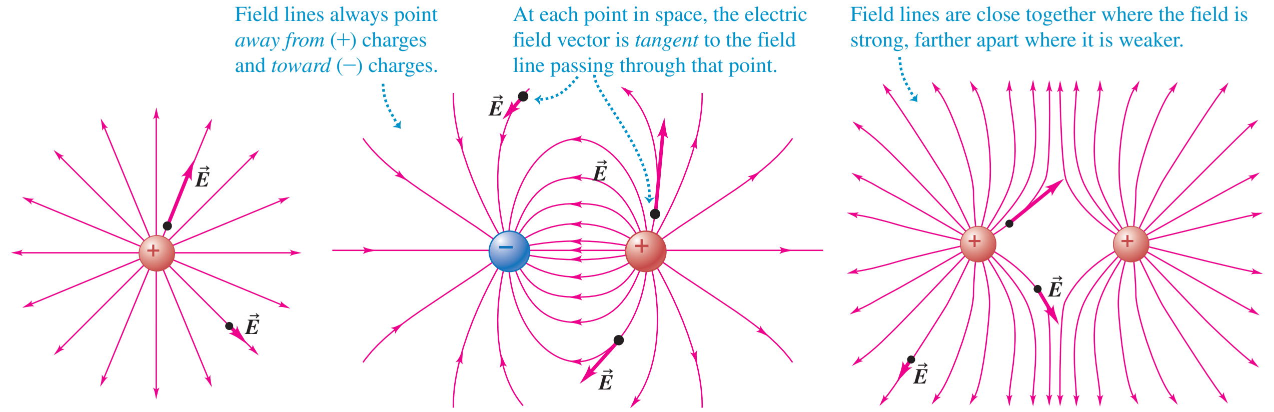
Electric field lines show the direction of $\vec{E}$ at each point, and their spacing gives a general idea of the *magnitude* of $\vec{E}$ at each point. Where $\vec{E}$ is strong, we draw lines bunched closely together; where $\vec{E}$ is weaker, they are farther apart. At any particular point, the electric field has a unique direction, so only one field line can pass through each point of the field. In other words, *field lines never intersect.*

Electric field lines always have these characteristics:

1. At every point in space, the electric field vector $\vec{E}$ at that point is tangent to the electric field line through that point.
2. Electric field lines are close together in regions where the magnitude of $\vec{E}$ is large, farther apart where it is small.
3. Field lines point away from positive charges and toward negative charges.



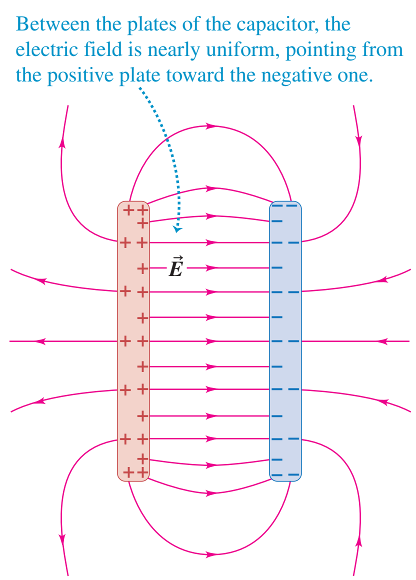

▲ **FIGURE 17.23** The direction of the electric field at any point is tangent to the field line through that point.




**(a)** A single positive charge **(b)** Two equal and opposite charges (a dipole) **(c)** Two equal positive charges

▲ **FIGURE 17.24** Electric field lines for different charge distributions.

Figure 17.24 shows some of the electric field lines in a plane containing (a) a single positive charge; (b) two equal charges, one positive and one negative (a dipole); and (c) two equal positive charges. These are cross sections of the actual three-dimensional patterns. The direction of the total electric field at every point in each diagram is along the tangent to the electric field line passing through the point. Arrowheads on the field lines indicate the sense of the $\vec{E}$ field vector along each line (showing that the field points away from positive charges and toward negative charges). In regions where the field magnitude is large, such as the space between the positive and negative charges in Figure 17.24b, the field lines are drawn close together. In regions where it is small, such as between the two positive charges in Figure 17.24c, the lines are widely separated.




▲ **FIGURE 17.25** The electric field produced by a parallel-plate capacitor (seen in cross section). Between the plates, the field is nearly uniform.

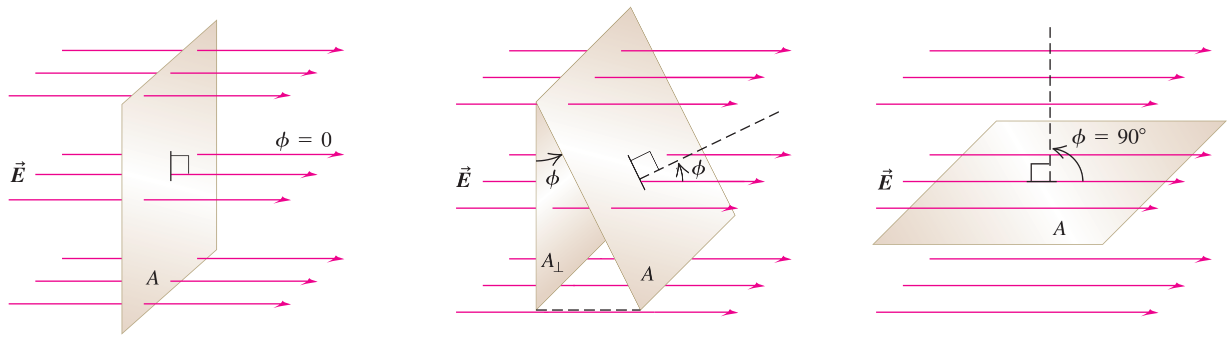
**NOTE** There may be a temptation to think that when a charged particle moves in an electric field, its path always follows a field line. Resist that temptation; the thought is erroneous. The direction of a field line at a given point determines the direction of the particle's *acceleration,* not its velocity. See Example 17.5.

### Parallel-plate capacitor

In a *uniform* electric field, the field lines are straight, parallel, and uniformly spaced, as in Figure 17.25. When two conducting sheets carry opposite charges and are close together compared with their size, the electric field in the region between them is approximately uniform. This arrangement is often used when a uniform electric field is needed, as in setups to deflect electron beams. A similar configuration of conductors, consisting of two sheets separated by a thin insulating layer, forms a device called a **parallel-plate capacitor,** which is widely used in electronic circuits and which we'll study in the next chapter.

## 17.8 Gauss's Law and Field Calculations

Gauss's law is an alternative formulation of the principles of electrostatics. It is mathematically equivalent to Coulomb's law, but for some problems it provides a useful alternative approach to calculating electric fields. Coulomb's law enables us to find the field at a *point* $P$ caused by a single *point* charge $q$. To calculate fields produced by an *extended* charge distribution, we have to represent that distribution as an assembly of point charges and use the superposition principle. Gauss's law takes a more global view. Given any general distribution of charge, we surround it with an imaginary closed surface (often called a *Gaussian surface*) that encloses the charge. Then we look at the electric field at various points on this imaginary surface. Gauss's law is a relationship between the field at *all* the points on the surface and the total charge enclosed within the surface.




Electric field $\vec{E}$ is perpendicular to area $A$; the angle between $\vec{E}$ and a line perpendicular to the surface is zero.
The flux is $\Phi_E = EA$.

**(a)**

Area $A$ is tilted at an angle $\phi$ from the perpendicular to $\vec{E}$.
The flux is $\Phi_E = EA\cos\phi$.

**(b)**

Area $A$ is parallel to $\vec{E}$ (tilted at 90° from the perpendicular to $\vec{E}$).
The flux is $\Phi_E = EA\cos 90° = 0$.

**(c)**

▲ **FIGURE 17.26** The electric flux through a flat surface at various orientations relative to a uniform electric field.

Gauss's law is part of the key to using symmetry considerations in electric-field calculations. Calculations with a system that has symmetry properties can nearly always be simplified if we can make use of the symmetry, and Gauss's law helps us do just that.

## Electric flux

In formulating Gauss's law, we'll use the concept of **electric flux,** also called *flux of the electric field.* We'll define this concept first, and then we'll discuss an analogy with fluid flow that will help you to develop intuition about it.

The definition of electric flux involves an area $A$ and the electric field at various points in the area. The area needn't be the surface of a real object; in fact, it will usually be an imaginary area in space. Consider first a small, flat area $A$ perpendicular to a uniform electric field $\vec{E}$ (Figure 17.26a). We denote electric flux by $\Phi_E$; we define the electric flux $\Phi_E$ through the area $A$ to be the product of the magnitude $E$ of the electric field and the area $A$:

$$\Phi_E = EA.$$

Roughly speaking, we can picture $\Phi_E$ in terms of the number of field lines that pass through $A$. More area means more lines through the area, and a stronger field means more closely spaced lines and therefore more lines per unit area.

If the area element $A$ isn't perpendicular to the field $\vec{E}$, then fewer field lines pass through it. In this case, what counts is the area of the silhouette of $A$ that we see as we look along the direction of $\vec{E}$; this is the area $A_\perp$ in Figure 17.26, the *projection* of the area $A$ onto a surface perpendicular to $\vec{E}$. Two sides of the projected rectangle have the same length as the original one, but the other two are foreshortened by a factor $\cos\phi$; so the projected area $A_\perp$ is equal to $A\cos\phi$. We can generalize our definition of electric flux for a uniform electric field:

**Electric flux**

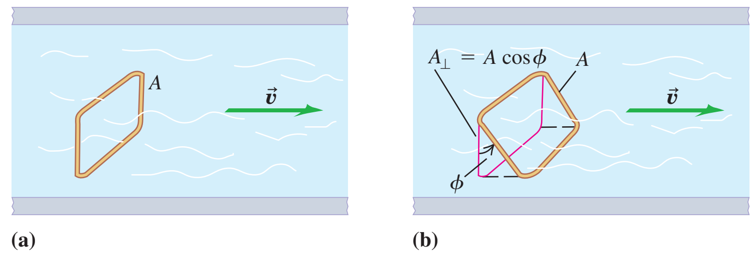
$$\Phi_E = EA\cos\phi. \tag{17.4}$$

Units: $\mathrm{N \cdot m^2/C}$
Notes:

- $\phi$ is the angle between $\vec{E}$ and a line that is perpendicular to the surface (the normal to the surface).
- $A$ is the area of the surface.
- If $\vec{E}$ is parallel to the surface then the flux is zero (Figure 17.26c).

▲ BIO Application
**Feeling my way.**

These African elephant-nose fish "feel" their way through their murky freshwater environment by producing an electric field and sensing how objects distort the field. The field is produced in pulses by an electric organ near each fish's tail; it is detected by receptors covering portions of the fish's skin. An object that conducts electricity better than fresh water, such as an animal or a plant, causes the nearby field lines to bunch together, creating a spot of stronger field on the fish's skin. An object that conducts less well than water, such as a rock, causes the field lines to spread apart, which the fish sense as a spot of weaker field. By integrating the information from their receptors, the fish can perceive their surroundings. Several groups of fish generate and use electric fields in this way.




▶ **FIGURE 17.27** The volume flow rate of water through the wire rectangle is $vA\cos\phi$, just as the electric flux through an area $A$ is $EA\cos\phi$.

We can identify $E\cos\phi$ as the component of the vector $\vec{E}$ perpendicular to the area. If we call this component $E_\perp$, Equation 17.4 can be rewritten as

$$\Phi_E = E_\perp A. \tag{17.5}$$

The word *flux* comes from a Latin word meaning "flow." Even though an electric field is *not* a flow, an analogy with fluid flow will help to develop your intuition about electric flux. Imagine that $\vec{E}$ is analogous to the velocity of flow $\vec{v}$ of water through the imaginary area bounded by the wire rectangle in Figure 17.27. The flow rate through the area $A$ is proportional to $A$ and $v$, and it also depends on the angle between $\vec{v}$ and a line perpendicular to the plane of the rectangle. When the area is perpendicular to the flow velocity $\vec{v}$ (Figure 17.27a), the volume flow rate is just $vA$. When the rectangle is tilted at an angle $\phi$ (Figure 17.27b), the area that counts is the silhouette area that we see when looking in the direction of $\vec{v}$. That area is $A\cos\phi$, as shown, and the volume flow rate through $A$ is $vA\cos\phi$. This quantity is called the *flux* of $\vec{v}$ through the area $A$; *flux* is a natural term because it represents the volume rate of flow of fluid through the area. In the electric-field situation, *nothing is flowing,* but the analogy to fluid flow may help you to visualize the concept.

## EXAMPLE 17.8 Electric flux through a disk

Now let's use Equation 17.4 to calculate the electric flux through a disk with radius 0.10 m. The disk is oriented with its axis (the line through the center, perpendicular to the disk's surface) at an angle of 30° to a uniform electric field $\vec{E}$ with magnitude $2.0 \times 10^3$ N/C (Figure 17.28). **(a)** What is the total electric flux through the disk? **(b)** What is the total flux through the disk if it is turned so that its plane is parallel to $\vec{E}$? **(c)** What is the total flux through the disk if it is turned so that its axis (marked by the dashed line perpendicular to the disk in the figure) is parallel to $\vec{E}$?

Video Tutor Solution

**SOLUTION**

**SET UP AND SOLVE Part (a):** The area is $A = \pi(0.10\text{ m})^2 = 0.0314\text{ m}^2$. From Equation 17.4,

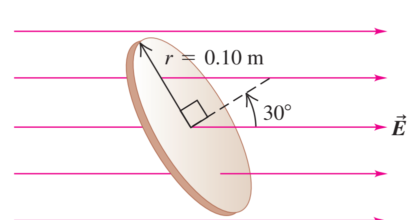
$$\Phi_E = EA\cos\phi = (2.0\times10^3\text{ N/C})(0.0314\text{ m}^2)(\cos 30^\circ) = 54\text{ N}\cdot\text{m}^2/\text{C}.$$

**Part (b):** The axis of the disk is now perpendicular to $\vec{E}$, so $\phi = 90^\circ$, $\cos\phi = 0$, and $\Phi_E = 0$.

**Part (c):** The axis of the disk is parallel to $\vec{E}$, so $\phi = 0$, $\cos\phi = 1$, and, from Equation 17.4,

$$\Phi_E = EA\cos\phi = (2.0\times10^3\text{ N/C})(0.0314\text{ m}^2)(1) = 63\text{ N}\cdot\text{m}^2/\text{C}.$$

**REFLECT** The flux through the disk is greatest when its axis is parallel to $\vec{E}$, and it is zero when $\vec{E}$ lies in the plane of the disk. That is, it is greatest when the most electric field lines pass through the disk, and it is zero when no lines pass through it.




▲ **FIGURE 17.28**

**Practice Problem:** What is the flux through the disk if its axis makes an angle of 45° with $\vec{E}$? *Answer:* 44 $\text{N}\cdot\text{m}^2/\text{C}$.

## EXAMPLE 17.9 Electric flux through a sphere

Video Tutor Solution

We can also calculate the flux through a surface that encloses a charge. Suppose a positive point charge with magnitude $3.0\ \mu\text{C}$ is placed at the center of a sphere with radius 0.20 m (Figure 17.29). Find the electric flux through the sphere due to this charge.

**SOLUTION**

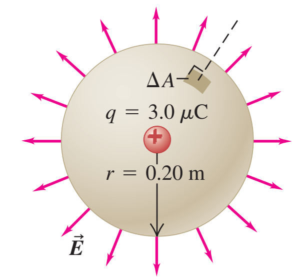
**SET UP AND SOLVE** At any point on the sphere, the magnitude of $\vec{E}$ is

$$E = \frac{kq}{r^2} = \frac{(8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)(3.0 \times 10^{-6}\ \text{C})}{(0.20\ \text{m})^2}$$
$$= 6.75 \times 10^5\ \text{N/C}.$$

From symmetry, the field is perpendicular to the spherical surface at every point (so that $E_\perp = E$), and it has the same magnitude at every point. The flux through any area element $\Delta A$ on the sphere is just $E\,\Delta A$, and the flux through the entire surface is $E$ times the total surface area $A = 4\pi r^2$. Thus, the total flux coming out of the sphere is

$$\Phi_E = EA = (6.75 \times 10^5\ \text{N/C})(4\pi)(0.20\ \text{m})^2$$
$$= 3.4 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}.$$

**REFLECT** The symmetry of the sphere plays an essential role in this calculation. We made use of the facts that $E$ has the same value at every point on the surface and that at every point $\vec{E}$ is perpendicular to the surface.




▲ **FIGURE 17.29**

**Practice Problem:** Repeat this calculation for the same charge but a radius of 0.10 m. You should find that the result is the same as the one you obtained earlier. We would have obtained the same result with a sphere of radius 2.0 m or 200 m. There's a good physical reason for this, as we'll soon see. *Answer:* $3.4 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}$.

## Gauss's law

**Gauss's law** relates the net electric flux through a closed surface, such as a sphere, to the total charge contained within the volume of the surface. It was formulated by Karl Friedrich Gauss (1777–1855), one of the greatest mathematicians of all time. Many areas of mathematics, from number theory and geometry to the theory of differential equations, bear the mark of his influence, and he made equally significant contributions to theoretical physics.

**Gauss's law**

The total electric flux $\Phi_E$ coming out of any closed surface (that is, a surface enclosing a definite volume) is proportional to the total (net) electric charge $Q_{\text{encl}}$ inside the surface according to the relationship

$$\Phi_E = 4\pi k Q_{\text{encl}}. \tag{17.6}$$

The flux can be independently determined by dividing the surface up into small elements of area $\Delta A$ and then performing the sum $\Phi_E = \sum E_\perp \Delta A$. Thus Gauss's law gives us a relationship between the enclosed charge and the field at the surface.

Units: $\text{N} \cdot \text{m}^2/\text{C}$
Notes:

- If there is no net charge inside the surface, then $\Phi_E = 0$.
- $k$ is the Coulomb's law constant, and $E_\perp$ is the component of the electric field that is perpendicular to the surface.
- Even if $\Phi_E = 0$, the electric field is not necessarily zero.

To develop Gauss's law, we'll start with the field due to a single positive point charge $q$. The field lines radiate out equally in all directions. We place this charge at the center of an imaginary spherical surface with radius $R$. The magnitude $E$ of the electric field at every point on the surface is given by

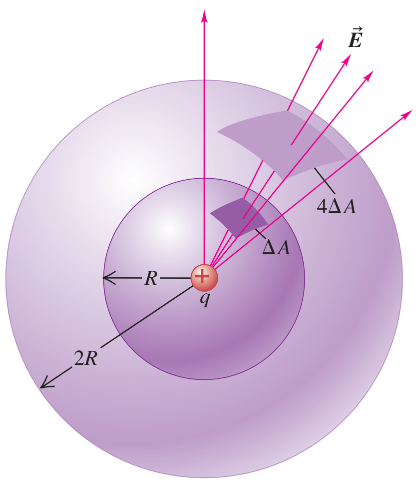
$$E = k\frac{q}{R^2}.$$

At each point on the surface, $\vec{E}$ is perpendicular to the surface, and its magnitude is the same at every point, just as in Example 17.9. The total electric flux is just the product of the field magnitude $E$ and the total area $A = 4\pi R^2$ of the sphere:

$$\Phi_E = EA = k\frac{q}{R^2}(4\pi R^2) = 4\pi kq \quad \text{(spherical surface)}. \tag{17.7}$$

We see that *the flux is independent of the radius R of the sphere.* It depends only on the charge $q$ enclosed by the sphere.

We can also interpret this result in terms of field lines. We consider two spheres with radii $R$ and $2R$ (Figure 17.30). According to Coulomb's law, the field magnitude is one-fourth as great on the larger sphere as on the smaller, so the number of field lines per unit area should be one-fourth as great. But the area of the larger sphere is four times as great, so the *total* number of field lines passing through is the same for both spheres.




▲ **FIGURE 17.30** Projection of an element of area $\Delta A$ on a spherical surface of radius $R$ onto a sphere of radius $2R$. The projection multiplies each linear dimension by 2, so the area element on the larger sphere is $4\Delta A$. The same number of field lines and the same flux pass through each area element.

We've derived Equation 17.7 only for spherical surfaces, but we can generalize it to *any* closed surface surrounding an electric charge. We imagine the surface as being divided into small elements of area $\Delta A$. If the electric field $\vec{E}$ is perpendicular to a particular element of area, then the number of field lines passing through that area is proportional to $E\,\Delta A$—that is, to the flux through $A$. If $\vec{E}$ is not perpendicular to the given element of area, we take the component of $\vec{E}$ perpendicular to $\Delta A$; we call this component $E_\perp$, as before. Then the number of lines passing through $\Delta A$ is proportional to $E_\perp \Delta A$. (We don't consider the component of $\vec{E}$ *parallel* to the surface because it doesn't correspond to any lines passing *through* the surface.)

To get the *total* number of field lines passing outward through the surface, we add up all the products $E_\perp \Delta A$ for all the surface elements that together make up the whole surface. This sum is the total flux through the entire surface. The total number of field lines passing through the surface is the same as that for the spherical surfaces we have discussed. Therefore, this sum is again equal to $4\pi kq$, just as in Equation 17.7, and our generalized relationship is

$$\Phi_E = \Sigma E_\perp \Delta A = 4\pi kq \quad \text{(for any closed surface)}. \tag{17.8}$$

There is one further detail: We have to keep track of which lines point *into* the surface and which ones point *out;* we may have both types in some problems. Let's agree that $E_\perp$ and $\Phi_E$ are positive when the vector $\vec{E}$ has a component pointing *out of* the surface and negative when the component points *into* the surface.

Here's another generalization: Suppose the surface encloses not just one point charge $q$, but several charges $q_1, q_2, q_3, \ldots$. Then the total (resultant) electric field $\vec{E}$ at any point is the vector sum of the $\vec{E}$ fields of the individual charges. Let $Q_{\text{encl}} = q_1 + q_2 + q_3 + \cdots$ be the *total* charge enclosed by the surface, and let $E_\perp$ be the component of the *total* field perpendicular to $\Delta A$. Then the general statement of Gauss's law is

$$\Sigma E_\perp \,\Delta A = 4\pi k Q_{\text{encl}}. \tag{17.9}$$

Gauss's law is usually written in terms of the constant $\epsilon_0$ we introduced in Section 17.4, defined by the relationship $k = 1/4\pi\epsilon_0$. In terms of $\epsilon_0$,

$$\Sigma E_\perp \Delta A = \frac{Q_{\text{encl}}}{\epsilon_0} \quad \text{(for any closed surface)}. \tag{17.10}$$

In Equations 17.6, 17.9, and 17.10, $Q_{encl}$ is always the algebraic sum of all the (positive and negative) charges enclosed by the surface, and $\vec{E}$ is the *total* field at each point on the surface. Also, note that this field is in general caused partly by charges inside the surface and partly by charges outside. The outside charges don't contribute to the total (net) flux through the surface, so Equation 17.10 is still correct even when there are additional charges outside the surface that contribute to the electric field at the surface. When $Q_{encl} = 0$, the total flux through the surface must be zero, even though some areas may have positive flux and others negative.

**NOTE** The surface used for applications of Gauss's law need not be a real physical surface; in fact, it is usually an imaginary surface, enclosing a definite volume and a definite quantity of electric charge.

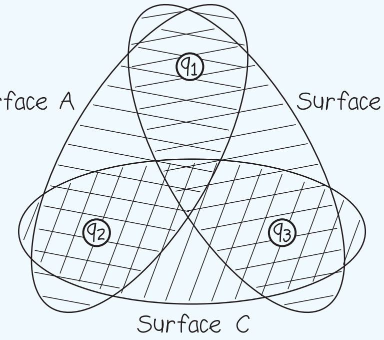
**QUANTITATIVE ANALYSIS 17.7**

### Gaussian surfaces and electric flux

Figure 17.31 shows three point charges: $q_1$ has a charge of $+5$ nC, $q_2$ has a charge of $-3$ nC, and $q_3$ has a charge of $-5$ nC. The figure also shows three Gaussian surfaces: Surface $A$ encloses $q_1$ and $q_2$, surface $B$ encloses $q_1$ and $q_3$, and surface $C$ encloses $q_2$ and $q_3$. Through which Gaussian surface does the electric flux point into the surface?

A. Surface *A* B. Surface *B* C. Surface *C*

**SOLUTION** From Gauss's law, we know that the larger the amount of charge enclosed by the Gaussian surface, the greater the magnitude of the electric flux. In addition, if the total enclosed charge is positive, the electric flux is out of the surface; if the enclosed charge is negative, the electric flux is into the surface. Surface $A$ encloses $q_1 + q_2 = +2$ nC, so the net flux is out of surface $A$. Surface $B$ encloses $q_1 + q_3 = 0$ nC, so there is no net flux through surface $B$. Finally, surface $C$ encloses $q_2 + q_3 = -8$ nC, so the net flux flows into surface $C$. Therefore, the correct answer is C.

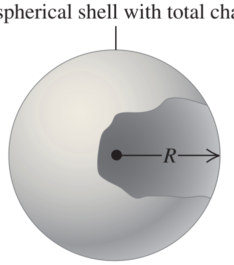



▲ **FIGURE 17.31**

**EXAMPLE 17.10** **Field due to a spherical shell of charge**

**Video Tutor Solution**

In this example we will apply Gauss's law to calculate the electric field of a specific, nontrivial charge distribution. Suppose a positive charge $q$ is spread uniformly over a thin spherical shell of radius $R$ (Figure 17.32). Find the electric field at points inside and outside the shell.




◀ **FIGURE 17.32**

**SOLUTION**

**SET UP** The system is spherically symmetric. This means that it is unchanged if we rotate it through any angle about an axis through its center. The field pattern of the rotated system must be identical to that of the original system. If the field had a component at some point that was perpendicular to the radial direction, that component would have to be different after at least some rotations. Thus, there can't be such a component, and the field must be radial.

We conclude that at every point outside the shell, the electric field due to the charge on the shell must be along a radial line—that is, along a line from the center of the shell to the field point. For the same reason, the magnitude $E$ of the field depends only on the distance $r$ from the center. Thus, the magnitude $E$ is the same at all points on a spherical surface with radius $r$, concentric with the conductor.

CONTINUED

**SOLVE** Because of the spherical symmetry, we take our Gaussian surface to be an imaginary sphere with radius $r$ and concentric with the shell. We'll locate this surface first inside and then outside the shell of charge.

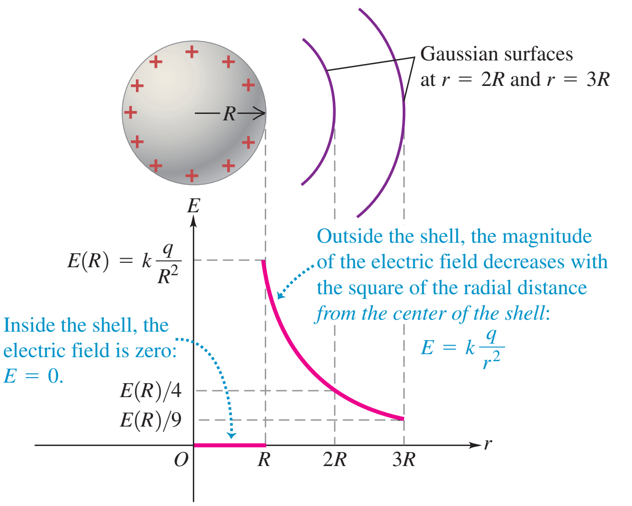
**Inside the shell ($r < R$):** The Gaussian surface has area $4\pi r^2$. Since, by symmetry, the electric field is uniform over the Gaussian sphere and perpendicular to it at each point, the electric flux is $\Phi_E = EA = E(4\pi r^2)$. The Gaussian surface is inside the shell and encloses none of the charge on the shell, so $Q_{\text{encl}} = 0$. Gauss's law $\Phi_E = Q_{\text{encl}}/\epsilon_0$ then says that $\Phi_E = E(4\pi r^2) = 0$, so $E = 0$. The electric field is zero at all points inside the shell.

**Outside the shell ($r > R$):** Again, $\Phi_E = E(4\pi r^2)$. But now all of the shell is inside the Gaussian surface, so $Q_{\text{encl}} = q$. Gauss's law $\Phi_E = Q_{\text{encl}}/\epsilon_0$ then gives $E(4\pi r^2) = q/\epsilon_0$, and it follows that

$$E = \frac{q}{4\pi\epsilon_0 r^2} = k\frac{q}{r^2}.$$

**REFLECT** Figure 17.33 shows a graph of the field magnitude $E$ as a function of $r$. The electric field is zero at all points inside the shell. At points outside the shell, the field drops off as $1/r^2$. Note that the magnitude of the electric field due to a point charge $q$ is $E = kq/r^2$, so at points outside the shell the field is the same as if all the charge were concentrated at the center of the shell.

**Practice Problem:** What total charge $q$ must be distributed uniformly over a spherical shell of radius $R = 0.50$ m to produce an electric field with magnitude 680 N/C at a point just outside the surface of the shell? *Answer:* $\pm 19$ nC.

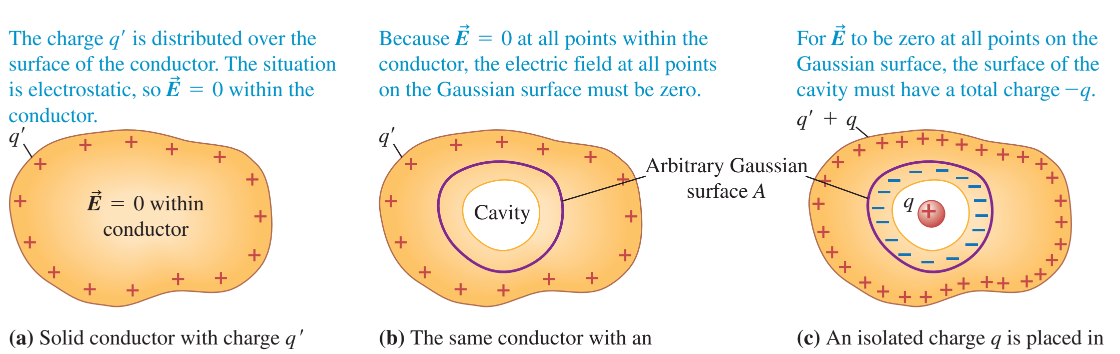



▶ **FIGURE 17.33** The electric field of a charged spherical shell as a function of distance from the center. Outside the sphere, the field is the same as though the sphere's charge were all located at the center of the sphere.

## 17.9 Charges on Conductors

Early in our discussion of electric fields, we made the point that in an electrostatic situation (where there is no net motion of charge), the electric field at every point within a conductor is zero. (If it were not, the field would cause the conductor's free charges to move.) We've also learned that the charge on a solid conductor is located entirely on its surface, as shown in Figure 17.34a. But what if there is a cavity inside the conductor (Figure 17.34b)? If there is no charge in the cavity, we can use a Gaussian surface such as $A$ to show that the net charge on the surface *of the cavity* must be zero because $\vec{E} = 0$ everywhere on the Gaussian surface. In fact, for this situation, we can prove not only that the *total* charge on the cavity surface is zero, but also that there can't be any charge *anywhere* on the cavity surface.

Suppose we place a small object with a charge $q$ inside a cavity in a conductor, making sure that it does not touch the conductor (Figure 17.34c). Again, $\vec{E} = 0$ everywhere




**(a)** Solid conductor with charge $q'$

**(b)** The same conductor with an internal cavity

**(c)** An isolated charge $q$ is placed in the cavity

▲ **FIGURE 17.34** The charge on a solid conductor, on a conductor with a cavity, and on a conductor with a cavity that contains a charge.

on the Gaussian surface $A$ (because the situation is still electrostatic), so, according to Gauss's law, the *total* charge inside this surface must be zero. Therefore, there must be a total charge $-q$ on the cavity surface. Of course, the *net* charge on the conductor (counting both the inner and outer surfaces) must remain unchanged, so a charge $+q$ must appear on its outer surface.

To see that this charge must be on the outer surface and not in the material, imagine first shrinking surface $A$ so that it's just barely bigger than the cavity. The field everywhere on $A$ is still zero, so, according to Gauss's law, the total charge inside $A$ is zero. Now let surface $A$ expand until it is just inside the outer surface of the conductor. The field is still zero everywhere on surface $A$, so the total charge enclosed is still zero. We have not enclosed any additional charge by expanding surface $A$; therefore, there must be no charge in the interior of the material. We conclude that the charge $+q$ must appear on the outer surface. By the same reasoning, if the conductor originally had a charge $q'$, then the total charge on the outer surface after the charge $q$ is inserted into the cavity must be $q + q'$.

### EXAMPLE 17.11 Location of net charge on conductors

In this example we will apply Gauss's law to a conducting shell that surrounds a charge. This problem is spherically symmetric, which makes it an ideal candidate for a Gauss's law analysis. A hollow conductor carries a net charge of $+7$ nC. In its cavity, insulated from the conductor, is a small, isolated sphere with a net charge of $-5$ nC. How much charge is on the outer surface of the hollow conductor? How much is on the inner surface of the cavity?

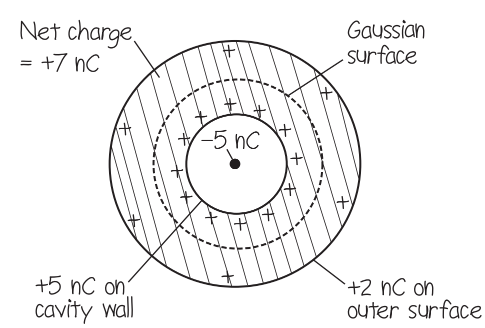
Video Tutor Solution

**SOLUTION**

**SET UP** Figure 17.35 shows our sketch. We know that in this electrostatic situation the electric field in the conducting material must be zero. We draw a Gaussian surface within the material of the conductor and apply Gauss's law.

**SOLVE** We apply Gauss's law $\Phi_E = Q_{\text{encl}}/\epsilon_0$ to the Gaussian surface shown in Figure 17.35. The Gaussian surface lies within the conducting material, so $E = 0$ everywhere on that surface. By Gauss's law, $\Phi_E = Q_{\text{encl}}/\epsilon_0$. Thus, $\Phi_E = 0$, so $Q_{\text{encl}} = 0$. But then, in order to have $Q_{\text{encl}} = 0$, there must be a charge of $+5$ nC on the inner surface of the cavity, to cancel the charge in the cavity. The conductor carries a total charge of $+7$ nC, and all of its net charge is on its surfaces. So, if there is $+5$ nC on the inner surface, the remaining $+2$ nC must be on the outer surface, as shown in our sketch.

**REFLECT** Field lines pass between the $+5$ nC on the inner surface of the cavity and the $-5$ nC on the object in the cavity. Each field line going to the $-5$ nC charge originated on the $+5$ nC charge; the field lines don't continue into the conducting material because $E = 0$ there. There is an electric field outside the conductor due to the $+2$ nC on its surface.

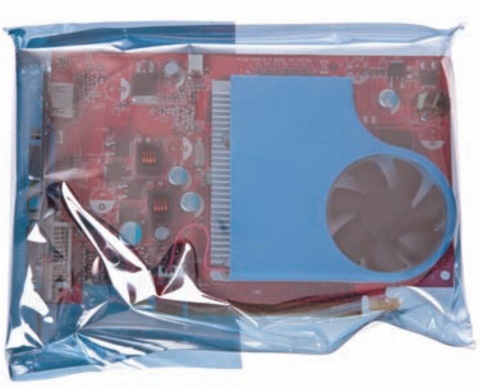



▲ **FIGURE 17.35** Our sketch for this problem.

**Practice Problem:** Repeat this example for the case where the conductor has a net charge of $+3$ nC. *Answers:* Inner surface $+5$ nC, outer surface $-2$ nC.

## Electrostatic shielding

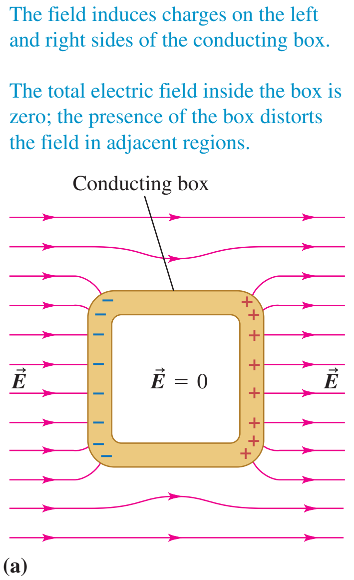
Suppose we have a highly sensitive electronic instrument that we want to protect from stray electric fields that might give erroneous measurements. We surround the instrument with a conducting box, or we line the walls, floor, and ceiling of the room with a



► Application **A Faraday cage when you need one.**

Detectives use a forensic Faraday bag, such as this one, to isolate a confiscated cell phone from the cellular network. The bag has a conductive coating that blocks the electric field component of a cell phone signal.

▶ **FIGURE 17.36** (a) The effect of putting a conducting box (an electrostatic shield) in a uniform electric field. (b) The conducting cage keeps the operator of this exhibit perfectly safe.





conducting material such as sheet copper. The external electric field redistributes the free electrons in the conductor, leaving a net positive charge on the outer surface in some regions and a net negative charge in others (Figure 17.36). This charge distribution causes an additional electric field such that the *total* field at every point inside the box is zero. The charge distribution on the box also alters the shapes of the field lines near the box, as the figure shows. Such a setup is often called a *Faraday cage*. Your microwave oven, in fact, is a Faraday cage in reverse. The metallic grid on the door window and the metallic walls of the interior are designed to keep the electromagnetic radiation that heats your food from escaping the oven.

## FREQUENTLY ASKED QUESTIONS

**Q:** *Can a particle have a charge that is only a fraction of that of the proton or electron?*

**A:** No. All charged objects have a net charge that is some integer multiple of the fundamental unit of charge, $e$.

**Q:** *Coulomb's law (Equation 17.1) has absolute value signs around the product of the charges. So what changes if I replace one of the charges with its negative value?*

**A:** Nothing changes in terms of the magnitude of the electric force. However, the *direction* of the force on each charge will be the opposite of what it was.

**Q:** *I understand that Coulomb's law (Equation 17.1) gives me the magnitude of the electric force. Which charge does this force actually act on, $q_1$ or $q_2$?*

**A:** The force acts on both charges! Equation 17.1 gives you the force that $q_2$ produces on $q_1$, which is equal in magnitude but opposite in direction to the force that $q_1$ produces on $q_2$.

**Q:** *How do I determine the direction of the force between two charges?*

**A:** If the charges are of the same sign, then they repel each other with a force magnitude given by Coulomb's law. If they are of the opposite sign, then they attract each other. For a given charge, the specific direction of the electric force and its corresponding components depend on the location of the charges in your coordinate system.

**Q:** *How do I use Equation 17.3 to calculate the electric field of a point charge?*

**A:** First, Equation 17.3 gives you the magnitude of the electric field at a distance $r$ from the charge. Remember that the charge produces electric field in all space, but the electric field gets weaker as you move away from the charge. If the charge is positive, then the electric field vector points *away* from the charge with a magnitude that is given by Equation 17.3. If the charge is negative, then the field points directly toward the charge.

**Q:** *From Gauss's law I know that if the net enclosed charge is zero, then the electric flux through the surface that encloses the charge is zero. Does it follow that the electric field at the surface is also zero?*

**A:** No. If electric field lines are flowing into and out of the surface, then the ingoing flux may cancel the outgoing flux. But there would still be electric field at the surface. However, if the charge distribution is spherically symmetric, then it is true that zero flux implies that the field at the surface is also zero.

## Bridging Problem

Video Tutor Solution

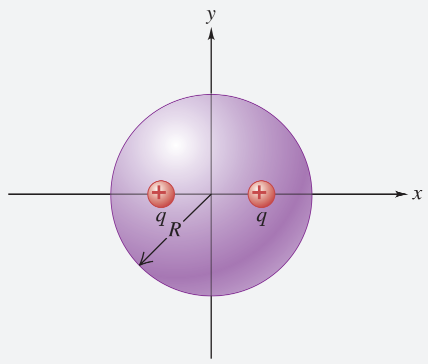
An imaginary sphere of radius $R$ is centered at the origin, as shown in Figure 17.37. A charge $q$ is rigidly fixed to the $x$ axis at $x = +R/2$ and a second charge $q$ is at $x = -R/2$. Finally, a proton (of mass $m$ and charge $+e$) is released from rest on the $y$ axis. In terms of $e$, $m$, $R$, $k$, and $q$, (a) calculate the magnitude and direction of the acceleration of the proton at the moment it is released from $y = +R/4$. (b) What are the magnitude and direction of the electric field at the origin, just before the proton is released? Calculate the net electric flux through the sphere (c) at the moment the proton is released from $y = +R/4$ and (d) a long time after it has been released (ignore the effects of gravity).

**Setup**

- Draw a free-body diagram for the proton.
- Determine the magnitude of each of the forces on the proton.
- Identify all the sources of electric field at the origin.
- Determine the net charge inside the sphere.

**Solve**

- Determine the $x$ and $y$ components of the forces on the proton.
- Use Newton's second law to find the $x$ and $y$ components of the proton's acceleration.
- Find the electric field contributions at the origin using vector addition.
- Use Gauss's law to calculate the final flux through the sphere, after the proton has moved a significant distance.




▲ **FIGURE 17.37**

**Reflect**

- What is the direction of the net electric field on the proton in part (a)? Is it in the same direction as the proton's acceleration?
- Are any electric forces acting on the charge at $x = +R/2$? Explain.
- Is the acceleration of the proton constant? Explain.
- How does the electric flux through the sphere depend on the position of the proton?

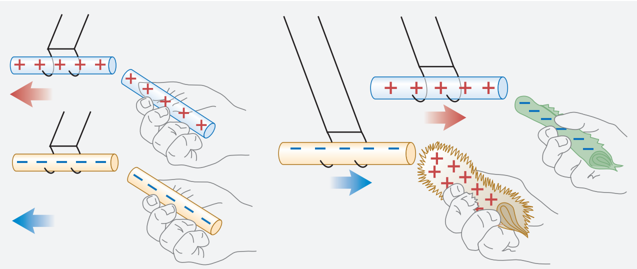
# CHAPTER 17 SUMMARY

### Electric Charge; Conductors and Insulators

(Sections 17.1–17.3) The fundamental entity in electrostatics is electric charge. There are two kinds of charge: positive and negative. Like charges repel each other; unlike charges attract. **Conductors** are materials that permit electric charge to move within them. **Insulators** permit charge to move much less readily. Most metals are good conductors; most nonmetals are insulators.

All ordinary matter is made of atoms, which consist of protons, neutrons, and electrons. The protons and neutrons form the nucleus of the atom; the electrons surround the nucleus at distances much greater than its size. Electrical interactions are chiefly responsible for the structure of atoms, molecules, and solids.

Electric charge is conserved: It can be transferred between objects, but isolated charges cannot be created or destroyed. Electric charge is quantized: Every amount of observable charge is an integer multiple of the charge of an electron or proton.

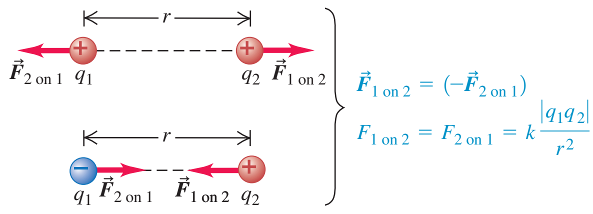



### Coulomb's Law

(Section 17.4) **Coulomb's law** is the basic law of interaction for point electric charges. For point charges $q_1$ and $q_2$ separated by a distance $r$, the magnitude $F$ of the force each charge exerts on the other is

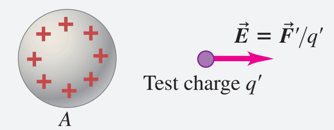
$$F = k\frac{|q_1 q_2|}{r^2}. \qquad (17.1)$$

The force on each charge acts along the line joining the two charges. It is repulsive if $q_1$ and $q_2$ have the same sign, attractive if they have opposite signs. The forces form an action–reaction pair and obey Newton's third law.

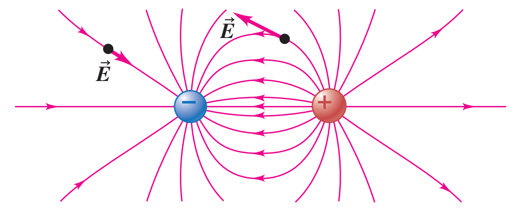


**Electric Field and Electric Forces**
(Sections 17.5 and 17.6) **Electric field,** a vector quantity, is the force per unit charge exerted on a test charge at any point, provided that the test charge is small enough that it does not disturb the charges that cause the field. The principle of superposition states that the electric field due to any combination of charges is the vector sum of the fields caused by the individual charges. From Coulomb's law, the magnitude of the electric field produced by a point charge is

$$E = k\frac{|q|}{r^2}. \qquad (17.3)$$

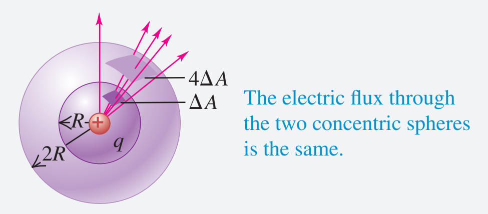



**Electric Field Lines**
(Section 17.7) **Field lines** provide a graphical representation of electric fields. A field line at any point in space is tangent to the direction of $\vec{E}$ at that point, and the number of lines per unit area (perpendicular to their direction) is proportional to the magnitude of $\vec{E}$ at the point. Field lines point away from positive charges and toward negative charges.

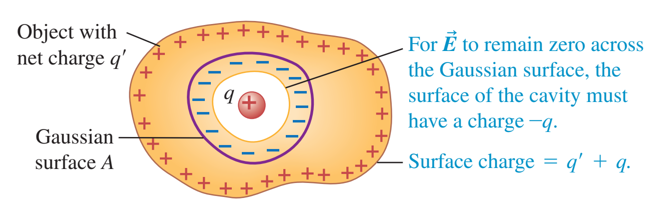



**Gauss's Law**
(Section 17.8) For a uniform electric field with component $E_\perp$ perpendicular to area $A$, the **electric flux** through the area is $\Phi_E = E_\perp A$ (Equation 17.5). **Gauss's law** states that the total electric flux $\Phi_E$ out of any closed surface (that is, a surface enclosing a definite volume) is proportional to the total electric charge $Q_{\text{encl}}$ inside the surface, according to the relationship

$$\Phi_E = 4\pi k Q_{\text{encl}}. \qquad (17.6)$$




**Charges on Conductors**
(Section 17.9) In a static configuration with no net motion of charge, the electric field is always zero within a conductor. The charge on a solid conductor is located entirely on its outer surface. If there is a cavity containing a charge $+q$ within the conductor, the surface of the cavity has a total induced charge $-q$.

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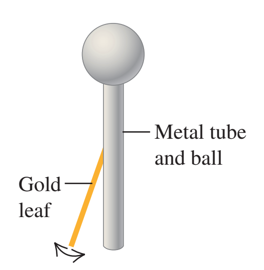
## Conceptual Questions

1. Bits of paper are attracted to an electrified comb or rod even though they have no net charge. How is this possible?
2. When you walk across a nylon rug and then touch a large metal object, you may get a spark and a shock. What causes this to happen?
3. What similarities does the electric force have to the gravitational force? What are the most significant differences?
4. In a common physics demonstration, a rubber rod is first rubbed vigorously on silk or fur. It is then brought close to a small Styrofoam™ ball, which it attracts. If you then touch the ball with the rod, it suddenly repels the ball. Why does it first attract the ball, and why does it then repel the same ball?
5. A gold leaf electroscope, which is often used in physics demonstrations, consists of a metal tube with a metal ball at the top and a sheet of extremely thin gold leaf fastened at the other end. (See Figure 17.38.) The gold leaf is attached in such a way that it can pivot about its upper edge. (a) If a charged rod is brought close to (but does not touch) the ball at the top, the gold leaf pivots outward, away from the tube. Why? (b) What will the gold leaf do when the charged rod is removed? Why? (c) Suppose that the charged rod touches the metal ball for a second or so. What will the gold leaf do when the rod is removed in this case? Why?




▲ **FIGURE 17.38** Question 5.

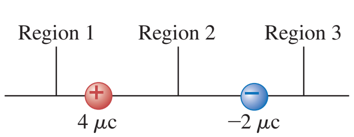
6. Show how it is possible for *neutral* objects to attract each other electrically.
7. Suppose you have a hollow spherical conductor. Is it possible for the net charge on the sphere to be negative, but the charge on

the outer surface of the conductor to be positive? Explain your answer.

8. If an electric dipole is placed in a uniform electric field, what is the net force on it? Will the same thing necessarily be true if the field is *not* uniform?
9. *Why* do electric field lines point away from positive charges and toward negative charges?
10. A lightning rod is a pointed copper rod mounted on top of a building and welded to a heavy copper cable running down into the ground. Lightning rods are used to protect buildings from lightning; the lightning current runs through the copper rather than through the building. Why does it do this?
11. A rubber balloon has a single point charge in its interior. Does the electric flux through the balloon depend on whether or not it is fully inflated? Explain your reasoning.
12. Explain how the electric force plays an important role in understanding each of the following: (a) the friction force between two objects, (b) the hardness of steel, and (c) the bonding of amino acids to form proteins.

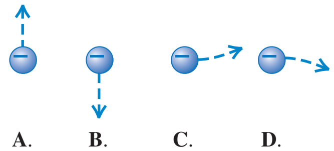
## Multiple-Choice Problems

1. Just after two identical point charges are released when they are a distance $D$ apart in outer space, they have an acceleration $a$. If you release them from a distance $D/2$ instead, their acceleration will be
   A. $a/4$. B. $a/2$. C. $2a$. D. $4a$.
2. If the electric field is $E$ at a distance $d$ from a point charge, its magnitude will be $2E$ at a distance
   A. $d/4$. B. $d/2$. C. $d/\sqrt{2}$.
   D. $d\sqrt{2}$. E. $2d$.
3. Two *unequal* point charges are separated as shown in Figure 17.39. The electric field due to this combination of charges can be zero
   A. only in region 1.
   B. only in region 2.
   C. only in region 3.
   D. in both regions 1 and 3.

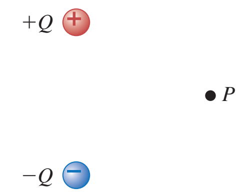



▲ **FIGURE 17.39** Multiple-Choice Problem 3.

4. A spherical balloon contains a charge $+Q$ uniformly distributed over its surface. When it has a diameter $D$, the electric field at its surface has magnitude $E$. If the balloon is now blown up to twice this diameter without changing the charge, the electric field at its surface is
   A. $4E$. B. $2E$. C. $E$.
   D. $E/2$. E. $E/4$.
5. An electron is moving horizontally in a laboratory when a uniform electric field is suddenly turned on. This field points vertically downward. Which of the paths shown will the electron follow, assuming that gravity can be neglected?

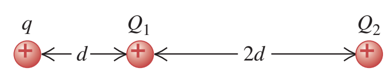



6. Point $P$ in Figure 17.40 is equidistant from two point charges $\pm Q$ of equal magnitude. If a negative point charge is placed at $P$ without moving the original charges, the net electric force the charges $\pm Q$ will exert on it is
   A. directly upward.
   B. directly downward.
   C. zero.
   D. directly to the right.
   E. directly to the left.



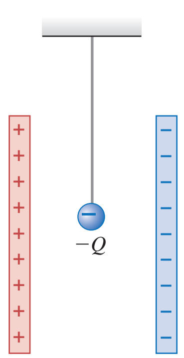

▲ **FIGURE 17.40** Multiple-Choice Problem 6.

7. A hollow conductor carries a net charge of $+3Q$. A small charge of $-2Q$ is placed inside the cavity in such a way that it is isolated from the conductor. How much charge is on the outer surface of the conductor?
   A. $+Q$ B. $-Q$ C. $+5Q$ D. $-5Q$
8. Three equal point charges are held in place as shown in Figure 17.41. If $F_1$ is the force on $q$ due to $Q_1$ and $F_2$ is the force on $q$ due to $Q_2$, how do $F_1$ and $F_2$ compare?
   A. $F_1 = 2F_2$ B. $F_1 = 3F_2$
   C. $F_1 = 4F_2$ D. $F_1 = 9F_2$




▲ **FIGURE 17.41** Multiple-Choice Problem 8.

9. An electric field of magnitude $E$ is measured at a distance $R$ from a point charge $Q$. If the charge is doubled to $2Q$ and the electric field is now measured at a distance of $2R$ from the charge, the new measured value of the field will be
   A. $2E$. B. $E$. C. $E/2$. D. $E/4$.
10. A very small ball containing a charge $-Q$ hangs from a light string between two vertical charged plates, as shown in Figure 17.42. When released from rest, the ball will
    A. swing to the right.
    B. swing to the left.
    C. remain hanging vertically.




▲ **FIGURE 17.42** Multiple-Choice Problem 10.

11. A point charge $Q$ at the center of a sphere of radius $R$ produces an electric flux of $\Phi_E$ coming out of the sphere. If the charge remains the same but the radius of the sphere is doubled, the electric flux coming out of it will be
    A. $\Phi_E/2$. B. $\Phi_E/4$. C. $2\Phi_E$.
    D. $4\Phi_E$. E. $\Phi_E$.

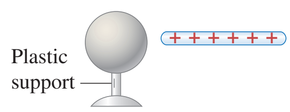
12. Two charged small spheres are a distance $R$ apart and exert an electrostatic force $F$ on each other. If the distance is halved to $R/2$, the force exerted on each sphere will be
   A. $4F$. B. $2F$. C. $F/2$. D. $F/4$.

# Problems

## 17.1 Electric Charge

## 17.2 Conductors and Insulators

1. A positively charged glass rod is brought close to a *neutral* sphere that is supported on a nonconducting plastic stand as shown in Figure 17.43. Sketch the distribution of charges on the sphere if it is made of (a) aluminum, (b) nonconducting plastic.



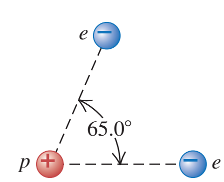

▲ **FIGURE 17.43** Problem 1.

2. A positively charged rubber rod is moved close to a *neutral* copper ball that is resting on a nonconducting sheet of plastic. (a) Sketch the distribution of charges on the ball. (b) With the rod still close to the ball, a metal wire is briefly connected from the ball to the earth and then removed. After the rubber rod is also removed, sketch the distribution of charges (if any) on the copper ball.
3. Two iron spheres contain excess charge, one positive and the other negative. (a) Show how the charges are arranged on these spheres if they are *very* far from each other. (b) If the spheres are now brought close to each other, but do not touch, sketch how the charges will be distributed on their surfaces. (c) In part (b), show how the charges would be distributed if both spheres were negative.
4. **Electrical storms.** During an electrical storm, clouds can build up very large amounts of charge, and this charge can induce charges on the earth's surface. Sketch the distribution of charges at the earth's surface in the vicinity of a cloud if the cloud is positively charged and the earth behaves like a conductor.

## 17.3 Conservation and Quantization of Charge

## 17.4 Coulomb's Law

5. In ordinary laboratory circuits, charges in the $\mu$C and nC range are common. How many excess electrons must you add to an object to give it a charge of (a) $-2.50\ \mu$C, (b) $-2.50$ nC?
6. **Signal propagation in neurons.** *Neurons* are components of the nervous system of the body that transmit signals as electrical impulses travel along their length. These impulses propagate when charge suddenly rushes into and then out of a part of the neuron called an *axon.* Measurements have shown that, during the inflow part of this cycle, approximately $5.6 \times 10^{11}$ $Na^+$ (sodium ions) per meter, each with charge $+e$, enter the axon. How many coulombs of charge enter a 1.5 cm length of the axon during this process? **BIO**
7. **Particles in a gold ring.** You have a pure (24-karat) gold ring with mass 17.7 g. Gold has an atomic mass of 197 g/mol and an atomic number of 79. (a) How many protons are in the ring, and what is their total positive charge? (b) If the ring carries no net charge, how many electrons are in it?
8. Two equal point charges of $+3.00 \times 10^{-6}$ C are placed 0.200 m apart. What are the magnitude and direction of the force each charge exerts on the other?
9. The repulsive force between two electrons has a magnitude of 4.00 N. (a) What is the distance between the electrons? (b) At what distance would the force be 1.00 N? (c) Calculate the ratio of the distance you found in (b) to that you found in (a).
10. A negative charge of $-0.550\ \mu$C exerts an upward 0.200 N force on an unknown charge 0.300 m directly below it. (a) What is the unknown charge (magnitude and sign)? (b) What are the magnitude and direction of the force that the unknown charge exerts on the $-0.550\ \mu$C charge?
11. **Forces in an atom.** The particles in the nucleus of an atom are approximately $10^{-15}$ m apart, while the electrons in an atom are about $10^{-10}$ m from the nucleus. (a) Calculate the electrical repulsion between two protons in a nucleus if they are $1.00 \times 10^{-15}$ m apart. If you were holding these protons, do you think you could feel the effect of this force? How many pounds would the force be? (b) Calculate the electrical attraction that a proton in a nucleus exerts on an orbiting electron if the two particles are $1.00 \times 10^{-10}$ m apart. If you were holding the electron, do you think you could feel the effect of this force?
12. (a) What is the total negative charge, in coulombs, of all the electrons in a small 1.00 g sphere of carbon? One mole of C is 12.0 g, and each atom contains 6 protons and 6 electrons. (b) Suppose you could take out all the electrons and hold them in one hand, while in the other hand you hold what is left of the original sphere. If you hold your hands 1.50 m apart at arm's length, what force will each of them feel? Will it be attractive or repulsive?
13. As you walk across a synthetic-fiber rug on a cold, dry winter day, you pick up an excess charge of $-55\ \mu$C. (a) How many excess electrons did you pick up? (b) What is the charge on the rug as a result of your walking across it?
14. Two small plastic spheres are given positive electric charges. When they are 15.0 cm apart, the repulsive force between them has magnitude 0.220 N. What is the charge on each sphere (a) if the two charges are equal? (b) if one sphere has four times the charge of the other?
15. An astronaut holds two small aluminum spheres, each having mass 0.0250 kg, 80.0 cm apart. Each sphere carries a charge of $+4.00\ \mu$C. (a) What is the repulsive force between the spheres? (b) What is the acceleration of each sphere the moment after the astronaut releases them?
16. Two small spheres spaced 20.0 cm apart have equal charge. How many excess electrons must be present on each sphere if the magnitude of the force of repulsion between them is $4.57 \times 10^{-21}$ N?
17. A 1 kg sphere having a charge of $+5\ \mu$C is placed on a scale, which measures its weight in newtons. A second sphere having a charge of $-4\ \mu$C is positioned directly above the first sphere. The distance between the two spheres is 0.3 m. What is the reading on the scale?
18. If a proton and an electron are released when they are $2.0 \times 10^{-10}$ m apart (typical atomic distances), find the initial acceleration of each of them.
19. Three point charges are arranged on a line. Charge $q_3 = +5.00$ nC and is at the origin. Charge $q_2 = -3.00$ nC and is at $x = +4.00$ cm. Charge $q_1$ is at $x = +2.00$ cm. What is $q_1$ (magnitude and sign) if the net force on $q_3$ is zero?
20. If two electrons are each $1.50 \times 10^{-10}$ m from a proton, as shown in Figure 17.44, find the magnitude and direction of the net electric force they will exert on the proton.
21. Two point charges are located on the $y$ axis as follows: charge $q_1 = -1.50$ nC

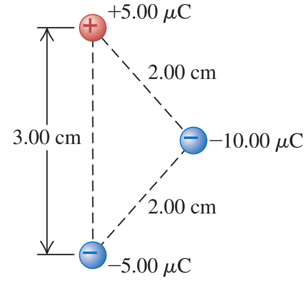



▲ **FIGURE 17.44** Problem 20.

at $y = -0.600$ m, and charge $q_2 = +3.20$ nC at the origin ($y = 0$). What is the net force (magnitude and direction) exerted by these two charges on a third charge $q_3 = +5.00$ nC located at $y = -0.400$ m?

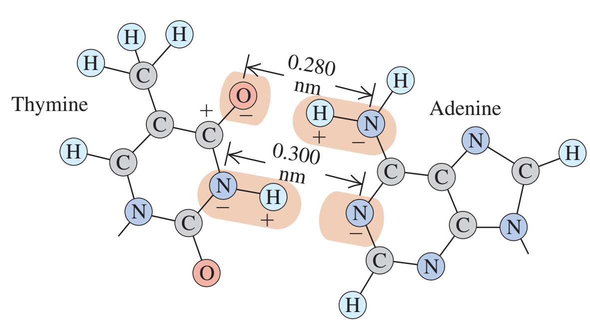
22. || Two point charges are placed on the $x$ axis as follows: Charge $q_1 = +4.00$ nC is located at $x = 0.200$ m, and charge $q_2 = -5.00$ nC is at $x = +0.400$ m. What are the magnitude and direction of the net force exerted by these two charges on a third point charge $q_3 = -0.600$ nC placed at the origin?

23. || Three charges are at the corners of an isosceles triangle as shown in Figure 17.45. The $\pm 5.00\ \mu$C charges form a dipole. (a) Find the magnitude and direction of the net force that the $-10.0\ \mu$C charge exerts on the dipole. (b) For an axis perpendicular to the line connecting the two charges of the dipole at its midpoint and perpendicular to the plane of the paper, find the magnitude and direction of the torque exerted on the dipole by the $-10.0\ \mu$C charge.



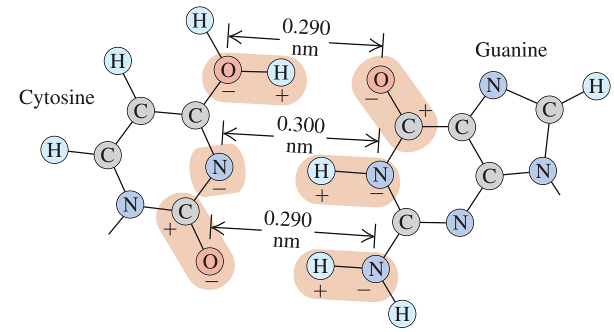

▲ **FIGURE 17.45** Problem 23.

24. || **Base pairing in DNA, I.** The two sides of the DNA double helix **BIO** are connected by pairs of bases (adenine, thymine, cytosine, and guanine). Because of the geometric shape of these molecules, adenine bonds with thymine and cytosine bonds with guanine. Figure 17.46 shows the thymine–adenine bond. Each charge shown is $\pm e$, and the H—N distance is 0.110 nm. (a) Calculate the *net* force that thymine exerts on adenine. Is it attractive or repulsive? To keep the calculations fairly simple, yet reasonable, consider only the forces due to the O—H—N and the N—H—N combinations, assuming that these two combinations are parallel to each other. Remember, however, that in the O—H—N set, the $O^-$ exerts a force on both the $H^+$ and the $N^-$, and likewise along the N—H—N set. (b) Calculate the force on the electron in the hydrogen atom, which is 0.0529 nm from the proton. Then compare the strength of the bonding force of the electron in hydrogen with the bonding force of the adenine–thymine molecules.



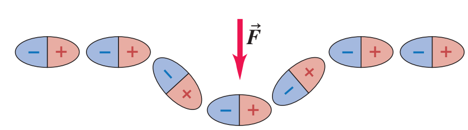

▲ **FIGURE 17.46** Problem 24.

25. || **Base pairing in DNA, II.** Refer to the previous problem. Fig- **BIO** ure 17.47 shows the bonding of the cytosine and guanine molecules. The O—H and H—N distances are each 0.110 nm. In this case, assume that the bonding is due only to the forces along the O—H—O, N—H—N, and O—H—N combinations, and assume also that these three combinations are parallel to each other. Calculate the *net* force that cytosine exerts on guanine due to the preceding three combinations. Is this force attractive or repulsive?



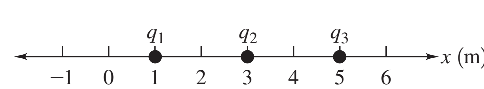

▲ **FIGURE 17.47** Problem 25.

26. || **Surface tension.** Surface tension is the force that causes the surface of water (and other liquids) to form a "skin" that resists penetration. Because of this force, water forms into beads, and insects such as water spiders can walk on water. As we shall see, the force is electric in nature. The surface of a polar liquid, such as water, can be viewed as a series of dipoles strung together in the stable arrangement in which the dipole moment vectors are parallel to the surface, all pointing in the same direction. Suppose now that something presses inward on the surface, distorting the dipoles as shown in Figure 17.48. Show that the two slanted dipoles exert a net upward force on the dipole between them and hence oppose the downward external force. Show also that the dipoles attract each other and thus resist being separated. Notice that the force between dipoles opposes penetration of the liquid's surface and is a simple model for surface tension.




▲ **FIGURE 17.48** Problem 26.

27. || Consider the charges in Figure 17.49. Find the magnitude and direction of the net force on $q_2 = +5.00\ \mu$C if (a) $q_1 = q_3 = +5.00\ \mu$C, (b) $q_1 = +5.00\ \mu$C and $q_3 = -5.00\ \mu$C.




▲ **FIGURE 17.49** Problem 27.

28. || Two unequal charges repel each other with a force $F$. If both charges are doubled in magnitude, what will be the new force in terms of $F$?

29. || In an experiment in space, one proton is held fixed and another proton is released from rest a distance of 2.50 mm away. (a) What is the initial acceleration of the proton after it is released? (b) Sketch qualitative (no numbers!) acceleration–time and velocity–time graphs of the released proton's motion.

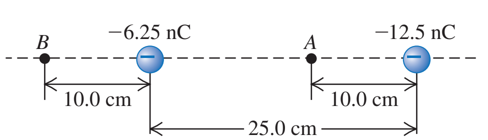
30. ||| A charge $+Q$ is located at the origin and a second charge, $+4Q$, is at distance $d$ on the $x$ axis. Where should a third charge, $q$, be placed, and what should be its sign and magnitude, so that all three charges will be in equilibrium?

## 17.5 Electric Field and Electric Forces

31. | A small object carrying a charge of $-8.00$ nC is acted upon by a downward force of 20.0 nN when placed at a certain point in an electric field. (a) What are the magnitude and direction of the electric field at the point in question? (b) What would be the magnitude and direction of the force acting on a proton placed at this same point in the electric field?
32. | (a) What must the charge (sign and magnitude) of a 1.45 g particle be for it to remain balanced against gravity when placed in a downward-directed electric field of magnitude 650 N/C ? (b) What is the magnitude of an electric field in which the electric force it exerts on a proton is equal in magnitude to the proton's weight?
33. || A uniform electric field exists in the region between two oppositely charged plane parallel plates. An electron is released from rest at the surface of the negatively charged plate and strikes the surface of the opposite plate, 3.20 cm distant from the first, in a time interval of $1.5 \times 10^{-8}$ s. (a) Find the magnitude of this electric field. (b) Find the speed of the electron when it strikes the second plate.

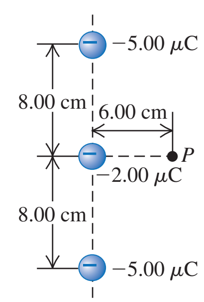
## 17.6 Calculating Electric Fields

34. | A particle has a charge of $-3.00$ nC. (a) Find the magnitude and direction of the electric field due to this particle at a point 0.250 m directly above it. (b) At what distance from the particle does its electric field have a magnitude of 12.0 N/C ?
35. | The electric field caused by a certain point charge has a magnitude of $6.50 \times 10^3$ N/C at a distance of 0.100 m from the charge. What is the magnitude of the charge?
36. | At a distance of 16 m from a charged particle, the electric field has a magnitude of 100 N/C. (a) At what distance is the electric field 400 N/C? (b) At what distance is it 10 N/C?
37. | **Electric fields in the atom.** (a) **Within the nucleus.** What strength of electric field does a proton produce at the distance of another proton, about $5.0 \times 10^{-15}$ m away? (b) **At the electrons.** What strength of electric field does this proton produce at the distance of the electrons, approximately $5.0 \times 10^{-10}$ m away?
38. || A proton is traveling horizontally to the right at $4.50 \times 10^6$ m/s. (a) Find the magnitude and direction of the weakest electric field that can bring the proton uniformly to rest over a distance of 3.20 cm. (b) How much time does it take the proton to stop after entering the field? (c) What minimum field (magnitude and direction) would be needed to stop an electron under the conditions of part (a)?
39. || Two point charges are separated by 25.0 cm (see Figure 17.50). Find the net electric field these charges produce at (a) point $A$, (b) point $B$. (c) What would be the magnitude and direction of the electric force this combination of charges would produce on a proton at $A$?

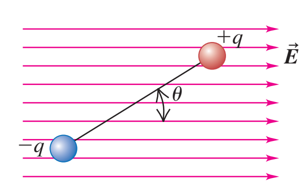



▲ **FIGURE 17.50** Problem 39.

40. || A point charge of $-4.00$ nC is at the origin, and a second point charge of $+6.00$ nC is on the $x$ axis at $x = 0.800$ m. Find the magnitude and direction of the electric field at each of the following points on the $x$ axis: (a) $x = 20.0$ cm, (b) $x = 1.20$ m, (c) $x = -20.0$ cm.
41. || In a rectangular coordinate system, a positive point charge $q = 6.00$ nC is placed at the point $x = +0.150$ m, $y = 0$, and an identical point charge is placed at $x = -0.150$ m, $y = 0$. Find the $x$ and $y$ components and the magnitude and direction of the electric field at the following points: (a) the origin; (b) $x = 0.300$ m, $y = 0$; (c) $x = 0.150$ m, $y = -0.400$ m; (d) $x = 0$, $y = 0.200$ m.
42. || Two particles having charges of $+0.500$ nC and $+8.00$ nC are separated by a distance of 1.20 m. (a) At what point along the line connecting the two charges is the net electric field due to the two charges equal to zero? (b) Where would the net electric field be zero if one of the charges were negative?
43. || Three negative point charges lie along a line as shown in Figure 17.51. Find the magnitude and direction of the electric field this combination of charges produces at point $P$, which lies 6.00 cm from the $-2.00\ \mu$C charge measured perpendicular to the line connecting the three charges.



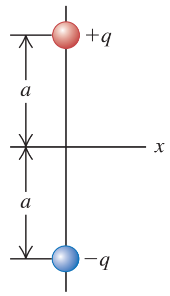

▲ **FIGURE 17.51** Problem 43.




▲ **FIGURE 17.52** Problem 44.

44. || **Torque and force on a dipole.** An electric dipole is in a uniform external electric field $\vec{E}$ as shown in Figure 17.52. (a) What is the net force this field exerts on the dipole? (b) Find the orientations of the dipole for which the torque on it about an axis through its center perpendicular to the plane of the figure is zero. (c) Which of the orientations in part (b) is stable, and which is unstable? (*Hint:* Consider a small displacement away from the equilibrium position, and see what happens.) (d) Show that, for the stable orientation in part (c), the dipole's own electric field *opposes* the external field for points between the charges.
45. || (a) An electron is moving east in a uniform electric field of 1.50 N/C directed to the west. At point $A$, the velocity of the electron is $4.50 \times 10^5$ m/s toward the east. What is the speed of the electron when it reaches point $B$, 0.375 m east of point $A$? (b) A proton is moving in the uniform electric field of part (a). At point $A$, the velocity of the proton is $1.90 \times 10^4$ m/s, east. What is the speed of the proton at point $B$?
46. || A $+20$ nC point charge is placed at the origin, and a $+5$ nC charge is placed on the $x$ axis at $x = 1$ m. At what position on the $x$ axis is the net electric field zero? (Be careful to keep track of the direction of the electric field of each particle.)

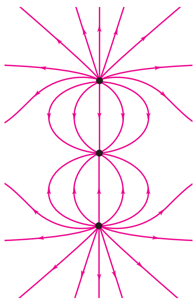
47. ||| For the dipole shown in Figure 17.53, show that the electric field at points on the $x$ axis points vertically downward and has magnitude $kq(2a)/(a^2 + x^2)^{3/2}$. What does this expression reduce to when the distance between the two charges is much less than $x$?




▲ **FIGURE 17.53** Problem 47.

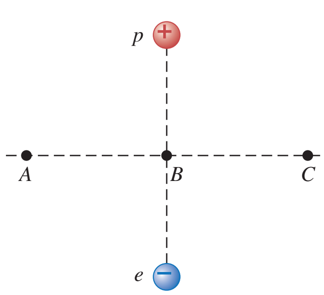
## 17.7 Electric Field Lines

48. | Figure 17.54 shows some of the electric field lines due to three point charges arranged along the vertical axis. All three charges have the same magnitude. (a) What are the signs of the three charges? Explain your reasoning. (b) At what point(s) is the magnitude of the electric field the smallest? Explain your reasoning. Explain how the fields produced by each individual point charge combine to give a small net field at this point or points.



▲ **FIGURE 17.54** Problem 48.

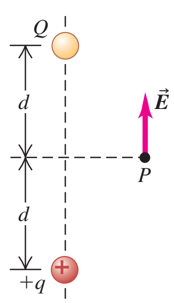
49. | A proton and an electron are separated as shown in Figure 17.55. Points $A$, $B$, and $C$ lie on the perpendicular bisector of the line connecting these two charges. Sketch the direction of the net electric field due to the two charges at (a) point $A$, (b) point $B$, and (c) point $C$.




▲ **FIGURE 17.55** Problem 49.

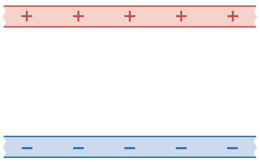
50. || Sketch electric field lines in the vicinity of two charges, $Q$ and $-4Q$, located a small distance apart on the $x$ axis.

51. | Two point charges $Q$ and $+q$ (where $q$ is positive) produce the net electric field shown at point $P$ in Figure 17.56. The field points parallel to the line connecting the two charges. (a) What can you conclude about the sign and magnitude of $Q$? Explain your reasoning. (b) If the lower charge were negative instead, would it be possible for the field to have the direction shown in the figure? Explain your reasoning.




▲ **FIGURE 17.56** Problem 51.

52. || Two very large parallel sheets of the same size carry equal magnitudes of charge spread uniformly over them, as shown in Figure 17.57. In each of the cases that follow, sketch the net pattern of electric field lines in the region between the sheets, but far from their edges. (*Hint:* First sketch the field lines due to each sheet, and then add these fields to get the net field.) (a) The top sheet is positive and the bottom sheet is negative, as shown, (b) both sheets are positive, (c) both sheets are negative.

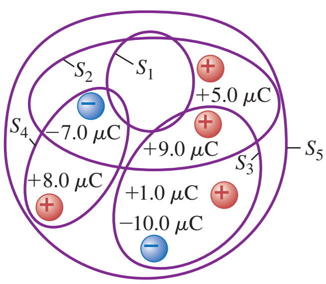


▲ **FIGURE 17.57** Problem 52.

## 17.8 Gauss's Law and Field Calculations

53. | (a) A closed surface encloses a net charge of 2.50 $\mu$C. What is the net electric flux through the surface? (b) If the electric flux through a closed surface is determined to be 1.40 $N \cdot m^2/C$, how much charge is enclosed by the surface?

54. | Figure 17.58 shows cross sections of five *closed* surfaces $S_1$, $S_2$, and so on. Find the net electric flux passing through each of these surfaces.



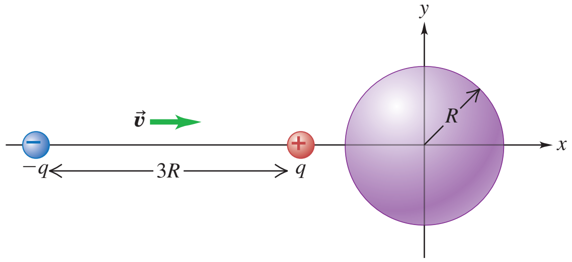

▲ **FIGURE 17.58** Problem 54.

55. || A point charge 8.00 nC is at the center of a cube with sides of length 0.200 m. What is the electric flux through (a) the surface of the cube, (b) one of the six faces of the cube?

56. | A charged paint is spread in a very thin uniform layer over the surface of a plastic sphere of diameter 12.0 cm, giving it a charge of $-15.0$ $\mu$C. Find the electric field (a) just inside the paint layer, (b) just outside the paint layer, and (c) 5.00 cm outside the surface of the paint layer.

57. | (a) How many excess electrons must be distributed uniformly within the volume of an isolated plastic sphere 30.0 cm in diameter to produce an electric field of 1150 N/C just outside the surface of the sphere? (b) What is the electric field at a point 10.0 cm outside the surface of the sphere?

58. || An electric dipole consists of charges $q$ and $-q$ separated by a distance $3R$. The dipole lies on the negative $x$ axis and is moving in the positive $x$ direction as shown in Figure 17.59. The dipole passes through a sphere of radius $R$ centered at the origin. Make a sketch of the electric flux through the sphere as a function of time.




▲ **FIGURE 17.59** Problem 58.

59. || A total charge of magnitude $Q$ is distributed uniformly within a *thick* spherical shell of inner radius $a$ and outer radius $b$. (a) Use Gauss's law to find the electric field within the cavity ($r \leq a$). (b) Use Gauss's law to prove that the electric field outside the shell ($r \geq b$) is exactly the same as if all the charge were concentrated as a point charge $Q$ at the center of the sphere. (c) Explain why the result in part (a) for a *thick* shell is the same as that found in Example 17.10 for a *thin* shell. (*Hint:* A thick shell can be viewed as infinitely many thin shells.)

## 17.9 Charges on Conductors

60. | During a violent electrical storm, a car is struck by a falling high-voltage wire that puts an excess charge of $-850\ \mu C$ on the metal car. (a) How much of this charge is on the inner surface of the car? (b) How much is on the outer surface?

61. | A neutral conductor completely encloses a hole inside of it. You observe that the outer surface of this conductor carries a charge of $-12\ \mu C$. (a) Can you conclude that there is a charge inside the hole? If so, what is this charge? (b) How much charge is on the inner surface of the conductor?

62. || An irregular neutral conductor has a hollow cavity inside of it and is insulated from its surroundings. An excess charge of $+16$ nC is sprayed onto this conductor. (a) Find the charge on the inner and outer surfaces of the conductor. (b) Without touching the conductor, a charge of $-11$ nC is inserted into the cavity through a small hole in the conductor. Find the charge on the inner and outer surfaces of the conductor in this case.

# General Problems

63. || Three point charges are arranged along the $x$ axis. Charge $q_1 = -4.50$ nC is located at $x = 0.200$ m, and charge $q_2 = +2.50$ nC is at $x = -0.300$ m. A positive point charge $q_3$ is located at the origin. (a) What must the value of $q_3$ be for the net force on this point charge to have magnitude $4.00\ \mu N$? (b) What is the direction of the net force on $q_3$? (c) Where along the $x$ axis can $q_3$ be placed and the net force on it be zero, other than the trivial answers of $x = +\infty$ and $x = -\infty$?

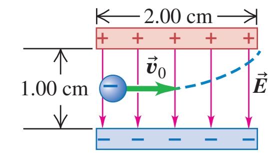
64. || An electron is released from rest in a uniform electric field. The electron accelerates vertically upward, traveling 4.50 m in the first $3.00\ \mu s$ after it is released. (a) What are the magnitude and direction of the electric field? (b) Are we justified in ignoring the effects of gravity? Justify your answer quantitatively.

65. || A charge $q_1 = +5.00$ nC is placed at the origin of an $x$-$y$ coordinate system, and a charge $q_2 = -2.00$ nC is placed on the positive $x$ axis at $x = 4.00$ cm. (a) If a third charge $q_3 = +6.00$ nC is now placed at the point $x = 4.00$ cm, $y = 3.00$ cm, find the $x$ and $y$ components of the total force exerted on this charge by the other two charges. (b) Find the magnitude and direction of this force.

66. || A charge of $-3.00$ nC is placed at the origin of an $x$-$y$ coordinate system, and a charge of 2.00 nC is placed on the $y$ axis at $y = 4.00$ cm. (a) If a third charge, of 5.00 nC, is now placed at the point $x = 3.00$ cm, $y = 4.00$ cm, find the $x$ and $y$ components of the total force exerted on this charge by the other two charges. (b) Find the magnitude and direction of this force.

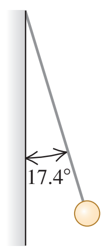
67. || Point charges of 3.00 nC are situated at each of three corners of a square whose side is 0.200 m. What are the magnitude and direction of the resultant force on a point charge of $-1.00\ \mu C$ if it is placed (a) at the center of the square, (b) at the vacant corner of the square?

68. || An electron is projected with an initial speed $v_0 = 5.00 \times 10^6$ m/s into the uniform field between the parallel plates in Figure 17.60. The direction of the field is vertically downward, and the field is zero except in the space between the two plates. The electron enters the field at a point midway between the plates. If the electron just misses the upper plate as it emerges from the field, find the magnitude of the electric field.




▲ **FIGURE 17.60** Problem 68.

69. || A small 12.3 g plastic ball is tied to a very light 28.6 cm string that is attached to the vertical wall of a room. (See Figure 17.61.) A uniform horizontal electric field exists in this room. When the ball has been given an excess charge of $-1.11\ \mu C$, you observe that it remains suspended, with the string making an angle of 17.4° with the wall. Find the magnitude and direction of the electric field in the room.




▲ **FIGURE 17.61** Problem 69.

70. || A $-5.00$ nC point charge is on the $x$ axis at $x = 1.20$ m. A second point charge $Q$ is on the $x$ axis at $-0.600$ m. What must be the sign and magnitude of $Q$ for the resultant electric field at the origin to be (a) 45.0 N/C in the $+x$ direction, (b) 45.0 N/C in the $-x$ direction?

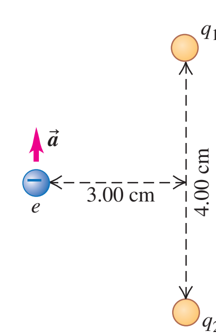
71. || DATA A physics student uses induction to give two small spheres the same charge $q$. He then proceeds to measure the magnitude of the repulsive force between the spheres as a function of their separation. The findings are given in the table.

| Separation (cm) | Force (N) |
|---|---|
| 2.0 | $2.3 \times 10^{-1}$ |
| 3.8 | $6.2 \times 10^{-2}$ |
| 6.3 | $2.3 \times 10^{-2}$ |
| 8.1 | $1.4 \times 10^{-2}$ |
| 9.6 | $9.8 \times 10^{-3}$ |

Make a linearized plot of the data, in which force is on the $y$ axis. Using a "best fit" to the data, determine the charge on each of the spheres.

72. || A $9.60\ \mu C$ point charge is at the center of a cube with sides of length 0.500 m. (a) What is the electric flux through one of the six faces of the cube? (b) How would your answer to part (a) change if the sides were 0.250 m long? Explain.

73. || Two point charges $q_1$ and $q_2$ are held 4.00 cm apart. An electron released at a point that is equidistant from both charges (see Figure 17.62) undergoes an initial acceleration of $8.25 \times 10^{18}$ m/s$^2$ directly upward in the figure, parallel to the line connecting $q_1$ and $q_2$. Find the magnitude and sign of $q_1$ and $q_2$.




▲ **FIGURE 17.62** Problem 73.

74. || An early model of the hydrogen atom viewed it as an electron orbiting a proton in a circular path with a radius of $5.29 \times 10^{-11}$ m. What would be the centripetal acceleration of the electron in this

model? How does this number compare with the acceleration of gravity, $g$? (You may need to review Chapter 6 on circular motion.)

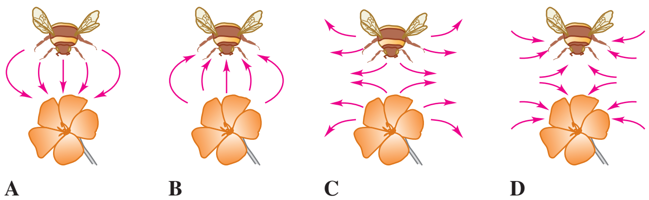
## Passage Problems

**BIO Electric bees.** Flying insects such as bees may accumulate a small positive electric charge as they fly. In one experiment, the mean electric charge of 50 bees was measured to be +30 pC per bee. Researchers also observed the electrical properties of a plant consisting of a flower on top of a long stem. The charge on the stem was measured as a positively charged bee approached, landed, and flew away. Plants are normally electrically neutral, so the measured net electric charge on the stem was zero when the bee was very far away. As the bee approached the flower, a small net positive charge was detected in the stem, even before the bee landed. Once the bee landed, the whole plant became positively charged, and this positive charge remained on the plant after the bee flew away. By creating artificial flowers with various charge values, experimenters found that bees can distinguish between charged and uncharged flowers and may use the positive electric charge left by an earlier bee as a cue to indicate that a plant has already been visited (in which case, little pollen may remain).

75. Consider a bee with the mean electric charge found in the experiment. This charge represents roughly how many missing electrons?
A. $1.9 \times 10^8$ B. $3.0 \times 10^8$
C. $1.9 \times 10^{18}$ D. $3.0 \times 10^{18}$

76. What is the best explanation for the observation that the electric charge on the stem became positive as the charged bee approached (before it landed)?
A. Because air is a good conductor, the positive charge on the bee's surface flowed through the air from bee to plant.
B. Because the earth is a reservoir of large amounts of charge, positive ions were drawn up the stem from the ground toward the charged bee.
C. The plant became electrically polarized as the charged bee approached.
D. Bees that visited the plant earlier deposited a positive charge on the stem.

77. After one bee left a flower with a positive charge, that bee flew away and another bee with the same amount of positive charge flew close to the plant. Which diagram in Figure 17.63 best represents the electric field lines between the bee and the flower?




▲ **FIGURE 17.63** Problem 77.

78. In a follow-up experiment, a charge of +40 pC was placed at the center of an artificial flower at the end of a 30-cm-long stem. Bees were observed to approach no closer than 15 cm from the center of this flower before they flew away. This observation suggests that the smallest external electric field to which bees may be sensitive is closest to which of these values?
A. 2.4 N/C
B. 16 N/C
C. $2.7 \times 10^{-10}$ N/C
D. $4.8 \times 10^{-10}$ N/C

**Space radiation shielding.** One of the hazards facing humans in space is space radiation: high-energy charged particles emitted by the sun. During a solar flare, the intensity of this radiation can reach lethal levels. One proposed method of protection for astronauts on the surface of the moon or Mars is an array of large, electrically charged spheres placed high above areas where people live and work. The spheres would produce a strong electric field $\vec{E}$ to deflect the charged particles that make up space radiation. The spheres would be similar in construction to a Mylar balloon, with a thin, electrically conducting layer on the outside surface on which a net positive or negative charge would be placed. A typical sphere might be 5 m in diameter.

79. Suppose that to repel electrons in the radiation from a solar flare, each sphere must produce an electric field $\vec{E}$ of magnitude $1 \times 10^6$ N/C at 25 m from the center of the sphere. What net charge on each sphere is needed?
A. −0.07 C
B. −8 mC
C. −80 $\mu$C
D. $-1 \times 10^{-20}$ C

80. What is the magnitude of $\vec{E}$ just outside the surface of such a sphere?
A. 0
B. $10^6$ N/C
C. $10^7$ N/C
D. $10^8$ N/C

81. What is the direction of $\vec{E}$ just outside the surface of such a sphere?
A. Tangent to the surface of the sphere
B. Perpendicular to the surface, pointing toward the sphere
C. Perpendicular to the surface, pointing away from the sphere
D. There is no electric field just outside the surface.

82. Which of the following is true about $\vec{E}$ inside a negatively charged sphere as described here?
A. It points from the center of the sphere to the surface and is largest at the center.
B. It points from the surface to the center of the sphere and is largest at the surface.
C. It is zero.
D. It is constant but is not zero.