Contents

PREFACE

R REVIEW OF PREALGEBRA SKILLS  R.1-1
   R.1 Simplifying Fractions
   R.2 Multiplying and Dividing Fractions
   R.3 Adding and Subtracting Fractions
   R.4 Decimals and Percents

1 THE REAL NUMBER SYSTEM  1.1-1
   1.1 Real Numbers and the Number Line
   1.2 Exponents and Order of Operations
   1.3 Variables, Expressions, and Equations
   1.4 Adding Real Numbers
   1.5 Subtracting Real Numbers
   1.6 Multiplying and Dividing Real Numbers
   1.7 Properties of Real Numbers
   1.8 Simplifying Expressions
   1.9 Applications of Real Numbers

2 EQUATIONS, INEQUALITIES, AND APPLICATIONS  2.1-1
   2.1 Solving Linear Equations: The Addition Property of Equality
   2.2 Solving Linear Equations: The Multiplication Property of Equality
   2.3 Solving Linear Equations: Fractions, Decimals, and More
   2.4 Algebraic Skills Involving Formulas, Geometry, and Proportions
   2.5 Introduction to Problem Solving and Linear Applications: Numbers, Geometry, and More
   2.6 Applications of Linear Equations: Proportion and Percent Problems
   2.7 Solving Linear Inequalities
   2.8 Applications of Linear Inequalities
3 GRAPHING EQUATIONS AND INEQUALITIES 3.1-1
3.1 Linear Equations in Two Variables: $Ax + By = C$
3.2 Graphing Linear Equations
3.3 Slope of a Line
3.4 Equations of Lines
3.5 Graphing Linear Inequalities in Two Variables
3.6 Applications of Linear Equations and Inequalities and Their Graphs
3.7 Introduction to Functions

4 SYSTEMS OF LINEAR EQUATIONS AND INEQUALITIES 4.1-1
4.1 Solving Systems of Linear Equations: Graphing Method
4.2 Solving Systems of Linear Equations: Substitution Method
4.3 Solving Systems of Linear Equations: Addition Method
4.4 Solving Systems of Linear Inequalities
4.5 Applying Systems of Linear Equations

5 EXPONENTS AND POLYNOMIALS 5.1-1
5.1 The Product Rule and Power Rules for Exponents
5.2 Integer Exponents and the Quotient Rule
5.3 Introduction to Polynomials
5.4 Adding and Subtracting Polynomials
5.5 Multiplying Polynomials
5.6 Special Products
5.7 Dividing Polynomials
5.8 Applying Exponents: Scientific Notation
5.9 Applying Polynomials

6 FACTORING POLYNOMIALS 6.1-1
6.1 The Greatest Common Factor
6.2 Factoring Trinomials of the Form $x^2 + bx + c$
6.3 Factoring Trinomials of the Form $ax^2 + bx + c$
6.4 Special Factoring Techniques
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5</td>
<td>A General Strategy for Factoring Polynomials</td>
</tr>
<tr>
<td>6.6</td>
<td>Solving Quadratic Equations by Factoring</td>
</tr>
<tr>
<td>6.7</td>
<td>Applying Quadratic Equations</td>
</tr>
<tr>
<td>7</td>
<td>RATIONAL EXPRESSIONS AND APPLICATIONS 7.1-1</td>
</tr>
<tr>
<td>7.1</td>
<td>Simplifying Rational Expressions</td>
</tr>
<tr>
<td>7.2</td>
<td>Multiplying and Dividing Rational Expressions</td>
</tr>
<tr>
<td>7.3</td>
<td>Adding and Subtracting Rational Expressions with the Same Denominator</td>
</tr>
<tr>
<td>7.4</td>
<td>Adding and Subtracting Rational Expressions with Different Denominators</td>
</tr>
<tr>
<td>7.5</td>
<td>Simplifying Complex Fractions</td>
</tr>
<tr>
<td>7.6</td>
<td>Solving Equations Containing Rational Expressions</td>
</tr>
<tr>
<td>7.7</td>
<td>Applying Rational Equations: Numbers, Rates, and Work</td>
</tr>
<tr>
<td>7.8</td>
<td>Applying Rational Equations: Variation</td>
</tr>
<tr>
<td>8</td>
<td>ROOTS AND RADICALS 8.1-1</td>
</tr>
<tr>
<td>8.1</td>
<td>Introduction to Radicals</td>
</tr>
<tr>
<td>8.2</td>
<td>Simplifying Radicals</td>
</tr>
<tr>
<td>8.3</td>
<td>Adding and Subtracting Radicals</td>
</tr>
<tr>
<td>8.4</td>
<td>Multiplying and Dividing Radicals</td>
</tr>
<tr>
<td>8.5</td>
<td>Solving Equations Containing Radicals</td>
</tr>
<tr>
<td>8.6</td>
<td>Applying Radicals</td>
</tr>
<tr>
<td>9</td>
<td>QUADRATIC EQUATIONS 9.1-1</td>
</tr>
<tr>
<td>9.1</td>
<td>Solving Quadratic Equations Using the Square Root Property</td>
</tr>
<tr>
<td>9.2</td>
<td>Solving Quadratic Equations by Completing the Square</td>
</tr>
<tr>
<td>9.3</td>
<td>Solving Quadratic Equations Using the Quadratic Formula</td>
</tr>
<tr>
<td>9.4</td>
<td>Graphing Quadratic Equations in Two Variables</td>
</tr>
<tr>
<td>9.5</td>
<td>Applying Quadratic Equations</td>
</tr>
</tbody>
</table>

GLOSSARY

ANSWERS

INDEX
**A Letter from the Authors**

**Dear Student,**

Welcome to your algebra course! We have created *MyMathLab for INTRO Algebra* with you in mind. Using your **Worktext** and **eText** with the **Roadmap to Learning Algebra**, outlined below, will help you master the important algebra concepts needed to succeed in your course. You’ll also find that many of these algebra concepts will apply to your daily life, so you will benefit from this course outside of the classroom as well.

**USING YOUR WORKTEXT AND ETTEXT**
*MyMathLab for INTRO Algebra* offers both a printed Worktext and an online eText (available in MyMathLab) to help you learn the math!

**In your Worktext, you can**
- Take notes.
- Show your work and record your answers to exercises.
- Create your own portable study guide by adding your solutions, answers, and notes and putting everything in a binder.

**In your eText, you can**
- Check your answers.
- Get additional information.
- Watch videos and animations.
- Work exercises.

**YOUR ROADMAP TO LEARNING ALGEBRA**
For every section in the Worktext and eText:

**Learn It.**
- Read the **Main Idea**, **Process**, and **Important Notes** for each Objective.
- Read each **Example** and its worked-out **Solution**. If you’re not sure you understand, click to watch a video (in the eText) of one of us solving this problem.
- Each Example is followed by a similar **Your Turn** exercise for you to try. Work out the solution to the **Your Turns** in the space provided in the Worktext. Enter your answer in the eText to see if you are correct. If your answer is incorrect, you can receive additional help.
- If necessary, work **More Practice** problems in MyMathLab to make sure you understand the math concept.
- Solve the **On Your Own** exercises and keep your work in your course notebook. Check your answers to these exercises in the back of your Worktext.
- To make sure you understood the Objective, try the **Reading Check** exercise in the eText.

**Check It.**
- Once you have finished the section, complete the **Learning Check** assignment in MyMathLab to check your understanding of the material you just covered.

**Do It.**
- Complete your homework assignment at the end of each section. You’ll also find some additional mid-chapter and end-of-chapter assignments in your MyMathLab course.
A LETTER FROM THE AUTHORS

Learn It

Worktext
Streamlined content with a guided approach.

eText
Core content enhanced with media resources and additional information.

Check It

Do It

MyMathLab®
A learning path in MyMathLab with assignments ready to go!

GUIDE TO THE ICONS
As you read your Worktext and eText, you will see icons indicating that there are more resources available to you in the eText. When you are in the eText, click on these icons to access the following:

VIDEOS. Every example has a corresponding video showing one of us working through the solution step-by-step. Sometimes, “More Information,” “FAQs,” and “Reminder” pop-ups are videos, too.

MORE INFORMATION. Click here for a more in-depth explanation of the concept.

FAQs. This icon answers common questions we know students have at certain points in the course.

REMINDERS. Here, you’ll find previously discussed content that is related to the current concept.

PRACTICE. Click here to complete a Your Turn, More Practice, Reading Check exercise, or assignments in MyMathLab.

In your eText, you can also click on links for additional resources and information!

Show your work when you see the Worktext icon next to the Your Turns and On Your Own exercises.

We wish you success in your course!

—Andreana Grimaldo and Denise Robichaud
Dear Instructor,

We firmly believe that students learn math by doing math, that solving problems in structured ways leads to mastery, and that technology can offer flexibility and interactivity that put students in control of their learning. However, after embracing technology and happily watching our students become more active learners, we noticed that the students were not recording their work in an organized, step-by-step fashion. We created *MyMathLab for INTRO Algebra* to give our students a balance of pencil and paper, and technology, using flexible learning tools in the form of a structured Worktext and an interactive MyMathLab course.

Our *MyMathLab for INTRO Algebra* package, consisting of a media-rich MyMathLab course and a printed Worktext, offers the best of both paper and online learning. The Worktext encourages organized, written work while guiding students through the learning process. The Worktext is paired with the MyMathLab eText, which provides an interactive, online environment with additional media resources including example videos, animations, and helpful pop-ups.

For additional information, please see the letter to the student on page vi.

These resources are flexible enough to be used in any type of math class. We have used this program in traditional lecture, self-paced, and online-only courses, as well as in computer-based learning labs. Instructors who choose *MyMathLab for INTRO Algebra* will be giving their students a complete learning package in which paper and technology work together seamlessly.
For your convenience, we have provided the *MyMathLab for INTRO Algebra* MyMathLab course with a variety of preassigned homework, quizzes, and tests for every section to get you through the course. Of course, you can revise as necessary to suit the way you teach your course. If you are new to MyMathLab or would like additional guidance for setting up your course, please refer to the *Annotated Instructor’s Edition, Instructor’s Resource Manual, Instructor’s Solutions Manual*, and the instructional support video.

We hope you enjoy teaching with *MyMathLab for INTRO Algebra*, and we would love to hear about your experience.

*Best of luck this semester!*

—Andreana Grimaldo and Denise Robichaud
STUDENT SUPPLEMENTS

By Paul Lorczak
Contains fully worked-out solutions to Your Turn exercises, On Your Own exercises, and odd-numbered Additional Exercises.

INSTRUCTOR SUPPLEMENTS

Includes answers to all the exercises, printed in gray on the same page as the exercises.

By Paul Lorczak
Contains fully worked-out solutions to Your Turn exercises, On Your Own exercises, and even-numbered Additional Exercises.

By Ulises Poyser, Quinsigamond Community College
For each Worktext section, this manual includes one mini-lecture with an instructional overview, board-ready examples with answers, key teaching notes, and a table listing multimedia resources. Additional resources include:
- One activity per chapter
- One multiple-choice and one free-response quiz each for the first and second half of every chapter
- Two multiple-choice tests and two free-response tests per chapter
- Cumulative tests (after chapters 3, 6, and 9)
- Answers to all tests and quizzes

TestGen®
TestGen® (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the worktext. TestGen is algorithmically based, allowing instructors to create multiple, but equivalent, versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions. The software and test bank are available for download from Pearson Education’s online catalog.
Pearson Math Adjunct Support Center
(www.pearsonontutorservices.com/math-adjunct.html)
The Pearson Math Adjunct Support Center is staffed by qualified instructors with more than 100 years of combined experience at both the community college and university levels. Assistance is provided for faculty in the following areas:

• Suggested syllabus consultation

• Tips on using materials packed with your book

• Book-specific content assistance

• Teaching suggestions, including advice on classroom strategies
Acknowledgments

To Ray, Amy, Alicia, and Allison: thanks for cheering me on!
—Andreana Grimaldo

To Joe and Howie for their love and support.
—Denise Robichaud

Many people helped to shape MyMathLab for INTRO Algebra, turning our idea into the finished product in your hands today. We are very grateful for all the wonderful and thoughtful feedback we have received as we developed MyMathLab for INTRO Algebra. Writing a first edition is a collaborative effort, and we owe many thanks to the following people for participating in focus groups, reviewer projects, and class testing:

Gisela Acosta-Carrasquillo, Valencia Community College – East Campus
Brenda Alberico, College of DuPage
Chandra Allen, Guilford Technical Community College
Deborah M. Alspaugh, Ivy Tech Community College – Gary
Billie Anderson, Tyler Junior College
Robin Anderson, Southwestern Illinois College
Mary Jo Anhalt, Bakersfield College
David Arreazola, Laredo Community College
Alex Asare, Roxbury Community College
Mr. Christy Babu, Laredo Community College
Frank Bambic, Cuyahoga Community College
Susan Barbitta, Guilford Technical Community College
Susan Beane, University of Houston – Downtown
Linda Bellington, Tyler Junior College
Aemiro Beyene, Quinsigamond Community College
Barry Biddlecomb, Georgia Gwinnett College
Tom Blackburn, Northeastern Illinois University
Brenda Blue, San Jacinto College
Elena Bogardus, Camden County College
Sandy Bolster, Berea College
Branson Brade, Houston Community College – Katy Campus
Sue Brown, Guilford Technical Community College
Alice Cangemi Pollock, Lone Star College – Montgomery
Brian Carter, St. Louis Community College – Forest Park
J. Castillo, Broward Community College
Matthews Chakkanakuzhi, Palomar College
Florence Chambers, Southern Maine Community College
Jerry J. Chen, Suffolk County Community College
Bryan Cockerham, Front Range Community College
Alicia Collins, Mesa Community College
Ebony Conley, Pulaski Technical College
Christine Copple, Northwest State Community College
Dr. Patrick S. Cross, University of Oklahoma
Inez DeShields, Bishop State Community College
Elena Dilai, Monroe Community College
LaTonya H. Ellis, Bishop State University
Meghan Erwin, Indiana University of Pennsylvania
Nancy D. Eschen, Florida State College at Jacksonville – South
Anne A. Fischer, Tulsa Community College – Metro Campus
David French, Tidewater Community College – Chesapeake
Deborah Fries, Wor Wic Community College
Troy Furlough, El Centro College
Roberta Hinkle Gansman, Guilford Technical Community College
Christina Gawlik, Texas Woman’s University
Kim Ghiselin, State College of Florida
Marsha Gossett, North Central Texas College
Cheryl Grant, Bowling Green State University
Jack Green, Mt. Hood Community College
Jin Ha, Northeast Lakeview College
Karen Hale, Onondaga Community College
Dr. Richard “Bubba” Hall, Cochise College
Craig D. Hardesty, Ed.D., Hillsborough Community College – Southshore Campus
Mashid Hassani, Ivy Tech Community College – Gary
Kevin Hastings, Parkland College
Abdalla H. Hazaimeh, Ivy Tech Community College
Al Hemenway, Los Angeles Mission College
Andrew Henley, Guilford Technical Community College
Nancy Hixson, Columbia Southern University
Cheryl Hobneck, Illinois Valley Community College
Michelle Hollis, Bowling Green Community College
Laura Huerta, Laredo Community College – South Campus
Denise Hum, Canada College
Sharon Jackson, Brookhaven College
Jay M. Jahangiri, Ph.D., Kent State University
Darryl Jinkerson, Ph.D., Abilene Christian University
Brenda Jinks, Southwest Tennessee Community College
Kim Johnson, Mesa Community College
Leif Jordan, College of the Desert
Philip Kaatz, Mesalands Community College
Linda Kass, Bergen Community College
Harriet Higgins Kiser, Georgia Highlands College
Mary Ann Klicka, Bucks County Community College
Rosa Kontos, Bergen Community College
Nicole Lang, North Hennepin Community College
Judith F. Langer, Westchester Community College
Lonnie Larson, Sacramento City College
Humberto Laurel, Laredo Community College
Kathryn Lavelle, Westchester Community College
Jennifer LeGrand, St. Charles Community College
Keith Luoma, Augusta State University
Douglas Mace, Kirtland Community College
Walt Mackey, Owens Community College
Carol Marinas, Barry University
Mark Marino, Erie Community College
Steve Marsden, Glendale Community College
Dorothy Marshall, Edison State College – Fort Myers
Carolyn Martinson, Jefferson Community College
Philip McCartney, Northern Kentucky University
Michael McComas, Mountwest Community & Technical College
Terri McKnight, Rowan-Cabarrus Community College
Elizabeth McMillan-McCartney, Northern Kentucky University
Angela M. McNulty, Joliet Junior College
Valerie Melvin, Cape Fear Community College
Beverly Meyers, Jefferson College
Pam Miller, Phoenix College
Christine Mirbaha, Community College of Baltimore County – Dundalk
Shanna Moody, Weatherford College
Jill Mueller DeWitt, Muskegon Community College
Linda Murphy, Northern Essex Community College
Michael Murphy, Texas State Technical College – Harlingen
Kiruba Murugiah, Banker Hill Community College
Vidya Nahar, Athens Technical College
Jill Nelipovich, Imperial Valley College
Elsie Newman, Owens Community College
Louise Pack, Rowan-Cabarrus Community College
Debra Panasuk, Quincy College
Jim Pierce, Lincoln Land Community College
Kerri Pippin, Jones County Junior College
David Platt, Front Range Community College
Elaine Previte, Bristol Community College
Thomas G. Pulver, Waubonsee Community College
Brooke Quinlan, Hillsborough Community College
George Reed, Angelina College
Jose Rico, Laredo Community College
Christopher Riola, Moraine Valley College
Diann Robinson, Ivy Tech Community College
Malinni Roeun, Coastline Community College
Patricia Rome, Delgado Community College
Jason Rosenberry, Harrisburg Area Community College
Linda Rottmann, University of Maine
Elizabeth Rourke, College of Southern Maryland
Elizabeth Russell, Glendale Community College
Daria Santerre, Norwalk Community College
Christine Schultz, Iowa State University
Matt Seikel, Owens Community College
Jonathan Shands, Cape Fear Community College
Billie Shannon, Southwestern Oregon Community College
Lisa Sheppard, Lorain County Community College
Deborah Simonson, Fox Valley Technical College
David R. Slauenwhite, Darton College
Craig Slocum, Moraine Valley Community College
Shannon Solis, San Jacinto College
Yvonne Stallings, Lone Star College – Montgomery
Larry Stone, Dakota County Technical College
Latasha Subramanian, Potomac State College of West Virginia University
Sandra Tannen, Camden County College
Kenneth Takvorian, Mt. Wachusett Community College
Linda Tansil, Southeastern Missouri State University
Kay Teague, Tarleton State University
Cassonda Thompson, York Technical College
Yan Tian, Palomar College
Susan Twigg, Wor Wic Community College
Diane Valade, Piedmont Virginia Community College
Jennifer Vanden Eynen, Grossmont College
Walter Wang, Baruch College
Meredith Watts, Mass Bay Community College
Leslie Wenzel, Garden City Community College
Steve Whittle, Augusta State University
Suzanne Williams, Central Piedmont Community College
Mary Williams, Roosevelt University – Chicago
Alice Williamson, Sussex County Community College
Emily Woods, Southern Maine Community College
Susan C. Working, Grossmont College
Yoshi Yamato, Pasadena Community College

We would also like to thank Courtney Slade for coordinating this project initially and for accuracy checking the main-text page proofs, and Kristin Ruscetta at PreMediaGlobal for overseeing production. Thanks to Helen Medley for preparing the answer section and to Jon Stockdale at Math Made Visible for accuracy checking the manuscript. In addition, thanks to Ulises Poyser for preparing the Instructo’s Resources Manual and to De Cook for checking it, to Paul Lorczak for preparing the solutions manuals and to Deana Richmond for checking the accuracy of the solutions manuals and the main-text page proofs.

Our thanks would not be complete without acknowledging the wonderful efforts of our team at Pearson Education: Cathy Cantin, Executive Editor; Dawn Nuttall, Senior Development Editor; Ron Hampton, Senior Production Supervisor; Carl Cottrell, Senior Media Producer; Kristina Evans, Associate Content Specialist; Kari Heen, Executive Content Editor; Katherine Minton, Associate Content Editor; Dona Kenly, Executive Market Development Manager; Michelle Renda, Executive Marketing Manager; Rachel Ross, Marketing Manager; Alicia Frankel, Associate Marketing Manager; Ashley Bryan, Marketing Assistant; Jon Wooding, Assistant Editor; and Kerianne Okie, Editorial Assistant.

—Andreana Grimaldo and Denise Robichaud
About the Authors

ANDREANA GRIMALDO
Andreana Grimaldo grew up in New Hampshire. She received her associate’s degree in business from Becker College and her bachelor of science in mathematics and master of science in applied mathematics from Worcester State College. Andreana has taught many levels of math, from developmental through calculus, beginning as an individual instructor to at-risk and non-English-speaking students. She started teaching as an adjunct at Quinsigamond Community College in 1990 and became a full-time professor in 2003. In 2007, she was awarded the NISOD Teaching and Leadership Excellence Award. Andreana is a Boston sports fan and her hobbies include golfing, reading, learning, and family activities. Her greatest passions are her family and “figuring out ways to excite students about mathematics.”

DENISE ROBICHAUD
Denise Robichaud was born and raised in Massachusetts. She received her bachelor of science in physics from the University of Lowell and her master of science in physics and master of arts in teaching from Tufts University. Denise started teaching math and physics part-time in 1992 and over the following ten years taught at a variety of institutions including a private high school, as well as 2-year and 4-year colleges. She has been a full-time professor at Quinsigamond Community College since 2002, and in 2007 she was awarded the NISOD Teaching and Leadership Excellence Award. When she is not instructing students, Denise enjoys reading, playing the guitar, exercising, cheering on Boston sports teams, and spending time with family and friends.
5.1 The Product Rule and Power Rules for Exponents

OBJECTIVES
A Evaluate exponential expressions
B Use the product rule for exponents
C Use the power rule for exponents
D Use the power rules for products and quotients

VOCABULARY
- exponential expression
- base
- exponent
- power

In this section, we will learn some exponent vocabulary and four exponent rules. In the next section, we will learn the remaining rules.

\[ a^m \cdot a^n = a^{m+n} \]
Evaluate Exponential Expressions

MAIN IDEA

An **exponential expression** is an algebraic expression that contains exponents. Recall that an exponent is the number of times the base is used as a factor. This is true whether the base is a number or a variable. For example:

- **base**: $2^3 = 2 \cdot 2 \cdot 2 = 8$
- **exponent, or power**: $a^n = a \cdot a \cdot a \cdot \ldots \cdot a$
- **$a$ is real; $n$ is an integer**
- **$a$ used as a factor $n$ times**

Because an exponential expression is just a special case of an algebraic expression, we will use the same process to evaluate each. For example, to evaluate $5x^3$ for $x = 2$:

- **Replace** $x$ with 2.
- **Simplify** the result using the correct order of operations.

PROCESS How to Evaluate Exponential Expressions

- **Replace** the variable(s) with the given value(s).
- **Simplify** the result using the correct order of operations.

IMPORTANT NOTES

- If an exponent is not shown, the exponent is understood to be 1. For example, $5$ is the same as $5^1$.

**EXAMPLE 1**

Evaluate each expression for the given value.

a) $4x^2$ when $x = -3$

   - **Rewrite the expression.** $4x^2$
   - **Replace** $x$ with $-3$.
   - **Simplify.** $4(-3)^2 = 36$

b) $-7p^5$ when $p = -2$

   - **Rewrite the expression.** $-7p^5$
   - **Replace** $p$ with $-2$.
   - **Simplify.** $-7(-2)^5 = 224$
Section 5.1

YOUR TURN 1

Evaluate each expression for the given value.

a) \(-2y^3\) when \(x = -4\)

b) \(-5a^2\) when \(a = -5\)

Put your base in parentheses so that you are less likely to forget the negative sign if there is one.

Go to the eText for more practice.

ON YOUR OWN 1

Evaluate each expression for the given value.

a) \(3x^4\) when \(x = -2\)

b) \(7y^3\) when \(y = \frac{2}{7}\)

c) \(5a^2b\) when \(a = -9\) and \(b = -3\)

Go to the eText for the online Reading exercise.

B Use the Product Rule for Exponents

MAIN IDEA

For any real number \(a\) and any integers \(m\) and \(n\):

<table>
<thead>
<tr>
<th>Product Rule</th>
<th>In Words</th>
<th>Example</th>
<th>Why It Works</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a^m \cdot a^n = a^{m+n})</td>
<td>When multiplying exponential expressions with same bases, add their exponents.</td>
<td>(2^3 \cdot 2^4 = 2^7)</td>
<td>Three 2’s times four 2’s results in seven 2’s.</td>
</tr>
</tbody>
</table>

PROCESS How to Use the Product Rule for Exponents

\[ a^m \cdot a^n = a^{m+n} \]  \((a \text{ is real; } m, n \text{ are integers.})

- Keep the base.
- Add the exponents.

IMPORTANT NOTES

- The bases must be the same to use this rule. For example, we cannot add the exponents in \(2^3 \cdot 5^4\).
- The expressions must be multiplied together to use this rule. For example, we cannot add the exponents in \(2^3 + 2^4\).
- Never multiply the bases together. For example, \(2^3 \cdot 2^4 = 2^7\), not \(4^7\).
- In this section, we will be using only positive exponents. We’ll see how the exponent rules work for zero and negative exponents in the next section.
Simplify. Express the final answer in exponential form.

a) \(2^3 \cdot 2^2\) 
   Rewrite the expression. \(2^3 \cdot 2^2 = 2^{3+2} = 2^5\)
   Keep the common base and add the exponents.

b) \(x^2 \cdot x^7\) 
   Rewrite the expression. \(x^2 \cdot x^7 = x^{2+7} = x^9\)
   Keep the common base and add the exponents.

c) \(2a^3 \cdot 3a^6\) 
   Rewrite the expression. \(2a^3 \cdot 3a^6 = 6a^{3+6} = 6a^9\)
   Multiply the 2 and 3.
   Keep the common base and add the exponents.

d) \(5x^3y^2 \cdot 3x^4y^5\) 
   Rewrite the expression. \(5x^3y^2 \cdot 3x^4y^5 = 15x^{3+4}y^{2+5} = 15x^7y^7\)
   Multiply the 5 and 3.
   Keep the common base \(x\) and add its exponents.
   Keep the common base \(y\) and add its exponents.

YOUR TURN 2

Simplify. Express the final answer in exponential form.

a) \((-4)^3 \cdot (-4)^3\) 
   Keep the entire base of \(-4\). Remember to show negative bases in parentheses.

b) \(m^3 \cdot m^4\)


c) \(-5x^2 \cdot 2x^4\)
   First, multiply the 8 and 2, then use the product rule on each base separately.

d) \(8x^5y^2 \cdot 2x^6y^3\)

Go to the eText for more practice.
Use the Power Rule for Exponents

**MAIN IDEA**

For any real number $a$ and any integers $m$ and $n$:

<table>
<thead>
<tr>
<th>Power Rule</th>
<th>In Words</th>
<th>Example</th>
<th>Why It Works</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(a^m)^n = a^{mn}$</td>
<td>When a power is raised to a power, multiply the exponents.</td>
<td>$(3^2)^3 = 3^6$</td>
<td>$(3^2)^3 = 3^6$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^6$</td>
<td>Multiplying three $3^2$’s results in six $3$’s.</td>
</tr>
</tbody>
</table>

**PROCESS** How to Use the Power Rule for Exponents

$(a^m)^n = a^{mn}$ ( $a$ is real; $m$, $n$ are integers.)

- **Keep** the base.
- **Multiply** the exponents.

**IMPORTANT NOTES**

- Use this rule when you have one base with a power being raised to another power.
- Remember, if you see two expressions with the same bases being multiplied together, use the product rule instead of the power rule.
- In this section, we will be using only positive exponents. We’ll see how the exponent rules work for zero and negative exponents in the next section.

---

ON YOUR OWN 2

Simplify. Express the final answer in exponential form.

a) $(-3)^4 \cdot (-3)^6$

b) $y^3 \cdot y^7 \cdot y^2$

c) $(-x^3y^5)(-3xy^6)$

The rule works the same here. Add all three exponents.

Remember to think of $x$ as $x^1$.
5.1-6

EXAMPLE 3

Simplify. Express the final answer in exponential form.

a) \((3^2)^6\)  
   Rewrite the expression. \((3^2)^6\)  
   Keep the base and multiply the exponents.  
   \(3^{2\cdot6} = 3^{12}\)

b) \((p^4)^5\)  
   Rewrite the expression. \((p^4)^5\)  
   Keep the base and multiply the exponents.  
   \(p^{4\cdot5} = p^{20}\)

YOUR TURN 3

Simplify. Express the final answer in exponential form.

a) \((2^3)^7\)

b) \(-(y^3)^2\)

For a power raised to a power, keep the base and multiply the exponents.

Notice that the negative sign is outside the parentheses.

ON YOUR OWN 3

Simplify. Express the final answer in exponential form.

a) \((2^3)^4\)  
   \(2^3\cdot2^3\cdot2^3\cdot2^3 = 2^{3+3+3+3} = 2^{12}\)

b) \((q^2)^9\)  
   \(q^{2\cdot9} = q^{18}\)

c) \(-(z^3)^7\)  
   \(-1\cdot(z^3)^7 = -z^{3\cdot7} = -z^{21}\)

Go to the eText for more practice.

Use the Power Rules for Products and Quotients

MAIN IDEA

For any real numbers \(a\) and \(b\) and any integer \(n\):

<table>
<thead>
<tr>
<th>Power Rule for …</th>
<th>In Words</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Products: ((ab)^n = a^n b^n)</td>
<td>When a product is raised to a power, each factor gets raised to the power.</td>
<td>((2 \cdot 3)^4 = 2^4 \cdot 3^4)</td>
</tr>
<tr>
<td>Quotients: (\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \quad b \neq 0)</td>
<td>When a quotient is raised to a power, both the numerator and denominator get raised to the power.</td>
<td>(\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4})</td>
</tr>
</tbody>
</table>
Section 5.1

**PROCESS** How to Use the Power Rules for Products and Quotients

\[(ab)^n = a^n b^n\] or \[\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\] (\(a, b\) are real, \(b \neq 0\); \(n\) is an integer.)

All factors or both the numerator and denominator get raised to the power.

**IMPORTANT NOTES**

- Use this rule when you see a product or quotient in parentheses being raised to a power.
- This rule works only for products and quotients, not for sums or differences. For example, \((3 + 4)^2\) does not equal \(3^2 + 4^2\).
- In this section, we will be using only positive exponents. We’ll see how the exponent rules work for zero and negative exponents in the next section.

**EXAMPLE 4**

Simplify. Completely evaluate any numbers with exponents for the final answer. Assume that all variables represent nonzero real numbers.

a) \((2x)^4\)

Rewrite the expression. 
\[(2x)^4\]

Raise all factors to the power. 
\[2^4 \cdot x^4\]

Evaluate. 
\[16x^4\]

b) \((3ab^2)^3\)

Rewrite the expression. 
\[(3ab^2)^3\]

Raise all factors to the power. 
\[3^3a^3(b^2)^3\]

Evaluate \(3^3\) and multiply the exponents for \(b\). 
\[27a^3b^6\]

c) \(\left(\frac{x}{y^2}\right)^3\)

Rewrite the expression. 
\[\left(\frac{x}{y^2}\right)^3\]

Raise both the numerator and denominator to the power. 
\[\frac{x^3}{(y^2)^3}\]

Multiply the exponents for \(y\). 
\[\frac{x^3}{y^6}\]

**YOUR TURN 4**

Simplify. Completely evaluate any numbers with exponents for the final answer. Assume that all variables represent nonzero real numbers.

a) \((4a)^3\)

Both factors get raised to the power.
Fill in the blanks to make true statements.

### KEY CONCEPTS

#### How to Evaluate Exponential Expressions

- __________ the variable(s) with the given value(s).
- __________ the result using the correct ________ ________.

#### How to Use the Product Rule for Exponents

\[ a^m \cdot a^n = a^{m+n} \quad (a \text{ is real; } m, n \text{ are ________}). \]
- Keep the ________.
- __________ the exponents.

#### How to Use the Power Rule for Exponents

\[ (a^m)^n = a^{m \cdot n} \quad (a \text{ is real; } m, n \text{ are integers}). \]
- Keep the ________.
- __________ the exponents.

### EXAMPLES

- Evaluate \(5x^3\) for \(x = 2\). \(5x^3\)
- Replace \(x\) with ____. \(5(2)^3\)
- Simplify. Evaluate ____. \(5 \cdot 8\)
- Multiply. \(40\)

\(2a^3 \cdot 3a^6\) __________ \(a^3 \cdot a^6\) \(6a^{3+6}\) __________ \(6a\) __________

\((p^3)^4\) \(p^{-1}\) \(p^12\)
Section 5.1

5.1-9

5.1 Additional Exercises

**KEY CONCEPTS**

How to Use the Power Rules for Products and Quotients

\((ab)^n = a^n b^n\) or \(\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\)

\((a, b \text{ are real, } b \neq 0; n \text{ is an integer.})\)

All factors or both the numerator and denominator get _________ to the _________.

**Examples**

**Vocabulary** exponential expression, base, exponent, power

**5.1 Additional Exercises**

**READING ✓**

Choose the best response.

1. In the expression \(5y^3\), 3 is the ___.
   a) base
   b) exponent
   c) numerical coefficient

2. In the expression \((-6)^3\), -6 is the ___.
   a) base
   b) exponent
   c) numerical coefficient

**For the following exponential expressions, determine whether you would a) add the exponents, b) multiply the exponents, or c) neither add nor multiply the exponents. Assume that all variables represent nonzero real numbers.**

3. \(3^3 \cdot 3^5\)
4. \(3^2 + 3^5\)
5. \((6^2)^3\)
6. \(- (4x^2)^2\)
7. \((2^3 \cdot 5^5)^7\)
8. \((\frac{3x^7}{y^3})^3\)

**PRACTICE PROBLEMS**

**A** Evaluate each expression for the given value. *See Example 1.*

9. \(3y^2\) when \(y = -2\)
10. \(5x^2\) when \(x = -3\)
11. \(-2y^3\) when \(y = 4\)
12. \(-5y^3\) when \(y = 3\)
13. \(7xy^2\) when \(x = -2, y = -3\)
14. \(8a^2b\) when \(a = -5, b = -2\)
5.1-10

B Simplify. Express the final answer in exponential form. See Example 2.

15. \(3^2 \cdot 3^5\) 16. \(2^6 \cdot 2^8\) 17. \((-5)^7(-5)^{12}\)
18. \((-3)^3(-3)^5\) 19. \(\left(\frac{2}{7}\right)^4 \left(\frac{2}{7}\right)^8\) 20. \(\left(\frac{9}{11}\right)^7 \left(\frac{9}{11}\right)^3\)
21. \(a^3 \cdot a^5\) 22. \(y^4 \cdot y^6\) 23. \(a^9 \cdot a^7\)
24. \(b^{10} \cdot b^4\) 25. \(r^5 \cdot r^8 \cdot r\) 26. \(t^4 \cdot t^7\)
27. \((3a^2)(5a^3)\) 28. \((4x^3)(2x^5)\) 29. \((-7a^3)(3a^8)\)
30. \((5x^5)(-6x^7)\) 31. \(- (11ab^3)(2a^3b^9)\) 32. \(-(12x^3y^5)(3xy^8)\)

C Simplify. Express the final answer in exponential form. See Example 3.

33. \((x^3)^2\) 34. \((y^4)^3\) 35. \((6^2)^9\)
36. \((5^3)^7\) 37. \((a^1)^3\) 38. \((b^1)^6\)
39. \(- (2^3)^{11}\) 40. \(- (5^4)^{12}\)

D Simplify. Completely evaluate any numbers with exponents for the final answer. See Example 4.

41. \((3a^2)^3\) 42. \((5a^3)^4\) 43. \((2x^5)^6\)
44. \((3x^4)^3\) 45. \((7r^2s)^3\) 46. \((9a^3b)^2\)
47. \((-5x^3y)^2\) 48. \((-7xy^3)^2\) 49. \((-ab^3c)^3\)
50. \((-x^2yz)^4\) 51. \(\left(\frac{r}{s}\right)^3, s \neq 0\) 52. \(\left(\frac{a}{b}\right)^5, b \neq 0\)
53. \(\left(\frac{xy^2}{5}\right)^3\) 54. \(\left(\frac{x^2y}{4}\right)^2\) 55. \(\left(\frac{-3xy^3}{a^2}\right)^2, a \neq 0\)
56. \(\left(\frac{-2ab^3}{c^4}\right)^2, c \neq 0\)

EXTENDING THE CONCEPTS

Simplify using one or more exponent rules.

57. \((-2x^3y)(3x^3y^2z)^2\) 58. \((-3x^3y)(5xy^2z^3)^3\) 59. \((-r^4s)^2(r^3s^5t)^4\)
60. \((-rs^3)^4(r^5s^2t^3)^2\) 61. \((2x^4)(8x^2)\) 62. \((13y^7)(2y^2)\)
63. \((-6a^2)^2(a^3)^4\) 64. \((-8x^4)^2(x^3)^5\) 65. \((25a^2b^3)(-3a^3b^5)\)
66. \((9a^6b^3)(-7a^2b^4)\) 67. \((7s^2t^3)^3\) 68. \((6a^5b)^2\)
69. \(\left(\frac{x}{5}\right)^3\) 70. \(\left(\frac{y}{4}\right)^2\)
Section 5.2

5.2 Integer Exponents and the Quotient Rule

OBJECTIVES
A Use zero as an exponent
B Use negative numbers as exponents
C Use the quotient rule for exponents
D Use combinations of rules

In this section, we will learn the remaining rule for exponents. We will also expand our use of the rules to include negative exponents and zero exponents. Then we will practice simplifying exponential expressions that require using combinations of rules.

A Use Zero as an Exponent

MAIN IDEA

Zero Exponent
$$a^0 = 1$$
(for $$a$$ a real number, $$a \neq 0$$)

In Words
Any nonzero number raised to the zero power equals 1.

Example
$$7^0 = 1$$

Why is any number (except zero) to the zero power equal to 1?

IMPORTANT NOTES

- It doesn’t matter what nonzero number is being raised to the zero power, the result will be 1. For example, $$(-89)^0 = 1$$, $$635^0 = 1$$.
- Zero to the zero power is undefined.
- Watch out for negative signs. If the sign is inside the parentheses, it’s part of the base and the exponent acts on it; if the sign is outside the parentheses—or if there are no parentheses—the sign is not part of the base and the exponent does not act on it. For example, $$(-2)^0 = 1$$, while $$-2^0 = -1$$. 
Evaluate. Assume that all variables represent nonzero real numbers.

**Example 1**

a) \( 5^0 \)  
\( 5^0 = 1 \)

b) \( z^0 \)  
\( z^0 = 1 \)

The zero exponent is acting on the entire base of \(-3\); so the answer is 1.

c) \((-3)^0 \)  
\((-3)^0 = 1 \)

d) \(-3^0 \)  
\(-3^0 = -1 \)

Think of this as “the opposite of \(3^0\).”

**Your Turn 1**

Evaluate. Assume that all variables represent nonzero real numbers.

a) \( x^0 \)

b) \( \left( \frac{2}{3} \right)^0 \)

c) \(-8^0 \)

d) \((-8)^0 \)

For which one is the negative sign part of the base?

Enter your answers in the eText.

**On Your Own 1**

Evaluate. Assume that all variables represent nonzero real numbers.

a) \((1.33)^0 \)

b) \( \left( \frac{-2}{7} \right)^0 \)

c) \( x^0 + y^0 \)

Go to the eText for more practice.

Use Negative Numbers as Exponents

**Main Idea**

<table>
<thead>
<tr>
<th>Negative Exponents</th>
<th>In Words</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a^{-n} = \frac{1}{a^n} ) and ( \frac{1}{a^{-n}} = a^n )</td>
<td>When a base has a negative exponent, take the reciprocal of the base and change the sign of its exponent.</td>
<td>( 3^{-2} = \frac{1}{3^2} ) and ( \frac{1}{3^{-2}} = 3^2 )</td>
</tr>
</tbody>
</table>

(for a real number, \( a \neq 0 \); n an integer)

Why do negative exponents work this way?
Section 5.2

**PROCESS** How to Simplify Negative Exponents

\[ a^{-n} = \frac{1}{a^n} \quad \text{and} \quad \frac{1}{a^{-n}} = a^n \quad (\text{for} \ a \ \text{a real number,} \ a \neq 0; \ n \ \text{an integer}) \]

- Take the **reciprocal** of the base.
- Change the sign of the exponent.

**IMPORTANT NOTES**

- A negative exponent does *not* mean a negative number.
  
  For example, \(3^{-2} = \frac{1}{3^2} = \frac{1}{9}\) is not a negative result.

- Take the reciprocal of only those bases that have a negative exponent. For example, in \(4x^{-5} = \frac{4}{x^5}\), the 4 stays as is because its exponent is +1.

- Remember, if an exponent is not shown, it is understood to be 1.

- All exponent rules work the same for positive and negative exponents.

- When simplifying exponential expressions, leave final answers with positive exponents only.

**EXAMPLE 2**

Simplify by writing with positive exponents. Evaluate if necessary. Assume that all variables represent nonzero real numbers.

a) \(5^{-2}\)  
   - Rewrite the expression. \(5^{-2}\)  
   - **Take the reciprocal** of the base and change the sign of the exponent. \(\frac{1}{5^2}\)  
   - Evaluate. \(\frac{1}{25}\)

b) \(-4^{-2}\)  
   - Rewrite the expression. \(-4^{-2}\)  
   - **Take the reciprocal** of the base and change the sign of the exponent. \(-\frac{1}{4^2}\)  
   - Evaluate. \(-\frac{1}{16}\)

c) \(\frac{1}{7x^{-3}}\)  
   - Rewrite the expression. \(\frac{1}{7x^{-3}}\)  
   - **Take the reciprocal** of the base and change the sign of its exponent. \(\frac{x^3}{7}\)
YOUR TURN 2

Simplify by writing with positive exponents. Evaluate if necessary. Assume that all variables represent nonzero real numbers.

a) \(4^{-3}\)

b) \(6a^{-5}\)

c) \(-\frac{1}{7^{-2}}\)

Take the reciprocal of the base and change the sign of the exponent.

What is the exponent on 6?

Make sure your final answer is still a negative number.

EXAMPLE 3

Simplify by writing with positive exponents. Evaluate if necessary. Assume that all variables represent nonzero real numbers.

a) \(\frac{5^{-3}}{2^{-4}}\)

Rewrite the expression.

Take the reciprocal of each base with a negative exponent and change the sign of its exponent.

Evaluate.

b) \(\left(\frac{2}{7}\right)^{-3}\)

Rewrite the expression.

Take the reciprocal of the base and change the sign of the exponent.

Raise both the 7 and 2 to the power of 3. (power rule for quotients)

Evaluate.

c) \(\frac{3a^{-5}}{b^{-2}}\)

Rewrite the expression.

Take the reciprocal of each base with a negative exponent and change the sign of its exponent.

Do not take the reciprocal of 3. It has an exponent of +1.
Section 5.2

5.2-15

YOUR TURN 3

Simplify by writing with positive exponents. Evaluate if necessary. Assume that all variables represent nonzero real numbers.

a) \( \frac{2^{-6}}{9^{-2}} \)

b) \( \left( \frac{3}{5} \right)^{-2} \)

c) \( \frac{2x^{-5}}{y^{-6}} \)

Note: Take the reciprocal of each base with a negative exponent and change the sign of its exponent.

The entire \( \frac{3}{5} \) is the base in this case.

Does 2 have a positive or negative exponent?

Go to the eText for more practice.

ON YOUR OWN 2

Simplify by writing with positive exponents. Evaluate if necessary. Assume that all variables represent nonzero real numbers.

a) \( x^{-6} \)

b) \( \left( \frac{2}{3} \right)^{-4} \)

c) \( \frac{5x^3y^{-4}}{z^{-2}} \)

d) \( 4^{-1} + 2^{-1} \)

Go to the eText for the online Reading exercise.

Video—See an example similar to On Your Own 2d worked out.

Use the Quotient Rule for Exponents

MAIN IDEA

For any real number \( a \) and any integers \( m \) and \( n \):

\[
\frac{a^m}{a^n} = a^{m-n} \quad \text{(for} \ a \neq 0) \quad \text{In Words: When dividing exponential expressions with the same base, subtract their exponents.}
\]

Example

\[
\frac{2^5}{2^3} = 2^{5-3} = 2^2
\]

Why It Works

\[
\frac{2^5}{2^3} = \frac{2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2}{2^1 \cdot 2^1 \cdot 2^1} = 2^2
\]
# PROCESS

**How to Use the Quotient Rule for Exponents**

\[ \frac{a^m}{a^n} = a^{m-n} \quad \text{(for } a \text{ a real number, } a \neq 0; m, n \text{ integers)} \]

- **Keep** the base.
- **Subtract** the exponents. (exponent of numerator – exponent of denominator)

## IMPORTANT NOTES

- The bases **must** be the same to use this rule.
  - For example, we cannot subtract the exponents in \( \frac{5^4}{2^3} \).
- **Never divide the bases. Keep the base and subtract the exponents.**
  - For example, \( \frac{2^7}{2^3} = 2^4 \), not \( 1^4 \).
- All exponent rules work the same for positive and negative exponents.
- When simplifying exponential expressions, leave final answers with positive exponents only.

## EXAMPLE 4

Simplify. Write your final answer using only positive exponents. Assume that all variables represent nonzero real numbers.

### a)

\[ \frac{6^8}{6^3} \]

Rewrite the expression.

**Keep** the common base and **subtract** the exponents.

\[ \frac{6^{8-3}}{6^3} = \frac{6^5}{6^3} \]

Take the reciprocal of \( x \). The other bases already have positive exponents.

### b)

\[ \frac{x^{-3}}{x^{-6}} \]

Rewrite the expression.

**Keep** the base and **subtract** the exponents.

\[ \frac{x^{-3-(-6)}}{x^{-6}} = \frac{x^3}{x^{-6}} \]

### c)

\[ \frac{15x^2y^6}{10x^5y^2} \]

Rewrite the expression.

Divide out a common factor of 5 from 15 and 10.

**Keep** the common base \( x \) and **subtract** its exponents.

\[ \frac{3x^2y^6}{2x^5y^2} = \frac{3x^{2-5}y^{6-2}}{2} \]

Take the reciprocal of \( x \) and change the sign of its exponent.

\[ \frac{3x^{-3}y^4}{2} = \frac{3y^4}{2x^3} \]

The expressions have the same base and are being divided; therefore, we use the quotient rule.

It’s always numerator exponent minus denominator exponent.

On the “**keep the base**” step, write the base in the numerator.
YOUR TURN 4

Simplify. Write your final answer using only positive exponents. Assume that all variables represent nonzero real numbers.

a) \( \frac{7^3}{7^5} \)

b) \( \frac{a^{-3}}{a^{-9}} \)

c) \( \frac{8r^3s^5}{6r^8s^2} \)

Remember to leave only positive exponents in final answers.

Will the subtracting exponents step be \(-3 - (-9)\) or \(-9 - (-3)\)?

Deal with numerical coefficients first, then with each base separately.

ON YOUR OWN 3

Simplify. Write your final answer using only positive exponents. Assume that all variables represent nonzero real numbers.

a) \( \frac{6^9}{6^{12}} \)

b) \( \frac{3^5m^{-2}}{3^2m^7} \)

c) \( \frac{12x^3y^8}{15x^2y^{10}} \)

d) \( \frac{5^{-1}a^{-2}b^4}{2ab^{-5}} \)

One way to simplify this is to rewrite with positive exponents first, then simplify.

Go to the eText for more practice.

Enter your answers in the eText

Use Combinations of Rules

MAIN IDEA

When simplifying exponential expressions, we have to decide whether to use one rule or more than one rule. The summary chart below shows all of the exponent rules we’ve learned. Use it to help identify which rule(s) to use.

Summary of Exponent Rules

For any real numbers \( a \) and \( b \) and any integers \( m \) and \( n \):

- **Product Rule** \( a^m \cdot a^n = a^{m+n} \)
- **Quotient Rule** \( \frac{a^m}{a^n} = a^{m-n} \) (\( a \neq 0 \))
- **Power Rule for Exponents** \( (a^m)^n = a^{mn} \)
- **Zero Exponent** \( a^0 = 1 \) (\( a \neq 0 \))
- **Power Rule for Products** \( (ab)^n = a^nb^n \)
- **Negative Exponent** \( a^{-n} = \frac{1}{a^n} \) and \( \frac{1}{a^{-n}} = a^n \) (\( a \neq 0 \))
- **Power Rule for Quotients** \( \left( \frac{a}{b} \right)^n = \frac{a^n}{b^n} \) (\( b \neq 0 \))
IMPORTANT NOTES

- For an exponential expression to be simplified, any of the above exponent rules that can be used must be used.
- Also remember to simplify fractions in final answers.

EXAMPLE 5

Simplify using one or more exponent rules. Write your final answer using only positive exponents. Assume that all variables represent nonzero real numbers.

\[
\frac{(4x^2)^{-3}}{2x^3}
\]

Rewrite the expression.

**Negative exponent:** Take the reciprocal of \( (4x^2)^{3} \) and change the sign of its exponent.

**Power rule:** Rewrite \( (4x^2)^{3} \) as \( 4^3x^{2\cdot3} = 64x^6 \).

Multiply 2 times 64.

**Product rule:** Rewrite \( x^5 \cdot x^6 \) as \( x^{5+6} = x^{11} \).

Notice that shifting the base of \( (4x^2)^{3} \) made the exponent of that entire base change from \(-3\) to \(+3\). It did not change the sign of the exponent in the \( x^2 \) that is part of the entire base.

YOUR TURN 5

Simplify using one or more exponent rules. Write your final answer using only positive exponents. Assume that all variables represent nonzero real numbers.

\[(2x^3y^2)^3(5y^{-4})\]

Start with the power rule on the first parentheses.

Next, multiply the coefficients together.

Then use the product rule for the base \( y \).

Go to the eText for more practice.
Section 5.2

ON YOUR OWN 4

Simplify using one or more exponent rules. Write your final answer using only positive exponents. Assume that all variables represent nonzero real numbers.

a) \((x^5)^3 \cdot x^{-2}\)

b) \((3x^{-3}y^2)^{-4}\)

c) \(\frac{(2xy)^{-3}}{(x^{-3}y^2)^2}\)

Go to the eText for the online Reading ✓ exercise.

Application Snapshots—Multiplying with Scientific Notation (Section 5.8, Objective C, Main Idea).

Go to the eText for your instructor’s online assignment.

5.2 Section Summary

Fill in the blanks to make true statements.

<table>
<thead>
<tr>
<th>KEY CONCEPTS</th>
<th>EXAMPLES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero Exponents</td>
<td></td>
</tr>
<tr>
<td>(a^0 = _) (for (a) a real number, (a \neq 0))</td>
<td>(2^0 = 1) ((-8)^0 = _) ((-6)^0 = _)</td>
</tr>
</tbody>
</table>

How to Simplify Negative Exponents

\(a^{-n} = \frac{1}{a^n}\) and \(\frac{1}{a^{-n}} = a^n\) (for \(a\) a real number, \(a \neq 0\); \(n\) an integer)

\(3^{-2} = \frac{1}{3^2}\) and \(\frac{1}{3^2} = 3^{-2}\)

• Take the _________ of the base.
• Change the sign of the _________.

How to Use the Quotient Rule for Exponents

\(\frac{a^m}{a^n} = a^{m-n}\) (for \(a\) a real number, \(a \neq 0\); \(m, n\) integers)

\(\frac{2^5}{2^3} = 2^{--} = 2^2\)

• Keep the _________.
• Subtract the _________.
  (exponent of numerator – exponent of denominator)

Summary of Exponent Rules

For any real numbers \(a\) and \(b\) and any integers \(m\) and \(n\):

Product Rule \(a^m \cdot a^n = a^{m+n}\)

Quotient Rule \(\frac{a^m}{a^n} = a^{m-n}\) (\(a \neq 0\))

Power Rule for Exponents \((a^m)^n = a^{mn}\)

Zero Exponent \(a^0 = 1\) (\(a \neq 0\))

Power Rule for Products \((ab)^n = a^nb^n\)

Negative Exponent \(a^{-n} = \frac{1}{a^n}\) and \(\frac{1}{a^n} = a^n\) (\(a \neq 0\))

Power Rule for Quotients \(\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\) (\(b \neq 0\))
In the following exercises, determine whether the expression simplifies to 0, 1, or 2. Assume that all variables represent nonzero real numbers.

1. \((-3)^0 + (-2)^0\)  
2. \((-6)^0 - (5)^0\)  
3. \((3^2) \cdot (3^{-2})\)  
4. \(x^{-6} \cdot x^2 \cdot x^4\)

5. \(\frac{2^7}{2^8}\)  
6. \(\frac{2^{-3}}{2^{-4}}\)  
7. \(\frac{(x^{-2})^{-2}}{(x^2)^2}\)  
8. \(\frac{0}{x^0}\)

**PRACTICE PROBLEMS**

**A** Evaluate. Assume that all variables represent nonzero real numbers. See Example 1.

9. \(3^0\)  
10. \(9^0\)  
11. \((-4)^0\)  
12. \((-10)^0\)

13. \(y^0\)  
14. \(z^0\)  
15. \(-6^0\)  
16. \(-5^0\)

17. \((3)^0 + (-19)^0\)  
18. \((-14)^0 + (2)^0\)

**B** Simplify by writing with positive exponents. Evaluate if necessary. Assume that all variables represent nonzero real numbers. See Examples 2 and 3.

19. \(6^{-3}\)  
20. \(5^{-2}\)  
21. \(-2^{-3}\)  
22. \(-3^{-4}\)

23. \(6x^{-2}\)  
24. \(5a^{-3}\)  
25. \(\frac{6^2}{3^4}\)  
26. \(\frac{4^2}{2^3}\)

27. \(\left(\frac{3}{4}\right)^{-2}\)  
28. \(\left(\frac{2}{7}\right)^{-3}\)  
29. \(x^{-3}y^5\)  
30. \(x^{-2}y^4\)

31. \(\frac{2}{x^5}\)  
32. \(\frac{3}{y^4}\)  
33. \(\frac{2a^{-1}}{4b^{-3}}\)  
34. \(\frac{6x^{-3}}{15y^{-4}}\)

35. \(\frac{8a^{-2}b^3}{6x^8y^{-6}}\)  
36. \(\frac{12x^{-3}y^2}{10a^4b^{-5}}\)  
37. \(3^{-2} + 2^{-2}\)  
38. \(5^{-1} + 2^{-3}\)

**C** Simplify. Write your final answer using only positive exponents. Assume that all variables represent nonzero real numbers. See Example 4.

39. \(\frac{5^7}{5^2}\)  
40. \(\frac{3^5}{3^2}\)  
41. \(\frac{x^6}{x^2}\)  
42. \(\frac{x^{12}}{x^5}\)

43. \(\frac{y^8}{y^8}\)  
44. \(\frac{a^{11}}{a^{11}}\)  
45. \(\frac{-6m^5}{-3m}\)  
46. \(\frac{-12x^4}{-4x}\)

47. \(\frac{3x^8}{9x^{12}}\)  
48. \(\frac{2m^4}{8m^{11}}\)  
49. \(\frac{5x^2}{x^{17}}\)  
50. \(\frac{9b^7}{b^{15}}\)
Section 5.2

51. \( \frac{x^{-3}}{x^5} \)  
52. \( \frac{z^{-2}}{z^7} \)  
53. \( \frac{y^{-3}}{x^4} \)  
54. \( \frac{ab^{-2}}{a^5} \)  
55. \( \frac{4^2z^{-2}}{4^2z^{-6}} \)  
56. \( \frac{3^4a^{-6}}{3^5a^{-8}} \)  
57. \( \frac{2^{-3}a^{-5}b^6}{2^{-7}a^6b^{-7}} \)  
58. \( \frac{4^{-2}x^{-2}y^4}{4^{-3}x^4y^{-3}} \)  

**D** Simplify using one or more exponent rules. Write your final answer using only positive exponents. Assume that all variables represent nonzero real numbers. See Example 5.

59. \( (2a^2b)^2(3a^4) \)  
60. \( (3xy^2)^3(8x^3) \)  
61. \( (-3x^{-2}y^2)^{-3} \)  
62. \( (-5a^{-3}b^2)^{-2} \)  
63. \( \frac{(-3x^3)^2}{2x^5} \)  
64. \( \frac{(2x^3)^4}{3x^4} \)  
65. \( \left( \frac{r^2s}{rs} \right)^{-2} \)  
66. \( \left( \frac{mn^3}{m^2n} \right)^{-3} \)  
67. \( (-2x^{-3}y)^{-3}(6x^{-2}y^3)^{-2} \)  
68. \( (3m^{-2}n)^{-1}(5m^3n^2)^{-3} \)  
69. \( \frac{a^{-3}m^2}{s^5b^3} \)  
70. \( \frac{(a^{-2}b^3)^4}{(3a^5b^{-3})^3} \)  

**APPLICATION SNAPSHOTS**

**Multiply with Scientific Notation**  Scientific notation is used to express very large and very small numbers by expressing the numbers as the product of a numerical coefficient and a power of ten. When calculations are done in scientific notation, the base of 10 is kept and an exponent rule is used to determine the final exponent.

Scientific notation for 5000:

\[ 5 \times 10^3 \]

- **numerical coefficient**
- **power of 10**

For Exercises 71–74, multiply the numerical coefficients, then use an exponent rule to determine the final exponent on 10. (Note: The \( \times \) indicates multiplication.)

71. \( (2 \times 10^3)(4 \times 10^5) \)  
72. \( (3 \times 10^2)(2 \times 10^4) \)  
73. \( (4 \times 10^{36})(2 \times 10^9) \)  
74. \( (3 \times 10^{25})(3 \times 10^{20}) \)  

**CHAPTER ✓**

Simplify. Evaluate if necessary. Write your final answer using only positive exponents. Assume that all variables represent nonzero real numbers.

75. \( 2y^5 \text{ when } y = 3 \)  
76. \( -6^2 \)  
77. \( -8a^2b^3(2a^3b^9) \)  
78. \( \left( \frac{-3m^2n}{s^3} \right)^2 \)  
79. \( \frac{8^6x^{-3}y^6}{8^3x^5y^{-2}} \)  
80. \( (-4x^{-1}y^3)^{-2}(2x^2y^2)^{-3} \)  
81. \( -12^0 + 6^0 \)  
82. \( -2^{-1} + 2^{-2} \)
5.3-22  Chapter 5

5.3  Introduction to Polynomials

OBJECTIVES
A  Review the vocabulary of algebraic expressions
B  Define polynomial, monomial, binomial, trinomial, and degree of a polynomial
C  Evaluate polynomials
D  Simplify polynomials by combining like terms

VOCABULARY

<table>
<thead>
<tr>
<th>term</th>
<th>degree of a polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>numerical coefficient</td>
<td>descending powers</td>
</tr>
<tr>
<td>degree of a term</td>
<td></td>
</tr>
<tr>
<td>polynomial</td>
<td></td>
</tr>
<tr>
<td>monomial</td>
<td></td>
</tr>
<tr>
<td>binomial</td>
<td></td>
</tr>
<tr>
<td>trinomial</td>
<td></td>
</tr>
</tbody>
</table>

In this section, we will learn about a special kind of algebraic expression called a polynomial. Much of the vocabulary and skills presented earlier for algebraic expressions will be reviewed and used here. This section will prepare us to add, subtract, multiply, and divide polynomials in future sections.

A  Review the Vocabulary of Algebraic Expressions

MAIN IDEA

The parts of an algebraic expression that are separated by addition or subtraction are called terms. The numerical factor of a term is called its numerical coefficient. For example:

\[3x^2 - 5x + 2xy^2 + 4\]

4 terms: \(3x^2, -5x, 2xy^2, 4\)
numerical coefficients: 3, -5, 2, 4

The degree of a term is the sum of the exponents of its variables. For example:

\[3x^2 - 5x + 2xy^2 + 4\]

degree of \(3x^2\): 2 because the exponent is 2
degree of \(-5x\): 1 because the exponent is 1
degree of \(2xy^2\): 3 because the sum of the exponents is 1 + 2 = 3
degree of 4: 0 because there is no variable (which can be thought of as a zero exponent, \(4x^0\))

IMPORTANT NOTES

- When a term does not have a variable, its degree is 0 because having no variable is the same as having a variable with an exponent of 0. For example: \(4 \rightarrow 4x^0 \rightarrow 4 \cdot 1 \rightarrow 4\)
- When no exponent is shown on a variable, the exponent is understood to be 1.
Section 5.3

**Example 1**

Complete the table. Place each term of the given expression in the table and identify the term’s coefficient and the degree of the term.

\[3x^5 - 2x^2y + 6x^4 - 5\]

<table>
<thead>
<tr>
<th>Term</th>
<th>3x^5</th>
<th>-2x^2y</th>
<th>6x^4</th>
<th>-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>3</td>
<td>-2</td>
<td>6</td>
<td>-5</td>
</tr>
<tr>
<td>Degree of term</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

Remember, when there’s no variable, the degree is 0.

**Your Turn 1**

Complete the table. Place each term of the given expression in the table and identify the term’s coefficient and the degree of the term.

\[-2x + 6x^3 - 3x^4y + 8\]

<table>
<thead>
<tr>
<th>Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
</tr>
<tr>
<td>Degree of term</td>
</tr>
</tbody>
</table>

**On Your Own 1**

For each expression identify the terms, the coefficient of each term, and the degree of each term.

a) \[-9x^3 - 2x^2 + 5x + 16\]  
b) \[6x^2 + 4x^3y^2\]  
c) \[-\frac{x}{2} + 2x^3 - 5\]

**B Define Polynomial, Monomial, Binomial, Trinomial, and Degree of a Polynomial**

**Main Idea**

Here are some vocabulary words we need to know about the special type of expression called a polynomial.

**Definition**

A polynomial in \(x\) is a sum of terms of the form \(ax^n\) where \(a\) is a real number and \(n\) is a whole number.

A monomial is a polynomial with exactly one term.

A binomial is a polynomial with exactly two terms.

A trinomial is a polynomial with exactly three terms.

**Example**

\[-6x^7 + 3x^2 - 5x + 4\]  
\[-6x^3\]  
\[x + 3\]  
\[\frac{2}{3}x^3 - 5x + 1\]
The degree of a polynomial is equal to the degree of its highest-degree term. 

\[-6x^7 + 3x^2 - 5x + 4\] has degree 7.

Polynomials are usually written with descending powers, meaning that going from left to right, the degrees of the terms are decreasing.

All of the above examples are written with descending powers.

### IMPORTANT NOTES

- Because polynomials are of the form \(ax^n\), where \(n\) is a whole number (non-negative integer), they never have variables in denominators.
- The variable in a polynomial doesn’t have to be \(x\). It can be any letter.
- Look at the prefix to tell you how many terms a polynomial has: monomial \(\text{mono}\) means one, binomial \(\text{bi}\) means two, trinomial \(\text{tri}\) means three, polynomial \(\text{poly}\) means many.
- A polynomial with more than three terms doesn’t have a special name and is simply called a polynomial.
- Polynomials can have more than one variable. For example, \(3x^2 - 2xy + 5y^2\) is a multivariable polynomial.

### EXAMPLE 2

For the polynomial \(-2x^2 + 4x^3 + 5x\):

a) Rewrite the polynomial with descending powers.

b) Determine the degree of the polynomial.

c) Identify the polynomial as a monomial, a binomial, a trinomial, or none of these.

a) The polynomial in descending powers is  

\[4x^3 - 2x^2 + 5x.\]

degree: 3 2 1

b) The degree of the polynomial equals the degree of its highest-degree term: 3.

c) The polynomial has 3 terms; therefore, it’s called a trinomial.

### YOUR TURN 2

For the polynomial \(-3x + 8x^8 + 5 - 2x^3\):

a) Rewrite the polynomial with descending powers.

b) Determine the degree of the polynomial.

c) Identify the polynomial as a monomial, a binomial, a trinomial, or none of these.

a) The polynomial in descending powers is ________________.

b) The degree of the polynomial equals the degree of its highest-degree term: ________.

c) The polynomial has _______ terms; therefore, it’s called ____________.

Go to the eText for more practice.
ON YOUR OWN 2

Rewrite each polynomial with descending powers; determine the degree of the polynomial; and identify the polynomial as a monomial, a binomial, a trinomial, or none of these.

a) \(27 - 7x^8\)  
b) \(43x\)  
c) \(16 - 2x^4 + 3x^2\)

Go to the eText for the online Reading ✓ exercise.

C Evaluate Polynomials

MAIN IDEA

If we need to evaluate a polynomial for a given value of the variable, we will use the same method we used when evaluating algebraic expressions earlier in the book.

PROCESS  How to Evaluate a Polynomial

1. **Replace** the variable with the given value.
2. **Simplify** the result using the correct order of operations.

### EXAMPLE 3

Evaluate the polynomial for the given value.

\(-3x^2 + 6x - 7; \ x = -2\)

**Rewrite the expression.**  
\(-3x^2 + 6x - 7\)

**Replace** \(x\) with \(-2\).

\(-3(-2)^2 + 6(-2) - 7\)

**Simplify.** Evaluate the exponent, \((-2)^2 = 4.\)

\(-3 \cdot 4 + 6(-2) - 7\)

**Multiply** \(-3 \cdot 4\).

\(-12 + 6(-2) - 7\)

**Multiply** \(6(-2)\).

\(-12 - 12 - 7\)

**Subtract.**

\(-24 - 7\)

\(-31\)

YOUR TURN 3

Evaluate the polynomial for the given value.

\(-4x^3 + 6x^2 - 10; x = -3\)

**Put** \(-3\) in parentheses to show that the exponents act on the entire base of \((-3).\)

**Use the correct order of operations.**

Enter your answer in the eText
5.3-26

ON YOUR OWN 3

Evaluate the following polynomials for \( x = -5 \) and \( y = 6 \).

a) \( x^2 + 3x - 6 \)

b) \(-2y^3 - 2y + 8\)

c) \(-x^3 - 3x^2 + 2x + 1\)

Simplify Polynomials by Combining Like Terms

MAIN IDEA

To simplify a polynomial containing like terms, we will use the same method we used when simplifying algebraic expressions earlier in our studies.

\[
\frac{5x^2 + 3x^2}{8x^2 - 7x} - 7x
\]

PROCESS How to Simplify Polynomials by Combining Like Terms

- **Identify** the like terms.
- **Combine** their numerical coefficients.

IMPORTANT NOTES

- Like terms have the same variables raised to the same powers.
- Remember, when combining numerical coefficients, we’re really using the Distributive Property. For example:

\[
\begin{align*}
6x + 3x & \quad 2x - 5x \\
(6 + 3)x & \quad (2 - 5)x \\
9x & \quad -3x \\
\end{align*}
\]

\[
\begin{align*}
-3x + 10x & \\
( -3 + 10)x & \\
7x & \\
\end{align*}
\]

EXAMPLE 4

Simplify by combining like terms.

Identify like terms. \(-7x^2 + 2x^2 + 6x^2 + 5\)

Combine their coefficients, \(-7 + 2 + 6 = 1\). \(1x^2 + 5\) or \(x^2 + 5\)
Section 5.3

YOUR TURN 4

Simplify by combining like terms. \(3y^6 - 12y^6 + 6y^3 - y^6\)

Combine all \(y\) terms that share the same exponent.

ON YOUR OWN 4

Simplify, if possible, by combining like terms.

\[\begin{align*}
a) & \quad 3ab - 10ab + 2ab \\
b) & \quad m^2 - 3mn + 5m^2 \\
c) & \quad \frac{1}{3}x^2 - 2x^2 \\
\end{align*}\]

Polynomials can have more than one variable.

5.3 Section Summary

Fill in the blanks to make true statements.

**KEY CONCEPTS**

- A(n) \(\text{in } x\) is a sum of terms of the form \(ax^n\) where \(a\) is a real number and \(n\) is a whole number.
- A(n) \(\text{is a polynomial with exactly one term.}\)
- A(n) \(\text{is a polynomial with exactly two terms.}\)
- A(n) \(\text{is a polynomial with exactly three terms.}\)

**EXAMPLES**

<table>
<thead>
<tr>
<th>(\text{KEY CONCEPTS})</th>
<th>(\text{EXAMPLES})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{in } x)</td>
<td>(-6x^7 + 3x^2 - 5x + 4)</td>
</tr>
<tr>
<td>(\text{is a polynomial with exactly one term.})</td>
<td>(-6x^3)</td>
</tr>
<tr>
<td>(\text{is a polynomial with exactly two terms.})</td>
<td>(x + 3)</td>
</tr>
<tr>
<td>(\text{is a polynomial with exactly three terms.})</td>
<td>(\frac{2}{3}x^3 - 5x + 1)</td>
</tr>
</tbody>
</table>

How to Evaluate a Polynomial

- \(\text{Simplify}\) the variable with the given value.
- \(\text{Simplify}\) the result using the correct ________ ________ ________.

\[\begin{align*}
2x^2 + 6x & \quad 2(\_\_)^2 + 6(\_\_) \\
2x^2 + 6x; \ x = 3 & \quad 2 \cdot \_\_ + 6(3) \\
& \quad 18 + \_\_ \\
& \quad 36
\end{align*}\]
5.3-28

How to Simplify Polynomials by Combining Like Terms

- Identify the ________ ________.
- ________ their numerical coefficients.

Vocabulary  term, numerical coefficient, degree of a term, polynomial, monomial, binomial, trinomial, degree of a polynomial, descending powers, multivariable polynomial

5.3 Additional Exercises

Reading ✓

Choose the best choice to complete the sentence from the word bank.

Word Bank:  terms  polynomial  order of operations
numerical coefficient  like terms  trinomial
replace  binomial  combining like terms
degree of term  descending powers  variables

1. The __________ of the term $-6x^2y$ is $-6$.
2. In the expression $x^2 - 2x + 3$, the ________ are $x^2$, $-2x$, and 3.
3. A polynomial with exactly three terms is a ________.
4. A polynomial with exactly two terms is a ________.
5. When evaluating a polynomial for a given value, we need to ________ the variable by that given value.
6. When evaluating a polynomial for a given value, we need to use the correct __________ to simplify.
7. Like terms have the same ________ raised to the same powers.
8. $4xy - 6xy + 8xy = 6xy$ is an example of ____________.

Practice Problems

For each expression, complete the table by identifying the terms, the numerical coefficient of each term, and the degree of each term. See Example 1.

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th>Degree of term</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2x^3 + 3x + 5x^2 - 8$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th>Degree of term</th>
</tr>
</thead>
<tbody>
<tr>
<td>$12 - 8x^7 + 3x^2 - 2x^5$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Section 5.3

11. \( \frac{1}{2}m^2 - 6m^5 + 4m \)

12. \( \frac{8}{7}m^2 - 20m^4 - 5 \)

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th>Degree of term</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

13. \( 5x - 32x^6 + 6x^3y \)

14. \( 13x - 27x^9 + 21xy \)

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th>Degree of term</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

B For each of the following: (See Example 2.)

a) Rewrite the polynomial with descending powers.
b) Determine the degree of the polynomial.
c) Identify the polynomial as a monomial, a binomial, a trinomial, or none of these.

15. \(-5x^2 + 16x^4 + 2\)
16. \(-2x^2 - 13x^4 + 8\)
17. \(-12z^5\)
18. \(13z^7\)
19. \(-2x^3 + x^2 - 6x^5 - x + 1 + 3x^4\)
20. \(x^3 - 2x^2 - 12x^5 - x + 5 - 11x^4\)
21. \(10 - 29x + 10x^2\)
22. \(-1 + 3x + 18x^2\)
23. \(-1 - a^4\)
24. \(-1 - a^8\)
25. \(-x^2 - 16 - x^4 + \frac{1}{2}x^3\)
26. \(\frac{2}{3}y^2 + 15 + x^4 - x^3\)

C Evaluate the polynomial for the given value. See Example 3.

27. \(3x - 9; x = 2\)
28. \(2x - 7; x = 3\)
29. \(6a + 12; a = -4\)
30. \(7a + 14; a = -5\)
31. \(r^2 - 4r - 6; r = 8\)
32. \(r^2 - 4r - 5; r = 7\)
33. \(-2x^2 + 5x + 3; x = -3\)
34. \(-3x^2 - 5x + 2; x = -2\)
35. \(-x^3 - 2x^2 + 4x - 6; x = -4\)
36. \(-x^3 + 3x^2 - 6x + 1; x = -5\)
37. \(1.3m^2 - 2.7m + 6; m = -1.2\)
38. \(2.1a^2 - 1.7a - 4; a = -1.1\)
D. Simplify by combining like terms. Write your final answer with descending powers. See Example 4.

39. \(18a - 12a\)  
40. \(17a - 11a\)

41. \(12m^4 + 18m^4\)  
42. \(6y^4 + 13y^4\)

43. \(2x^2 - 11 + 6x^2 + 7\)  
44. \(5x^2 + 16 + 3x^2 - 12\)

45. \(-8y^2 - 6y^2 + 14 - 12y^2\)  
46. \(-19y^2 - 3y^2 + 17 - 8y^2\)

47. \(-2x - 4 - x^2 + 12 - 5x + 6x^2\)  
48. \(-5x - 6 - x^2 - 16 - 3x + 7x^2\)

49. \(-2x^7 - 5x^5 + 6\)  
50. \(-x^6 + 4x^3 - 2\)

51. \(\frac{1}{3}y^2 - \frac{1}{4} + 5x - \frac{5}{6}x^2 + \frac{3}{8}\)  
52. \(\frac{1}{5}x^2 - \frac{3}{4} + 6x - \frac{3}{10}x^2 + \frac{1}{2}\)

53. \(6ab - 10ab + 2ab\)  
54. \(-4ab + 6ab - 9ab\)

55. \(m^2 + 2mn - mn + n^2\)  
56. \(4m^2 - 6mn - 3mn - 4n^2\)

APPLICATION SNAPSHOTS

57. **Area** The area of a garden with width \(x\) feet and length \((x + 3)\) feet is given by the polynomial \((x^2 + 3x)\). Use the polynomial to find the area (in square feet) of the garden when the width is 10 feet.

58. **Area** The area of a garden with width \(x\) feet and length \((x + 4)\) feet is given by the polynomial \((x^2 + 4x)\). Use the polynomial to find the area (in square feet) of the garden when the width is 20 feet.

59. **Stopping Distance of a Car** The stopping distance (in feet) of a car traveling at a speed of \(v\) miles per hour is approximated by the polynomial \(0.05v^2 + 2.25v\). The polynomial takes into account the reaction time of the driver and the braking distance. Use the polynomial to estimate the stopping distance of a car traveling 30 miles per hour.

60. **Stopping Distance of a Car** The stopping distance (in feet) of a car traveling at a speed of \(v\) miles per hour is approximated by the polynomial \(0.05v^2 + 2.25v\). The polynomial takes into account the reaction time of the driver and the braking distance. Use the polynomial to estimate the stopping distance of a car traveling 60 miles per hour.

CHAPTER ✓

Simplify. Express final answers using positive exponents. Assume that all variables represent nonzero real numbers.

61. \(m \cdot m^6 \cdot m^7\)  
62. \((-4xy^3)^2\)  
63. \(\left(\frac{ab^2}{4}\right)^3\)

64. \(\frac{2m^3}{8m^8}\)  
65. \(\left(\frac{x^2}{xy^3}\right)^{-2}\)  
66. \((-2m^2n^{-2})^{-3}\)

Evaluate the following polynomials for the given value.

67. \(-2x^2 + x + 3; x = -4\)  
68. \(x^3 + 2x^2 - 3x - 7; x = -1\)
OBJECTIVES

A Add polynomials
B Subtract polynomials
C Add and subtract more than two polynomials
D Add and subtract multivariable polynomials

We are now ready to focus on adding and subtracting polynomials. Then in the remaining sections, we will learn to multiply and divide polynomials and put all of our exponent and polynomial skills together to solve application problems.

Add Polynomials

MAIN IDEA

We already know how to add polynomials because adding polynomials is a special case of simplifying expressions that we learned earlier in the book.

**Simplify the Expression**

\[
2(x + 4) + 3(5x - 2) = 2x + 8 + 15x - 6 + 17x + 2
\]

**Add the Polynomials**

\[
(x + 5) + (x - 3) = x + 5 + x - 3 + 2x + 2
\]

Because the multipliers outside the parentheses are always 1 when we add polynomials, the end result is that we can remove the parentheses and just combine like terms.

PROCESS

To add polynomials, combine their like terms.
ON YOUR OWN 1

Find the sums for the following polynomials.

a) Add horizontally: \((2x^5 - 4x^2 + x) + (-4x^5 + 2x^2 + 10x)\)

b) Add vertically: \((-6x^2 + 2x - 8) + (4x^2 - 5x + 2)\)

c) Add \((-6x^2 - 5)\) to \((7x^4 + 3x^3 - 2x + 1)\) using whichever method you prefer.

Go to the eText for more practice.
Subtract Polynomials

MAIN IDEA

As with adding polynomials, subtracting polynomials is a special case of simplifying expressions.

Simplify the Expression

\[2(x + 4) - 3(5x - 2)\]

\[2x + 8 - 15x + 6\]

\[-13x + 14\]

\[\text{distribute}\]

\[\text{combine like terms}\]

\[\text{Order matters for subtraction.}\]

Subtract the Polynomial

\[x - 3\] from \[4x + 5\]

\[4x + 5 - (x - 3)\]

\[3x + 8\]

The parentheses around the subtracted polynomial are very important. They indicate that the negative sign gets distributed to each term. In other words, the entire polynomial is being subtracted, not just the first term. Because distributing this negative sign makes every term in the polynomial change sign, the end result is that when we subtract a polynomial, we’re really adding its opposite.

PROCESS

To subtract a polynomial, add its opposite.

IMPORTANT NOTES

• Whether subtracting horizontally or vertically, change to addition and change the sign of each term in the subtracted polynomial.

• The vertical method for subtraction is an important skill for dividing polynomials later in this chapter.

EXAMPLE 2

Subtract \((-x^2 - 5x + 2)\) from \((3x^3 + x^2 + 2x - 6)\) horizontally and then vertically.

Horizontal:

Write as subtraction.

\[(3x^3 + x^2 + 2x - 6) - (-x^2 - 5x + 2)\]

Add the opposite. Distribute, then combine like terms.

\[3x^3 + x^2 + 2x - 6 + x^2 + 5x - 2\]

\[3x^3 + x^2 + 2x - 6 + x^2 + 5x - 2\]

\[\frac{3x^3 + 2x^2 + 7x - 8}{3x^3 + 2x^2 + 7x - 8}\]

Vertical:

Set up as though subtracting numbers vertically, making sure you line up like terms.

Add the opposite. Distribute, then combine like terms.

\[3x^3 + x^2 + 2x - 6\]

\[-(-x^2 - 5x + 2)\]

\[3x^3 + x^2 + 2x - 6\]

\[+ x^2 + 5x - 2\]

\[\frac{3x^3 + 2x^2 + 7x - 8}{3x^3 + 2x^2 + 7x - 8}\]
YOUR TURN 2

Subtract \((-5y^3 - 2y^2)\) from \((13y^3 + 7y^2 - 10y)\) horizontally and then vertically.

Horizontal:
Distribute, then combine like terms.

Vertical:

Which one is being subtracted? Be careful of the order.
Remember to leave a gap under the \(y\) term.

Go to the eText for more practice.

ON YOUR OWN 2

Subtract the following polynomials.

a) Subtract \((-6x^2 - 3x - 15)\) from \((4x^2 + 2x - 6)\) horizontally.

b) Subtract vertically: \((-2p^2 - 3p - 17) - (5p^2 + 8p - 11)\)

c) Subtract \((8r^3 - 3r + 14)\) from \((-3r^3 - 6r^2 + 2r - 8)\) using whichever method you prefer.

Go to the eText for the online Reading ✓ exercise.

Add and Subtract More Than Two Polynomials

MAIN IDEA
When adding and subtracting more than two polynomials, we will use the same methods we just learned. These problems are more complicated to keep track of. Make sure you keep your work neat and organized.

PROCESS How to Add and Subtract More Than Two Polynomials

- Distribute to remove parentheses and to change any subtractions to “add the opposite.”
- Combine like terms.

IMPORTANT NOTES

- Anytime a polynomial is being subtracted, put parentheses around it to remind yourself to distribute the negative sign.
- If you’re given a word statement to translate, remember that order matters when subtracting.
5.4-35

Section 5.4

EXAMPLE 3

Perform the indicated operations horizontally and then vertically.

\[(5x^2 - 30x + 13) - (7x^2 - 8x + 9) + (6x^2 + 2x - 22)\]

**Horizontal:**

Rewrite the problem.

\[5x^2 - 30x + 13 - 7x^2 + 8x - 9 + 6x^2 + 2x - 22\]

Distribute to change any subtraction to “add the opposite.”

Combine like terms.

\[4x^2 - 20x - 18\]

**Vertical:**

Set up vertically.

\[
\begin{align*}
5x^2 - 30x + 13 \\
-(7x^2 - 8x + 9) \\
6x^2 + 2x - 22
\end{align*}
\]

Distribute to change any subtraction to “add the opposite.”

Combine like terms.

\[4x^2 - 20x - 18\]

YOUR TURN 3

Perform the indicated operations horizontally and then vertically.

\[(10u^2 - 6u + 15) + (-20u^2 + u - 12) - (5u^2 - 5)\]

**Horizontal:**

Distribute, then combine like terms.

**Vertical:**

Remember to leave a gap for any missing like terms.

Go to the eText for more practice.

Enter your answer in the eText

ON YOUR OWN 3

Perform the indicated operations using your preferred method.

\[
\begin{align*}
a) \quad -(m^2 + 6m - 3) & - (15m^2 - 11m + 8) + (11m^2 - 3m + 2) \\
b) \quad (a^2 + 1) & - (a^2 - 3) + (5a^2 + 2a - 9)
\end{align*}
\]

Go to the eText for the online Reading exercise.
Add and Subtract Multivariable Polynomials

**MAIN IDEA**
A multivariable polynomial has at least two different variables, such as $6x^2 + 4xy - 2y^2$. The process for adding and subtracting these polynomials is the same as for single-variable polynomials. It’s just a bit trickier to identify the like terms.

**PROCESS** How to Add and Subtract Multivariable Polynomials
- **Distribute** to remove parentheses and to change any subtractions to “add the opposite.”
- **Combine like terms.**

**IMPORTANT NOTES**
- Like terms contain the same variables raised to the same powers.
- The order of the variables in like terms does not matter because they are multiplied together. For example, $5x^2y$ and $-yx^2$ are like terms.
- Anytime a polynomial is being subtracted, put parentheses around it to remind yourself to distribute the negative sign.

**EXAMPLE 4**
Perform the indicated operation.

$$ (m^2 + 3mn - n^2) + (5m^2 - 4mn - 25n^2) $$

Rewrite the problem.

$$ (m^2 + 3mn - n^2) + (5m^2 - 4mn - 25n^2) $$

**Distribute** to remove parentheses.

$$ m^2 + 3mn - n^2 + 5m^2 - 4mn - 25n^2 $$

**Combine like terms.** Identify, then combine their coefficients.

$$ m^2 + 3mn - n^2 + 5m^2 - 4mn - 25n^2 $$

Try this one vertically to see if you get the same answer.

$$ 6m^2 - mn - 26n^2 $$

**YOUR TURN 4**
Perform the indicated operation.

$$ (9a^2b - 4a^2 + 2b) - (4a^2b - 3a^2 - 7b) $$

Distribute, then combine like terms.

$$ 6a^2b - 4a^2 + 2b - 4a^2b + 3a^2 + 7b $$

$$ 2a^2 + 9b $$
Section 5.4

ON YOUR OWN 4

Perform the indicated operation.

a) \((6p^2 - 5pq - q^2) - (-7p^2 - 4pq + 5q^2)\)

b) \((7xy^3 - 6x + 3y) + (2xy^3 - 13x + 8y)\)

5.4 Additional Exercises

Choose the best response.

1. The sum of \((-8a^2 + 3a + 6)\) and \((3a^2 - 8)\) can be set up vertically as __________.

   a) \(-8a^2 + 3a + 6\)
   b) \(3a^2 - 8\)
   c) Both answers are correct.
5.4-38

2. The sum of \((-5x^3 + 3x)\) and \((-3x^3 + 5x - 15)\) can be set up vertically as \_________.
   a) \(-5x^3 + 3x\)
   \hspace{1cm} \(-3x^3 + 5x - 15\)
   \hspace{1cm} \underline{\phantom{\phantom{-5x^3 + 3x}}}
   \hspace{1cm} \underline{\phantom{\phantom{-3x^3 + 5x - 15}}}
   b) \(-5x^3 + 3x\)
   \hspace{1cm} \(-3x^3 + 5x - 15\)
   \hspace{1cm} \underline{\phantom{\phantom{-5x^3 + 3x}}}
   \hspace{1cm} \underline{\phantom{\phantom{-3x^3 + 5x - 15}}}
   c) Both answers are correct.

3. Subtracting \((4m^2 - 5)\) from \((-m^2 + 10)\) can be set up horizontally as \_________.
   a) \((-m^2 + 10) - (4m^2 - 5)\)
   b) \((4m^2 - 5) - (-m^2 + 10)\)
   c) Both answers are correct.

4. Subtracting \((8x^3 - 5x)\) from \((7x^3 + 2x)\) can be set up horizontally as \_________.
   a) \((8x^3 - 5x) - (7x^3 + 2x)\)
   b) \((7x^3 + 2x) - (8x^3 - 5x)\)
   c) Both answers are correct.

5. Subtracting \((-5x^4 - 6x^2)\) from the sum of \((3x^4 - 14x^2)\) and \((5x^4 - 2x^2)\) can be written
   as \_________.
   a) \((-5x^4 - 6x^2) - (3x^4 - 14x^2) + (5x^4 - 2x^2)\)
   b) \((3x^4 - 14x^2) + (5x^4 - 2x^2) - (-5x^4 - 6x^2)\)
   c) Both answers are correct.

6. Subtracting \((-2x^3 + x)\) from the sum of \((-2x^3 - 3x)\) and \((3x^3 + 15x)\) can be written as \_________.
   a) \((-2x^3 - 3x) + (3x^3 + 15x) - (-2x^3 + x)\)
   b) \((-2x^3 + x) - (-2x^3 - 3x) + (3x^3 + 15x)\)
   c) Both answers are correct.

7. To write \((3p^2q^2 - 10pq - 5) - (p^2q^2 - pq)\) in a vertical setup, once the subtraction has been changed
   to “add the opposite,” it looks like \_________.
   a) \(3p^2q^2 - 10pq - 5\)
   \hspace{1cm} \(-p^2q^2 + pq\)
   b) \(3p^2q^2 - 10pq - 5\)
   \hspace{1cm} \(-p^2q^2 - pq\)
   c) \(3p^2q^2 - 10pq - 5\)
   \hspace{1cm} \(p^2q^2 - pq\)

8. To write \((13x^3y - 2xy + 6) - (2x^3y + xy)\) in a vertical setup, once the subtraction has been changed
   to “add the opposite,” it looks like \_________.
   a) \(13x^3y - 2xy + 6\)
   \hspace{1cm} \(-2x^3y + xy\)
   b) \(13x^3y - 2xy + 6\)
   \hspace{1cm} \(-2x^3y - xy\)
   c) \(13x^3y - 2xy + 6\)
   \hspace{1cm} \(2x^3y - xy\)
Section 5.4

PRACTICE PROBLEMS

A Find the sum. Add horizontally. See Example 1.

9. \((3x - 6) + (2x - 3)\)  
10. \((6x - 5) + (8x - 2)\)  
11. \((-7x^2 + 6) + (2x^2 + 3)\)  
12. \((-12x^2 + 5) + (3x^2 + 11)\)  
13. \((6x^3 - 7x + 12) + (-8x^2 - 4)\)  
14. \((7x^2 - 2x + 9) + (-9x^2 + 16)\)  
15. \((2x^3 - 11x^2 - 12) + (-5x^3 - 5x^2 + 6x)\)  
16. \((3x^3 - 14x^2 - 8) + (-7x^3 - 9x^2 + 12x)\)  
17. \(\left(\frac{1}{2}m^2 - \frac{2}{3}m + \frac{6}{7}\right) + \left(-\frac{3}{4}m^2 + \frac{1}{6}m - \frac{1}{2}\right)\)  
18. \(\left(-\frac{2}{5}p^2 + \frac{1}{2}p - \frac{1}{4}\right) + \left(\frac{1}{3}p^2 + \frac{5}{6}p + \frac{1}{5}\right)\)  

19. Find the sum of \((p^4 - 7p^2 + p)\) and \((-6p^2 + 11 - 6p^4 + 2p^3)\).
20. Find the sum of \((-12z^4 - 6z^2 + z)\) and \((7z^2 - 16 + z^4 - 3z^3)\).

Find the sum. Add vertically. See Example 1.

21. \(-12p^3 - p^2\) 
\(-p^3 + 6p^2\)  
22. \(5m^3 - 9m^2\) 
\(-6m^3 + 4m^2\)  
23. \(8x^2 - 3x - 6\) 
\(3x^2 - 12x + 8\)  
24. \(-2x^2 - x - 11\) 
\(-x^2 + 3x + 17\)

B Find the difference. Subtract horizontally. See Example 2.

25. \((6x - 8) - (-4x + 3)\)  
26. \((-8x - 7) - (2x - 15)\)  
27. \((3x^2 - 5x + 7) - (-2x - 24)\)  
28. \((-7x^2 + 4x - 6) - (3x - 21)\)  
29. \((2x^2 - 5x + 6) - (-3x^2 + 2x - 11)\)  
30. \((-5x^2 + 4x + 13) - (-2x^2 + 7x - 14)\)  
31. \((7a^3 - 10a + 4) - (-12 - 2a^3 + 3a^2)\)  
32. \((-4a^3 + 3a - 15) - (15 + 6a^3 + 5a^2)\)  
33. \((-0.9x^2 - 0.3x - 1.6) - (0.2x^2 - 1.05)\)  
34. \((1.2x^2 - 0.2x - 2.9) - (-0.6x^2 + 2.6)\)  
35. Subtract \((-3a + 6)\) from \((8a^2 - 4a + 13)\).  
36. Subtract \((6a - 7)\) from \((-9a^2 - 12a - 11)\).

Find the difference. Subtract vertically. See Example 2.

37. \(7x^2 + 10x - 1\) 
\(-(3x^2 + 11x - 2)\)  
38. \(2x^2 - 3x + 5\) 
\(-(x^2 - 2x + 9)\)  
39. \(12z^3 + 7z^2 - 2\) 
\(-(-6z^3 - 13z - 5)\)  
40. \(13y^3 + 7y^2 - 3\) 
\(-(-5y^3 + 17y - 2)\)

C Perform the indicated operations. See Example 3.

41. \((-11x + 2) + (14x - 3) - (-6x - 1)\)  
42. \((4x - 1) + (-6x - 7) - (-8x + 2)\)  
43. \((p^2 + 3p - 5) + (-3p^2 - 4p + 3) - (-6 + 9p)\)  
44. \((2p^2 - 4p + 3) + (-5p^2 + 8p - 7) - (10 - 7p)\)  
45. \((-t^3 - t^2 - 11) - (4t^3 - 5) + (8t^2 + 9)\)
5.4-40

46. \((-t^3 - t^2 - 17) - (-3t^3 - 5) + (4t^2 + 11)\)

47. Subtract \((5a - 2)\) from the sum of \((10a^2 - 2a + 4)\) and \((-2a^2 + 11a + 1)\).

48. Subtract \((7a + 12)\) from the sum of \((9a^2 - a + 5)\) and \((-12a^2 - 2a + 8)\).

D Perform the indicated operation. See Example 4.

49. \((3r^2 - 2rs + 4s^2) - (6r^2 - 5rs + 8s^2)\)

50. \((-2r^2 - 5rs + s^2) - (7r^2 - 2rs + 6s^2)\)

51. \((-x^2y^2 + 10) + (5x^2y^2 - 10xy - 4)\)

52. \((8x^2y^2 - 6) + (-12x^2y^2 + 3xy - 16)\)

53. \((12a^2b - 2ab - 8bc^2) - (-5e^2 + 4ab + 5a^2b)\)

54. \((-6a^2b + 5ab - 7bc^2) - (2bc^2 + 6ab + 9a^2b)\)

55. \((3x^2y - 2xy + 9y^2) + (-8y^2 - 5xy - xy^2)\)

56. \((-4x^2y + 10xy - 3y^2) + (-9y^2 - 2xy + 8xy^2)\)

APPLICATION SNAPSHOTS

57. Perimeter The perimeter of a rectangle is the sum of the lengths of all of its sides. Find the polynomial that represents the perimeter of the rectangle shown.

\[
\begin{align*}
(4x + 1) & \\
(x^2 + 2x) & \\
\end{align*}
\]

58. Perimeter The perimeter of a triangle is the sum of the lengths of all of its sides. Find the polynomial that represents the perimeter of the triangle shown.

\[
\begin{align*}
(x^2 + x) & \\
(x^2 + x) & \\
(2x^3 + 5x) & \\
\end{align*}
\]

59. Physics Two toy rockets were shot straight up from the ground at the same time. The first one was shot at a speed of 20 feet per second; the second, at 25 feet per second. Ignoring air resistance, the height (in feet) of each toy rocket at time \(t\) (in seconds) is given by the following:

First rocket: \(-16t^2 + 20t\) Second rocket: \(-16t^2 + 25t\)

Find a polynomial that represents the difference in height between the second rocket and the first rocket at time \(t\).

60. Physics Two toy rockets were shot straight up from the ground at the same time. The first one was shot at a speed of 20 feet per second; the second, at 30 feet per second. Ignoring air resistance, the height (in feet) of each toy rocket at time \(t\) (in seconds) is given by the following:

First rocket: \(-16t^2 + 20t\) Second rocket: \(-16t^2 + 30t\)

Find a polynomial that represents the difference in height between the second rocket and the first rocket at time \(t\).
5.5 Multiplying Polynomials

OBJECTIVES
A Multiply monomials
B Multiply a monomial by a polynomial
C Multiply two polynomials

In this section, we will focus on multiplying polynomials. Exponent rules will be used heavily here; so make sure you are comfortable with those rules before starting this section.

A Multiply Monomials

MAIN IDEA

When multiplying two monomials, look for like bases and apply the appropriate exponent rules. For example:

\((-2x^2y)(3x^2y^3z)\) \hspace{1cm} \text{think of as} \hspace{1cm} \frac{-2 \cdot 3 \cdot x^3 \cdot x^2 \cdot y \cdot y^3 \cdot z}{x^5} \hspace{1cm} y^4 \hspace{1cm} -6x^5y^4z

In this case, the product rule for exponents was used for the \(x\) base and the \(y\) base.

\(\text{Why is it okay to reorder the factors of the monomials like that?}\)
### PROCESS

**How to Multiply Monomials**

- Multiply the coefficients.
- Add the exponents for each like base separately.

### IMPORTANT NOTES

- The product rule for exponents states that when two exponential expressions with like bases are multiplied, you add their exponents. For example, \( x^3 \cdot x^2 = x^5 \).
- No matter how many like bases there are, take the like bases one at a time and add their exponents.
- This process also works for more than two monomials being multiplied together.
- Use the Exponent Rule Summary Chart as needed.

### EXAMPLE 1

Multiply \((-3a^3b)(2ab^4c)\).

Rewrite the problem. \((-3a^3b)(2ab^4c)\)

Multiply the coefficients. \(-6a^{3+1}b^{1+3}c\)

Add the exponents for each like base separately. \(-6a^4b^4c\)

### YOUR TURN 1

Multiply \((6r^3st)(-4rs^3t^2)\).

Deal with each like base separately.

### ON YOUR OWN 1

Multiply.

- \((2x^3)(3xy^2)\)
- \((-4ab^2)(-2ab^3c)\)
- \((-2x^2y^4)(-3xy^3)(5x^3y)\)

Go to the eText for more practice.
Multiply a Monomial by a Polynomial

**MAIN IDEA**

When a monomial is multiplied by a polynomial, the monomial distributes to each term in the polynomial. In other words, we use the Distributive Property. For example:

\[
3x^2(6x - 2) \rightarrow 3x^2 \cdot 6x - 3x^2 \cdot 2 \rightarrow 18x^3 - 6x^2
\]

**PROCESS**

**How to Multiply a Monomial by a Polynomial**

- Distribute and multiply the monomial to each term in the polynomial.

**IMPORTANT NOTES**

- If the monomial has a negative sign in front of it, the sign is part of the monomial; so the sign must distribute with the monomial.
- No matter how many terms are in the polynomial, the monomial distributes to every term.

**EXAMPLE 2**

Multiply \(-2a^3(5a^4 + 6a^3 - 7a + 12)\).

Rewrite the problem.

\(-2a^3(5a^4 + 6a^3 - 7a + 12)\)

Distribute the \(-2a^3\).

\((-2a^3) \cdot 5a^4 + (-2a^3) \cdot 6a^3 - (-2a^3) \cdot 7a + (-2a^3) \cdot 12\)

Multiply.

\(-10a^7 - 12a^6 + 14a^4 - 24a^3\)

**YOUR TURN 2**

Multiply \(5b^2(-3b^3 + 5b^2 - 7b - 3)\).

Distribute \(5b^2\) to every term.

**ON YOUR OWN 2**

Multiply.

a) \(2x(-5x^2 - 3x + 2)\)  
b) \(-5r^2(2r^2 - r + 4)\)  
c) \(6x^2y(2xy + 8)\)

The process is the same for multivariable polynomials.
Multiply Two Polynomials

MAIN IDEA
When two polynomials are multiplied, each term in the first polynomial distributes to each term in the second polynomial. For example:

\[(2x + 3)(x + 1)\]

\[= 2x \cdot x + 2x \cdot 1 + 3 \cdot x + 3 \cdot 1\]

\[= 2x^2 + 2x + 3x + 3\]

\[= 2x^2 + 5x + 3\]

Why does the distributing work this way?

PROCESS
How to Multiply a Polynomial by a Polynomial

- **Distribute and multiply** each term in the first polynomial by each term in the second polynomial.
- **Combine like terms.**

IMPORTANT NOTES
- Remember to combine like terms whenever possible.
- For large polynomials, see the vertical method in Example 4.

EXAMPLE 3
Multiply \((4x - 1)(2x + 7)\).

Rewrite the problem.

\[(4x - 1)(2x + 7)\]

Distribute the \(4x\), then the \(-1\).

\[4x \cdot 2x + 4x \cdot 7 - 1 \cdot 2x - 1 \cdot 7\]

Multiply.

\[8x^2 + 28x - 2x - 7\]

Combine like terms.

\[8x^2 + 26x - 7\]

See the Example Video for ideas on how to deal with the signs.

YOUR TURN 3
Multiply \((3y + 5)(y - 6)\).

Distribute the \(3y\); then distribute the \(5\).

Draw the distributing arrows if you find them helpful.

Enter your answer in the eText
EXAMPLE 4

Multiply \((2a + 4)(6a^2 - 2a - 6)\) horizontally and then vertically.

Horizontal:
Rewrite the problem.

\[
(2a + 4)(6a^2 - 2a - 6)
\]

Distribute the 2a, then the 4.

\[
2a \cdot 6a^2 - 2a \cdot 2a - 6 \cdot 2a - 6 \cdot 4 \cdot 6a^2 - 4 \cdot 2a - 4 \cdot 6
\]

Multiply.

\[
12a^3 - 4a^2 - 12a + 24a^2 - 8a - 24
\]

Combine like terms.

\[
12a^3 + 20a^2 - 20a - 24
\]

Vertical:
Set up the problem.

\[
\begin{array}{c}
6a^2 - 2a - 6 \\
2a + 4
\end{array}
\]

Multiply \(4(6a^2 - 2a - 6)\), then \(2a(6a^2 - 2a - 6)\).

\[
\frac{24a^2 - 8a - 24}{12a^3 - 4a^2 - 12a}
\]

Combine like terms.

\[
12a^3 + 20a^2 - 20a - 24
\]

YOUR TURN 4

Multiply \((3x - 7)(2x^2 + 4x - 6)\) horizontally and then vertically.

Horizontal:

Vertical:

When a trinomial is involved, you might find it easier to use the vertical method.

Enter your answer in the eText.

ON YOUR OWN 3

Multiply horizontally and then vertically.

\begin{align*}
\text{a) } & (5x + 6)(x + 2) \\
\text{b) } & (2a - b)(3a + 4b) \\
\text{c) } & (x + 3)(2x^2 - 5x + 8)
\end{align*}

Enter your answer in the eText.
5.5 Section Summary

Fill in the blanks to make true statements.

**KEY CONCEPTS** | **EXAMPLES**
--- | ---
How to Multiply Monomials | \((-3a^3b)(2ab^3c)\)
- Multiply the ________.
- Add the ________ for each like base separately.
- \(_a^3b^{1+3}c\)
- \(-6a-b-c\)

How to Multiply a Monomial by a Polynomial | \((-2a^3(5a^4 + 6a^3 - 7a + 12)\)
- Distribute and multiply the ________ to each
  ________ in the polynomial.
- \(-10a^7 - 12a^6 + 14a^5 - 24a^4\)

How to Multiply a Polynomial by a Polynomial | **Horizontal:**
- Distribute and multiply each ________ in the
  first polynomial to each ________ in the second
  polynomial.
- Combine like ________.
- **Vertical:**

**5.5 Additional Exercises**

**READING ✓**

Choose the best description for each example.

a) product of two polynomials
b) product of two monomials
c) product of a monomial and polynomial
d) product of a binomial and a trinomial

1. \((-ab^3)(-5ab^2)\)
2. \((3x^3y)(2xy^3)\)
3. \(-ab^2(3a^4 + 9a^2 - a)\)
4. \(-2xy^3(2x^3 - 6x + 8)\)
5. \((x^2 - 3x + 2)(2x^2 + 5x - 1)\)
6. \((6z^2 - 2z + 1)(-z^2 - z + 3)\)
## Section 5.5

### PRACTICE PROBLEMS

#### A Multiply. See Example 1.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.</td>
<td>$(3x^2)(4x^3)$</td>
</tr>
<tr>
<td>8.</td>
<td>$(2x^3)(5x^2)$</td>
</tr>
<tr>
<td>9.</td>
<td>$(-3m^5)(-6m^3)$</td>
</tr>
<tr>
<td>10.</td>
<td>$(-2a^2)(-9a^5)$</td>
</tr>
<tr>
<td>11.</td>
<td>$(4m^2n)(-3m^2n^3)$</td>
</tr>
<tr>
<td>12.</td>
<td>$(-5m^3n)(-2m^2n^5)$</td>
</tr>
<tr>
<td>13.</td>
<td>$(-7x^8y^3)(3x^2y^5)$</td>
</tr>
<tr>
<td>14.</td>
<td>$(2x^9y^2)(-6x^3y^6)$</td>
</tr>
<tr>
<td>15.</td>
<td>$(-7a^4b^2)(-a^2b^6)(3ab^2)$</td>
</tr>
<tr>
<td>16.</td>
<td>$(-3a^3b)(2ab^5)(-a^2b)$</td>
</tr>
<tr>
<td>17.</td>
<td>$(-2xy^3)^2$</td>
</tr>
<tr>
<td>18.</td>
<td>$(-4x^2y)^2$</td>
</tr>
</tbody>
</table>

#### B Multiply. See Example 2.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.</td>
<td>$3x(2x - 7)$</td>
</tr>
<tr>
<td>20.</td>
<td>$4x(7x + 6)$</td>
</tr>
<tr>
<td>21.</td>
<td>$-2x^2(2x^2 - 3)$</td>
</tr>
<tr>
<td>22.</td>
<td>$-5x^2(x^2 + 6)$</td>
</tr>
<tr>
<td>23.</td>
<td>$9a^2(-2a^3 + 3a^2 - 6)$</td>
</tr>
<tr>
<td>24.</td>
<td>$6b^2(-3b^3 - 6b^2 + 7)$</td>
</tr>
<tr>
<td>25.</td>
<td>$\frac{1}{2}a^3(4a^2 - 6a + 2)$</td>
</tr>
<tr>
<td>26.</td>
<td>$\frac{1}{3}b^2(6b^2 + 3b - 9)$</td>
</tr>
<tr>
<td>27.</td>
<td>$-x^3(-5x^4 + 3x^3 + 8)$</td>
</tr>
<tr>
<td>28.</td>
<td>$-y^3(-4y^4 + 5y^3 - 11)$</td>
</tr>
<tr>
<td>29.</td>
<td>$2r^3s(6r^3 - rs^2 + s)$</td>
</tr>
<tr>
<td>30.</td>
<td>$3r^2s(7r^3 + rs^2 - s)$</td>
</tr>
<tr>
<td>31.</td>
<td>$-x^2y(-2x^3 + xy^2 - 8y)$</td>
</tr>
<tr>
<td>32.</td>
<td>$-a^2b(-5a^3 + ab^2 - 3b)$</td>
</tr>
<tr>
<td>33.</td>
<td>$-5r^2t(-3r^3t^2 - 2r^2t^2 + 6rt - 8)$</td>
</tr>
<tr>
<td>34.</td>
<td>$-4a^2b(-2a^3b + 3a^2b^2 - 8ab + 2)$</td>
</tr>
</tbody>
</table>

#### C Multiply. See Example 3.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>35.</td>
<td>$(x + 3)(x + 2)$</td>
</tr>
<tr>
<td>36.</td>
<td>$(y + 5)(y + 1)$</td>
</tr>
<tr>
<td>37.</td>
<td>$(a - 3)(a + 8)$</td>
</tr>
<tr>
<td>38.</td>
<td>$(b + 9)(b - 2)$</td>
</tr>
<tr>
<td>39.</td>
<td>$(y - 9)(y - 11)$</td>
</tr>
<tr>
<td>40.</td>
<td>$(z - 4)(z - 12)$</td>
</tr>
<tr>
<td>41.</td>
<td>$\left( y + \frac{2}{3} \right) \left( y - \frac{1}{3} \right)$</td>
</tr>
<tr>
<td>42.</td>
<td>$\left( y + \frac{2}{5} \right) \left( y - \frac{3}{5} \right)$</td>
</tr>
<tr>
<td>43.</td>
<td>$(2r - 3)(3r - 5)$</td>
</tr>
<tr>
<td>44.</td>
<td>$(3t - 5)(2t - 6)$</td>
</tr>
<tr>
<td>45.</td>
<td>$(5x + 8)(2x - 3)$</td>
</tr>
<tr>
<td>46.</td>
<td>$(9x - 3)(3x + 5)$</td>
</tr>
<tr>
<td>47.</td>
<td>$(6a - 7)(6a + 7)$</td>
</tr>
<tr>
<td>48.</td>
<td>$(8x + 5)(8x - 5)$</td>
</tr>
<tr>
<td>49.</td>
<td>$(x - 6)^2$</td>
</tr>
<tr>
<td>50.</td>
<td>$(x + 3)^2$</td>
</tr>
<tr>
<td>51.</td>
<td>$(5d + 3)^2$</td>
</tr>
<tr>
<td>52.</td>
<td>$(2c - 5)^2$</td>
</tr>
</tbody>
</table>

### Multiply. Choose either the horizontal or vertical method. See Example 4.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>53.</td>
<td>$(x + 3)(x^2 + x - 2)$</td>
</tr>
<tr>
<td>54.</td>
<td>$(a - 2)(a^2 - a + 3)$</td>
</tr>
<tr>
<td>55.</td>
<td>$(y - 3)(y^2 - 2y + 6)$</td>
</tr>
<tr>
<td>56.</td>
<td>$(a + 5)(a^2 + 3a - 11)$</td>
</tr>
<tr>
<td>57.</td>
<td>$(2a - 3)(-a^3 - 2a + 11)$</td>
</tr>
<tr>
<td>58.</td>
<td>$(3x + 5)(-x^3 + 5x^2 - x)$</td>
</tr>
<tr>
<td>59.</td>
<td>$(r^3 + 3r)(r^2 - 4r + 5)$</td>
</tr>
<tr>
<td>60.</td>
<td>$(x^2 - 2x)(x^2 + 4x - 7)$</td>
</tr>
<tr>
<td>61.</td>
<td>$(2r - 1)(5r^3 + r^2 - 6r + 1)$</td>
</tr>
<tr>
<td>62.</td>
<td>$(3x + 2)(4x^3 + 3x^2 + x - 5)$</td>
</tr>
</tbody>
</table>
63. **Area** Find the polynomial that represents the area of the rectangle shown. (Hint: Area = Length • Width.)

\[
\begin{array}{c}
(2x) \\
(x + 5)
\end{array}
\]

64. **Area** Find the polynomial that represents the area of the rectangle shown. (Hint: Area = Length • Width.)

\[
\begin{array}{c}
(2x + 7) \\
(x + 3)
\end{array}
\]

65. **Money** A student deposits $200 in an account with an interest rate of \( r \). If the interest is compounded annually, after 2 years, the amount of money in the account is given by \( 200(1 + r)^2 \). Find the polynomial that represents the amount of money in the account after 2 years. (Hint: Multiply \( (1 + r)(1 + r) \); then distribute the 200.)

66. **Money** A student deposits $150 in an account with an interest rate of \( r \). If the interest is compounded annually, after 2 years, the amount of money in the account is given by \( 150(1 + r)^2 \). Find the polynomial that represents the amount of money in the account after 2 years. (Hint: Multiply \( (1 + r)(1 + r) \); then distribute the 150.)

### Chapter Exercises

Use the rules of exponents to simplify the following expressions. Express answers with positive exponents only. Assume that all variables represent nonzero real numbers.

67. \( b^4 \cdot b \cdot b^7 \)

68. \( (9x^3y)^2 \)

69. \( \frac{2r^{-1}}{4x^{-3}} \)

70. \( \frac{4^2a^{-2}}{4^4a^{-6}} \)

71. \( \frac{(-3z^4)^{-2}}{2z^5} \)

Evaluate the following expression for the given value.

72. \(-2y^2 + 5y + 3; y = -3\)

Simplify by combining like terms.

73. \(-8a^2 - 6a^2 + 14 - 12a^2\)

74. \(5x^2 + 16 + 3x^2 - 12\)

Perform the indicated operation.

75. Find the sum of \((c^4 - 7c^2 + c)\) and \((-6c^2 + 11 - 6c^4 + 2c^3)\).

76. Subtract \((-3p + 6)\) from \((7p^2 - 4p + 13)\).

77. \(3a^2b(7a^3 - ab^2 + b)\)

78. \((9b - 3)(3b + 4)\)
In this section, we will focus on multiplying binomials. First, we will learn the FOIL method, which is just a catchy word to help us remember the steps for multiplying two binomials. Next, we will examine three special cases of multiplying two binomials. Learning these special cases will provide useful shortcuts that we can use in the next chapter, where we’ll learn how to factor. Finally, we’ll learn what to do when multiplying three or more binomials.

**OBJECTIVES**

A Multiply two binomials using the FOIL method  
B Multiply the sum and difference of two terms  
C Square a binomial  
D Find greater powers of binomials

**VOCABULARY**

FOIL  
perfect square trinomials

**APPLICATION SNAPSHOT**

Area Problem

**A Multiply Two Binomials Using the FOIL Method**

**MAIN IDEA**

When multiplying two binomials, we can use the word FOIL to remember the multiplying process.

- **F** First terms
- **O** Outer terms
- **I** Inner terms
- **L** Last terms

\[(2x + 3)(x + 1)\]

\[= 2x \cdot x + 2x \cdot 1 + 3 \cdot x + 3 \cdot 1\]

\[= 2x^2 + 2x + 3x + 3\]

\[= 2x^2 + 5x + 3\]

**PROCESS**

How to Multiply Two Binomials Using FOIL

- **Multiply:** First terms  
  Outer terms  
  Inner terms  
  Last terms

- **Combine like terms.**

**IMPORTANT NOTES**

- We’re using the same distributing process here as in the general approach for multiplying two polynomials. **FOIL** is just a catchy word to help remember the approach.
• Remember to combine like terms whenever possible when multiplying polynomials.
• Practice doing FOIL in your head sometimes. (Then write it out to check your answer.) You will find this skill extremely helpful when we learn factoring in the next chapter.

**Example 1**

Find the product.

a) \((y - 4)(y + 3)\)  

Remember, this is still the distributing process for multiplying polynomials. It just has a catchy name.

**Rewrite the problem.** \((y - 4)(y + 3)\)

**Use FOIL.**  
\(y^2 + 3y - 4y - 12\)

**Combine like terms.**  
\(y^2 - y - 12\)

b) \((7a - 2)(5a - 3)\)

**Rewrite the problem.** \((7a - 2)(5a - 3)\)

**Use FOIL.**  
\(35a^2 - 21a - 10a + 6\)

**Combine like terms.**  
\(35a^2 - 31a + 6\)

**Your Turn 1**

Find the product.

a) \((x + 5)(x - 7)\)  

Use FOIL; then combine like terms.

b) \((8r - 3)(2r - 5)\)

**On Your Own 1**

Find the product.

a) \((x + 6)(x + 9)\)  

b) \((2x - 5)(4x + 1)\)  

c) \((3a - 2b)(2a - b)\)

**Multiply the Sum and Difference of Two Terms**

**Main Idea**

When we multiply the sum and difference of two terms, the result follows the same pattern. Therefore, we can write a formula for the situation.

- **Specific Example**  
  \((x + 7)(x - 7) = x^2 - 7x + 7x - 49 = x^2 - 49\)  
  the middle terms combine to zero

- **General Formula**  
  \((a + b)(a - b) = a^2 - b^2\)
Section 5.6

The middle terms *always* combine to zero when the sum and difference of two terms are multiplied because they are opposites. Therefore, the final result of the multiplying is always the difference of the squares of the terms.

**PROCESS** How to Multiply the Sum and Difference of Two Terms

\[(a + b)(a - b) = a^2 - b^2\]

- **Identify** the two terms \((a \text{ and } b)\) in the formula.
- **Find the difference** of the squares of the terms.

**IMPORTANT NOTES**

- This formula is helpful to memorize as a multiplying shortcut and for future topics.
- When writing the difference of the squares, show the terms in the same order they appear in the problem.
- You will get the same result if you use FOIL.
- Simplify your final answer if possible.

**EXAMPLE 2**

Find the product. Check your answer by the FOIL method.

a) \((x - 8)(x + 8)\)

Rewrite the problem. \((x - 8)(x + 8)\)

Identify the two terms. \(x \text{ and } 8\)

Find the difference of the squares of the terms. \(x^2 - 8^2\)

Simplify. \(x^2 - 64\)

b) \((2a + 3)(2a - 3)\)

Rewrite the problem. \((2a + 3)(2a - 3)\)

Identify the two terms. \(2a \text{ and } 3\)

Find the difference of the squares of the terms. \((2a)^2 - 3^2\)

Simplify. \(4a^2 - 9\)

See the FOIL checks in the eText.

**YOUR TURN 2**

Find the product. Check your answer by the FOIL method.

a) \((r + 4)(r - 4)\)

Identify the terms; then find the difference of their squares.

b) \((5a - 7)(5a + 7)\)

Remember to put parentheses around \(5a\) for squaring.

Check by FOIL:

Go to the eText for more practice.
ON YOUR OWN 2

Find the product. Check your answer by the FOIL method.

a) \((z - 6)(z + 6)\)  
b) \((x + 1)(x - 1)\)  
c) \((2t - 9)(2t + 9)\)

Go to the eText for the online Reading ✓ exercise.

Square a Binomial

MAIN IDEA

When we square a binomial, the result follows a pattern. Therefore, we can write a formula for the situation.

Specific Example

\((x + 3)^2 = (x + 3)(x + 3) = x^2 + 3x + 3x + 9 = x^2 + 6x + 9\)

there are two of these

General Formulas

\((a + b)^2 = a^2 + 2ab + b^2\)

\((a - b)^2 = a^2 - 2ab + b^2\)

The middle terms always double when a binomial is squared because the middle terms are identical. Therefore, the final result of the squaring contains twice the product of the two terms in the original binomial. The sign in front of the middle term is always the same as the sign between the terms in the original binomial.

PROCESS

How to Square a Binomial

\((a + b)^2 = a^2 + 2ab + b^2\)

\((a - b)^2 = a^2 - 2ab + b^2\)

- Identify the two terms \((a\) and \(b)\) in the formula.
- Apply the formula for the appropriate square of a binomial.

IMPORTANT NOTES

- The trinomials that result from squaring binomials are called perfect square trinomials.
- Notice in the formulas that if the binomial being squared is a sum, the resulting middle term of the trinomial is \(+\); if the binomial being squared is a difference, the resulting middle term of the trinomial is \(-\).
- You will get the same result if you use FOIL.
- Simplify your final answer if possible.
- Notice above that \((x + 3)^2\) does not equal \(x^2 + 3^2\).
Section 5.6

EXAMPLE 3

Find the product. Check your answer by the FOIL method.

a) \((a - 7)^2\)

Rewrite the problem. \((a - 7)^2\)

Identify the two terms. \(a\) and 7

Apply the formula. \((a - b)^2 = a^2 - 2ab + b^2\)

Substitute \(a\) for \(a\) and 7 for \(b\). \((a - 7)^2 = a^2 - 2(a \cdot 7) + 7^2\)

Simplify. \(a^2 - 14a + 49\)

b) \((3z + 2)^2\)

Rewrite the problem. \((3z + 2)^2\)

Identify the two terms. \(3z\) and 2

Apply the formula. \((a + b)^2 = a^2 + 2ab + b^2\)

Substitute \(3z\) for \(a\) and 2 for \(b\). \((3z + 2)^2 = (3z)^2 + 2(3z \cdot 2) + 2^2\)

Simplify. \(9z^2 + 12z + 4\)

See the FOIL checks in the eText.

YOUR TURN 3

Find the product. Check your answer by the FOIL method.

a) \((y + 2)^2\)

Check by FOIL:

b) \((2a - 5)^2\)

ON YOUR OWN 3

Find the product. Check your answer by the FOIL method.

a) \((p + 5)^2\)

Go to the eText for the online Reading ✓ exercise.
Find Greater Powers of Binomials

**MAIN IDEA**

When a binomial is raised to a power greater than 2, we can rewrite the problem to involve one or more squares of the binomial. Here’s the general idea of how it works. See Example 4 for the complete solutions.

A binomial cubed: \((a + 2)^3 = (a + 2)^2(a + 2) = \ldots\) Apply the formula for the binomial squared; then multiply the result by the third binomial.

A binomial to the 4th power: \((x - 3)^4 = (x - 3)^2(x - 3)^2 = \ldots\) Apply the formula twice for the binomial squared; then multiply those results together.

**PROCESS** How to Find Greater Powers of Binomials

- **Rewrite** the problem to involve one or more squares of the binomial.
- **Apply the formula(s)** for the appropriate square of a binomial.
- **Multiply** the resulting polynomials.

**IMPORTANT NOTES**

- This process works for powers greater than 4, too.
- Remember, you will get the same result if you use the general process for multiplying polynomials instead of the formulas.
- Memorizing and mastering the formulas in this section now will serve you well in the next chapter on factoring, where they are used throughout.
- Simplify your final answer if possible.

**EXAMPLE 4**

Find the product.

**a)** \((a + 2)^3\)

**Rewrite** the problem as a squared binomial times a binomial.

This rewrite works because \(x^3 = x^2 \cdot x\).

**Apply the formula** for the square of a binomial.

**Multiply** the resulting polynomials.

**b)** \((x - 3)^4\)

**Rewrite** the problem as two squared binomials.

**Apply the formula** for the square of a binomial.

The multiplying can be done horizontally or vertically.
Section 5.6

Multiply the resulting polynomials.

\[
\begin{align*}
&x^2 - 6x + 9 \\
&9x^2 - 54x + 81 \\
&- 6x^3 + 36x^2 - 54x \\
&x^4 - 6x^3 + 9x^2 \\
&x^4 - 12x^3 + 54x^2 - 108x + 81
\end{align*}
\]

YOUR TURN 4

Find the product.

a) \((a - 5)^3\)

b) \((x + 2)^4\)

Rewrite as \((a - 5)^3(a - 5)\).

Apply the formula \((a - b)^2 = a^2 - 2ab + b^2\).

Multiply the result by \((a - 5)\).

Rewrite as \((x + 2)^2(x + 2)^2\).

ON YOUR OWN 4

Find the product.

a) \((a - 1)^3\)  
b) \((3x - 2)^3\)  
c) \((x + 3)^4\)

Enter your answers in the eText

5.6 Section Summary

Fill in the blanks to make true statements.

**KEY CONCEPTS**

How to Multiply Two Binomials Using FOIL

- Multiply: ________ terms
  
  ________ terms
  
  ________ terms
  
  ________ terms

- ________ like terms.
### Key Concepts

#### How to Multiply the Sum and Difference of Two Terms

\[(a + b)(a - b) = a^2 - b^2\]

- Identify the two terms \((a\) and \(b) in the formula.
- Find the difference of the squares of the terms.

#### How to Square a Binomial

\[
\begin{align*}
(a + b)^2 &= a^2 + 2ab + b^2 \\
(a - b)^2 &= a^2 - 2ab + b^2
\end{align*}
\]

- Identify the two terms \((a\) and \(b) in the formula.
- Apply the formula for the appropriate square of a binomial.

#### How to Find Greater Powers of Binomials

- Rewrite the problem to involve one or more of the binomial.
- Apply the for the appropriate square of a binomial.
- Multiply the resulting polynomials.

### Vocabulary

**FOIL**, perfect square trinomials

---

### 5.6 Additional Exercises

**Reading ✓**

Match the expression in Column I with its correct polynomial in Column II. Try doing the problems in your head for numbers 1 through 6.

<table>
<thead>
<tr>
<th>Column I</th>
<th>Column II</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ((x + 7)(x - 1))</td>
<td>a) (x^3 + 21x^2 + 147x + 343)</td>
</tr>
<tr>
<td>2. ((x + 1)(x - 7))</td>
<td>b) (x^2 + 6x - 7)</td>
</tr>
<tr>
<td>3. ((x - 1)(x + 1))</td>
<td>c) (x^3 + 343)</td>
</tr>
<tr>
<td>4. ((x + 7)(x - 7))</td>
<td>d) (x^2 - 1)</td>
</tr>
<tr>
<td>5. ((x - 1)^2)</td>
<td>e) (x^3 - 21x^2 + 147x - 343)</td>
</tr>
<tr>
<td>6. ((x + 7)^2)</td>
<td>f) (x^2 - 49)</td>
</tr>
<tr>
<td>7. ((x + 7)^3)</td>
<td>g) (x^2 + 2x + 1)</td>
</tr>
<tr>
<td>8. ((x - 7)^3)</td>
<td>h) (x^2 + 49)</td>
</tr>
<tr>
<td></td>
<td>i) (x^2 + 14x + 49)</td>
</tr>
<tr>
<td></td>
<td>j) (x^2 - 6x - 7)</td>
</tr>
</tbody>
</table>
PRACTICE PROBLEMS

A Find the product using the FOIL method. See Example 1.

9. \((r + 2)(r + 3)\) 10. \((x + 5)(x + 1)\) 11. \((y - 2)(y + 6)\)
12. \((b + 8)(b - 3)\) 13. \((x - 9)(x - 7)\) 14. \((a - 8)(a - 6)\)
15. \((x + 12)(x - 2)\) 16. \((a - 11)(a + 5)\) 17. \((y - 5)(y - 10)\)
18. \((x - 6)(x - 9)\) 19. \((x + \frac{5}{6})(x - \frac{1}{5})\) 20. \((x - \frac{1}{4})(x + \frac{1}{6})\)
21. \((2x + 7)(x + 4)\) 22. \((3x + 6)(x + 9)\) 23. \((11 + 3a)(3 - 4a)\)
24. \((13 + 5b)(2 - 6b)\) 25. \((3x - y)(2x - y)\) 26. \((5x - y)(8x - y)\)
27. \((4x^2 + 3)(3x + 2)\) 28. \((7x^2 + 1)(2x + 5)\)

B Find the product using the formula for multiplying the sum and difference of two terms. Check your answer using the FOIL method. See Example 2.

29. \((x + 6)(x - 6)\) 30. \((a - 2)(a + 2)\)
31. \((a + 12)(a - 12)\) 32. \((m + 10)(m - 10)\)
33. \((x + \frac{3}{5})(x - \frac{3}{5})\) 34. \((x + \frac{2}{9})(x - \frac{2}{9})\)
35. \((3x - 1)(3x + 1)\) 36. \((2x + 2)(2x - 2)\)
37. \((5t - 8)(5t + 8)\) 38. \((6m + 7)(6m - 7)\)
39. \((2x - 5y)(2x + 5y)\) 40. \((3m - 9n)(3m + 9n)\)

C Find the product using one of the formulas for the square of a binomial. See Example 3.

41. \((x + 3)^2\) 42. \((x + 5)^2\) 43. \((a - 4)^2\)
44. \((a - 7)^2\) 45. \((r - 0.2)^2\) 46. \((r - 0.5)^2\)
47. \((a + \frac{2}{3})^2\) 48. \((b + \frac{2}{7})^2\) 49. \((3x - 2)^2\)
50. \((2x - 8)^2\) 51. \((5x + 9)^2\) 52. \((6x + 8)^2\)
53. \((2x^2 - 1)^2\) 54. \((3x^2 - 5)^2\) 55. \((5x - 3y)^2\)
56. \((6x - 2y)^2\)

D Find the product. See Example 4.

57. \((x + 5)^3\) 58. \((x + 6)^3\) 59. \((4m - 1)^3\)
60. \((3b - 1)^3\) 61. \((2x + 3)^4\) 62. \((5b + 2)^4\)
63. \((2k - 1)^4\) 64. \((3r - 2)^4\)
MIXED PRACTICE

Find the product of the binomials. See Examples 1–4.

65. \((2x + 5)(2x - 5)\)  
66. \((3x - 9)(3x + 9)\)  
67. \((1 - 3x)(2 + 5x)\)  
68. \((2 - x)(3 + 7x)\)  
69. \((4m + 5)(m + 1)\)  
70. \((5a + 2)(a + 2)\)  
71. \((4x + 3)^3\)  
72. \((2x + 7)^3\)  
73. \((5w - 2)(2w + 5)\)  
74. \((4a + 9)(2a - 1)\)

APPLICATION SNAPSHOTS

75. Area  Find the polynomial that represents the area of the square shown.

\[
\begin{array}{c}
(x + 4) \\
(x + 4)
\end{array}
\]

76. Area  Find the polynomial that represents the area of the square shown.

\[
\begin{array}{c}
(3x + 1) \\
(3x + 1)
\end{array}
\]

77. Money  A student deposits $200 in an account with an interest rate of \(r\). If the interest is compounded annually, after 2 years, the amount of money in the account is given by \(200(1 + r)^2\). Find the polynomial that represents the amount of money in the account after 2 years. Use one of the formulas for squaring a binomial.

78. Money  A student deposits $150 in an account with an interest rate of \(r\). If the interest is compounded annually, after 2 years, the amount of money in the account is given by \(150(1 + r)^2\). Find the polynomial that represents the amount of money in the account after 2 years. Use one of the formulas for squaring a binomial.

CHAPTER ✓

Evaluate each expression for the given value.

79. \(a^2 - 4a - 6; a = 8\)  
80. \(1.3m^2 - 2.7m + 6; m = -2.4\)

Simplify. Write your final answer using only positive exponents. Assume that all variables represent nonzero real numbers.

81. \(\frac{ab^{-2}}{a^3}\)  
82. \(\frac{(2x^2)^{-4}}{3x^4}\)

Perform the indicated operation.

83. Add: \(\frac{5m^3 - 9m^2}{-6m^3 + 4m^2}\)  
84. Subtract: \(-\frac{x^2 - 2x + 9}{x^2 - 2x}\)

85. \((x + 3)(2x^2 - 5x + 8)\)  
86. \((x^2 - 2x)(x^2 + 4x - 7)\)

87. \((9x - 3)(3x + 5)\)  
88. \((3x^2 - 5)^2\)
In this section, we will learn two methods for dividing polynomials. The first method works only when dividing by a monomial, while the second method (long division) works for all cases.

**A. Divide a Polynomial by a Monomial**

**Main Idea:**

First, let’s review some division vocabulary and ideas. There are many ways to show division. For example:

10 divided by 2 can be shown as $10 \div 2 = 5$ or $10 \div 2 = 5 \div 2 = 5$.

In each case, the **dividend** is what’s being divided (10 in this case).

the **divisor** is what’s doing the dividing (2 in this case).

the **quotient** is the result (5 in this case).

To check a result: The quotient times the divisor equals the dividend. $5 \times 2 = 10$ Yes

**New:**

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial and simplify. A number problem is also shown so that you can compare the steps side by side.

Rewrite as a fraction. $\frac{6x^2 + 4x}{2x}$

$\frac{6x^2 + 4x}{2x} = \frac{6x^2}{2x} + \frac{4x}{2x} = 3x + 2$.

Divide each term of the polynomial by the monomial.

Simplify. $3x + 2$

Check. $(3x + 2) \cdot 2 = 6x^2 + 4x$ Yes

**Process:**

- **Divide** each term of the polynomial by the monomial.
- **Simplify**.
**EXAMPLE 1**

Find the quotient. Assume that all variables represent nonzero real numbers.

a) \( \frac{36x^4 + 4x^2}{4x^2} \)  
Rewrite as a fraction. \( \frac{36x^4 + 4x^2}{4x^2} \)  
Divide each term of the polynomial by the monomial. \( \frac{36x^4}{4x^2} + \frac{4x^2}{4x^2} \)  
Simplify. \( 9x^2 + 1 \)

b) \( \frac{18x^6 - 9x^3 + 6x}{3x^2} \)  
Rewrite as a fraction. \( \frac{18x^6 - 9x^3 + 6x}{3x^2} \)  
Divide each term of the polynomial by the monomial. \( \frac{18x^6}{3x^2} - \frac{9x^3}{3x^2} + \frac{6x}{3x^2} \)  
Simplify. \( 6x^4 - 3x + \frac{2}{x} \)

See the checks in the eText.

**YOUR TURN 1**

Find the quotient. Assume that all variables represent nonzero real numbers.

a) \( \frac{20x^3 - 4x^3}{2x} \)  
Divide each term in the polynomial; then simplify.

b) \( \frac{30x^6 + 15x^4 - 5x}{5x^2} \)
Write the final answer with positive exponents only.

**ON YOUR OWN 1**

Find the quotient. Assume that all variables represent nonzero real numbers.

a) \( \frac{8x^3 - 2x}{2x} \)  
b) \( \frac{40x^4 - 20x^3 + 10x^2}{5x} \)  
c) \( \frac{6x^3 - 3x^2 + 2x}{3x^2} \)

Go to the eText for more practice.
Divide a Polynomial by a Polynomial

MAIN IDEA

We will use long division when dividing a polynomial by a polynomial. The process is similar to long division of whole numbers. Here is a side-by-side comparison of a number problem and a polynomial problem.

**176 ÷ 3**

<table>
<thead>
<tr>
<th>Round 1:</th>
<th>((x^2 + 5x - 14) ÷ (x + 2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Divide. (\frac{17}{3} \approx 5)</td>
<td>Divide. (\frac{x^2}{x} = x) (Put the x on top.)</td>
</tr>
<tr>
<td>(Put the 5 on top.)</td>
<td>(x + 2) (x^2 + 5x - 14)</td>
</tr>
<tr>
<td>Multiply. (5 \cdot 3 = 15)</td>
<td>Multiply. (x(x + 2) = x^2 + 2x)</td>
</tr>
<tr>
<td>Subtract the 15.</td>
<td>Subtract the (x^2 + 2x).</td>
</tr>
<tr>
<td>Bring down the 6.</td>
<td>(Distribute the minus sign.)</td>
</tr>
</tbody>
</table>

**Final Answer:** 58 R2

or

\(\frac{2}{3}\)

Check: The quotient times the divisor plus the **remainder** equals the dividend.

\(58 \cdot 3 + 2\)

174 + 2

176 Yes

**Video—See the division worked out.**

### PROCESS

**How to Divide a Polynomial by a Polynomial**

- Write the polynomial with descending powers.
- Add zeros for any missing terms.
- For as many rounds as necessary: Divide
  Multiply
  Subtract
  Bring down

**5.7-61**
IMPORTANT NOTES

- You’re done when there’s nothing left to bring down or when the degree of whatever is brought down is less than the degree of the divisor.
- This long division process also works for dividing by a monomial. Try it for \((36x^4 + 4x^2) \div 4x^2\) and see if you get \(9x^2 + 1\).
- Be patient while learning this process. It takes a while to become comfortable using it.

EXAMPLE 2

Find the quotient. There will be no remainder. \(x - 5x^2 - 3x - 10\)

Round 1:

Divide. \(\frac{x^2}{x} = x\)

(\text{Put the } x \text{ on top.})

Multiply. \(x(x - 5) = x^2 - 5x\)

Subtract the \(x^2 - 5x\).

(Distribute the minus sign.)

Bring down the \(-10\).

Round 2:

Divide. \(\frac{2x}{x} = 2\)

(\text{Put the } +2 \text{ on top.})

Multiply. \(2(x - 5) = 2x - 10\)

Subtract the \(2x - 10\).

(Distribute the minus sign.)

Final Answer: \(x + 2\)

See the check in the eText.

YOUR TURN 2

Find the quotient. There will be no remainder. \((2x^2 - 15x - 8) \div (x - 8)\)

Which polynomial goes inside the division symbol?

Set up the problem in long division format. \[
\begin{array}{c|ccccc}
\text{quotient} & \text{divisor} & \text{dividend} \\
\hline
\end{array}
\]

Do as many rounds of Divide, Multiply, Subtract, Bring down as necessary.

Check: \((\text{your answer}) \cdot (x - 8)\) should equal \(2x^2 - 15x - 8\).

Go to the eText for more practice.
EXAMPLE 3

Divide \((2x^2 + 4x - 18)\) by \((x + 4)\). There will be a remainder.

Round 1:

Divide. \(\frac{2x^2}{x} = 2x\)

(Put the \(2x\) on top.)

Multiply. \(2x(x + 4) = 2x^2 + 8x\)

Subtract the \(2x^2 + 8x\).

(Distribute the minus sign.)

Bring down the \(-18\).

Round 2:

Divide. \(-\frac{4x}{x} = -4\)

(Put the \(-4\) on top.)

Multiply. \(-4(x + 4) = -4x - 16\)

Subtract the \(-4x - 16\).

(Distribute the minus sign.)

Final Answer: \(2x - 4 R -2\) \(\rightarrow 2x - 4 - \frac{2}{x + 4}\)

See the check in the eText.

YOUR TURN 3

Find the quotient. There will be a remainder. \((4b^2 - 4b - 3) \div (2b - 1)\)

Check: \((\text{your answer}) \cdot (2b - 1) + (\text{the remainder})\) should equal \(4b^2 - 4b - 3\).

Go to the eText for more practice.

ON YOUR OWN 2

Find the quotient.

a) Divide \(2x^2 + 11x + 15\) by \(x + 3\).

b) \(\frac{6x^2 + 9x - 4}{3x + 6}\)

c) \((x^3 + 1) \div (x + 1)\)

To account for missing terms, rewrite this as \(x + 1)x^2 + 0x^1 + 0x + 1\) and follow the DMSB steps.

Go to the eText for your instructor's online assignment.

Application Snapshots—Find the Width Problem (Section 5.9, Example 4).

Go to the eText for your instructor's online assignment.
5.7 Section Summary

Fill in the blanks to make true statements.

<table>
<thead>
<tr>
<th>KEY CONCEPTS</th>
<th>EXAMPLES</th>
</tr>
</thead>
</table>
| **How to Divide a Polynomial by a Monomial** | $36x^4 + 4x^2$
| • Divide each _________ of the polynomial by the _________ | $\frac{4x^2}{4x^2}$
| • Simplify. | $9x^2 + _-$ |
| **How to Divide a Polynomial by a Polynomial** | $x - 5\frac{x^2}{9} - 3x - 10$
| • Write the polynomial with descending _________ | $\frac{\_ + 2}{\_ + 2}$
| • Add _________ for any missing terms. | $\frac{x - 5\frac{x^2}{9} - 3x - 10}{\frac{\_ - 10}{\_ - 10}}$
| • For as many rounds as necessary: _________ | _________ |
| Round 1 | _________ |
| Round 2 | _________ |

**Vocabulary**  
dividend, divisor, quotient, remainder

5.7 Additional Exercises

In each of the following, choose the first step in finding the quotient of the given example.

1. $(10x^4 - 5x^2 - x) \div 5x$
   a) $\frac{5x^4}{5x} - \frac{x}{5x}$
   b) $\frac{10x^4}{5x} - \frac{5x^2}{5x} - \frac{x}{5x}$
   c) $\frac{5x}{10x^4} - \frac{5x}{5x^2} - \frac{5x}{x}$

2. $(12y^3 - 4y^2 + y) \div 4y$
   a) $\frac{4y}{12y^3} - \frac{4y}{4y^2} + \frac{4y}{y}$
   b) $\frac{12y^3}{4y} - \frac{4y^2}{4y} + \frac{y}{4y}$
   c) $\frac{8y^3}{4y} + \frac{y}{4y}$

3. $2x - 3\frac{8x^3}{4x^2} + 2x + 2$  
   a) $\frac{2x}{2x} - 3\frac{8x^3}{4x^2} + 2x + 2$
   b) $\frac{4x^2}{4x^2} + 2x + 2$
   c) $\frac{x}{2x} - 3\frac{8x^3}{4x^2} + 2x + 2$

4. $2x + 3\frac{4x^3}{2x^2} - 8x + 3$
   a) $\frac{2x}{2x} + 3\frac{4x^3}{2x^2} - 8x + 3$
   b) $\frac{x}{x} + 3\frac{4x^3}{2x^2} - 8x + 3$
   c) $\frac{x}{x} + 3\frac{4x^3}{2x^2} - 8x + 3$
PRACTICE PROBLEMS

A. Find the quotient. Assume that all variables represent nonzero real numbers. See Example 1.

5. \( (12y^3 - 24y^2) \div 6y \)
6. \( (18a^3 + 27a^3) \div 9a^3 \)
7. \( \frac{-15c^4 - 5c^3}{-5c^2} \)
8. \( \frac{-45y^5 - 9y^4}{-9y^3} \)
9. \( (-21t^4 - t) \div 3t^2 \)
10. \( (-9y^4 - y) \div 3y^2 \)
11. \( \frac{12u^6 - 3u^5 - 2u^4}{-3u^4} \)
12. \( \frac{5m^5 - 10m^4 - 4m^3}{-5m^3} \)
13. \( \frac{35y^5 - 7y^3 + 28y^2}{-7y^2} \)
14. \( \frac{26a^5 + 16a^3 - 2a}{-2a^2} \)
15. \( (-15x^6 - 8x^4 - 6x^3) \div (-3x^3) \)
16. \( (12x^7 - 9x^6 - 4x^2) \div (-2x^3) \)
17. \( (-4x^3y^2 + x^2y^2 - xy^3) \div (-x^2y^2) \)
18. \( (-9a^6b^3 - a^2b^2 - ab^3) \div (-a^2b^3) \)

B. Find the quotient. See Examples 2 and 3.

19. \( (x^2 + 13x + 12) \div (x + 1) \)
20. \( (x^2 - 7x + 12) \div (x - 3) \)
21. \( (2x^2 + 3x - 2) \div (2x - 1) \)
22. \( (3x^2 + 11x + 6) \div (3x + 2) \)
23. \( (8x^2 + 2x - 3) \div (4x + 3) \)
24. \( (6x^2 + 5x - 6) \div (3x - 2) \)
25. \( (9x^2 + 6x + 1) \div (3x + 1) \)
26. \( (25x^2 + 20x + 4) \div (5x + 2) \)
27. \( \frac{3x^2 - x - 8}{x - 1} \)
28. \( \frac{3x^2 - x - 4}{x - 2} \)
29. \( \frac{5x^2 - 2x - 3}{x - 2} \)
30. \( \frac{2x^2 - 7x + 2}{x + 4} \)
31. \( (-13x + 3x^2 - 10) \div (x - 3) \)
32. \( (-2x + 5x^2 - 3) \div (x + 2) \)
33. \( (4x^3 + 8x^2 - 5x - 50) \div (x - 2) \)
34. \( (6x^3 - 25x^2 + 24x - 33) \div (2x - 7) \)
35. \( (4x^3 - 8x^2 + 13x - 7) \div (2x - 1) \)
36. \( (9x^3 + 9x^2 - 22x + 9) \div (3x - 2) \)
37. \( (8b^3 - 125) \div (2b - 5) \)
38. \( (27z^3 - 8) \div (3z - 2) \)
39. **Find the Width** The area of a rectangle is \((2x^2 + 6x)\) square inches, and its length is \((2x)\) inches. Find its width.

\[
\frac{(2x^2 + 6x)}{(2x)}
\]

40. **Find the Width** The area of a rectangle is \((9x^3 + 6x^2)\) square inches, and its length is \((3x)\) inches. Find its width.

\[
\frac{(9x^3 + 6x^2)}{(3x)}
\]

41. **Find the Length** The area of a rectangle is \((x^2 + 12x + 27)\) square inches, and its width is \((x + 3)\) inches. Find its length.

\[
\frac{(x^2 + 12x + 27)}{(x + 3)}
\]

42. **Find the Length** The area of a rectangle is \((x^2 + 7x + 10)\) square inches, and its width is \((x + 2)\) inches. Find its length.

\[
\frac{(x^2 + 7x + 10)}{(x + 2)}
\]

---

43. \(\left(\frac{m^2 n}{mm^3}\right)^{-3}\)

44. \((-3xy)^{-3}(2x^{-2}y^3)^{-2}\)

45. \((4x^3 - 11x^2 - 12) + (-8x^3 - 5x^2 + 6x)\)

46. \((9z^3 - 10z + 4) - (-12 - 3z^3 + 3z^2)\)

47. \(5b^2(-2b^3 + 3b^2 - 6)\)

48. \((2x + 9)(5x - 3)\)

49. \((2x - 11)^2\)

50. \(\left(r + \frac{2}{7}\right)\left(r - \frac{2}{7}\right)\)

51. \(\frac{24u^6 - 6u^5 - 4u^4}{-3u^4}\)

52. \(\frac{5q^2 - 2q - 3}{q - 2}\)
**5.8 Applying Exponents: Scientific Notation**

**OBJECTIVES**

A. Write numbers in scientific notation
B. Write scientific notation in standard form
C. Multiply and divide with scientific notation
D. Solve applications involving scientific notation

**MAIN IDEA**

*Scientific notation* is a way to express very large or very small numbers without having to take up a lot of space. In scientific notation, numbers are written as the product of a number and a power of ten. The way we usually write numbers (not in scientific notation) is called *standard form*.

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>Scientific Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>speed of light (in mi/sec)</td>
<td>186,000</td>
</tr>
<tr>
<td>gravitational constant (in lb $\cdot$ ft$^2$/slug$^2$)</td>
<td>0.0000000344</td>
</tr>
<tr>
<td>mass of an electron (in kg)</td>
<td>0.000000000000000000000000000000911</td>
</tr>
</tbody>
</table>

**Definition of Scientific Notation**

$a \times 10^n$, where $1 \leq a < 10$ and $n$ is an integer

**PROCESS** How to Write a Number in Scientific Notation

- **Write $a$** by moving the decimal point until there is exactly one nonzero digit to the left of the decimal.
- **Count the number of places** you moved the decimal point. This is the absolute value of the exponent, $n$.
- **Find the sign** of the exponent.
  - If the standard form number is larger than $a$, the exponent is positive.
  - If the standard form number is smaller than $a$, the exponent is negative.
- **Write the final answer in the form** $a \times 10^n$. 

**Video**—What does multiplying by 10 to a negative exponent mean?
IMPORTANT NOTES

- $a$ is called the coefficient.
- Once you’ve found $a$, delete any zeros to the right of its last nonzero number. Those zeros are unnecessary.
- The decimal moves the number of places equal to the absolute value of the exponent.
- A positive exponent means that the standard form number is larger than $a$. A negative exponent means that the standard form number is smaller than $a$.
- If the decimal does not move, the exponent is zero. Remember, $10^0 = 1$.
- Remember, if no decimal is shown, it is understood to come after the last digit in the number. For example, 2 is 2.0.

EXAMPLE 1

Write the following numbers in scientific notation.

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>Scientific Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 6,780,000</td>
<td>$6.78 	imes 10^6$</td>
</tr>
<tr>
<td>Value of $n$:</td>
<td>6 spaces to 6.78</td>
</tr>
<tr>
<td>Sign of $n$:</td>
<td>6,780,000 is larger than 6.78; so the exponent is $+6$.</td>
</tr>
<tr>
<td>b) 0.000034</td>
<td>$3.4 	imes 10^{-5}$</td>
</tr>
<tr>
<td>Value of $n$:</td>
<td>5 spaces to 3.4</td>
</tr>
<tr>
<td>Sign of $n$:</td>
<td>0.000034 is smaller than 3.4; so the exponent is $-5$.</td>
</tr>
</tbody>
</table>

YOUR TURN 1

Write the following numbers in scientific notation.

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>Scientific Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 0.008</td>
<td>$8 	imes 10^{-3}$</td>
</tr>
<tr>
<td>b) 24,500,000</td>
<td>$2.45 	imes 10^8$</td>
</tr>
</tbody>
</table>

Go to the eText for more practice.
**ON YOUR OWN 1**

Write the following numbers in scientific notation.

a) 0.0005  
b) 2,100,000  
c) 0.063

Go to the eText for the online Reading ✓ exercise.

**B Write Scientific Notation in Standard Form**

**MAIN IDEA**

When a positive number is multiplied by a positive power of ten, the result is a larger number.  
When a positive number is multiplied by a negative power of ten, the result is a smaller number.

<table>
<thead>
<tr>
<th>Scientific Notation</th>
<th>Standard Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6 \times 10^4$</td>
<td>60,000</td>
</tr>
<tr>
<td>make larger</td>
<td></td>
</tr>
<tr>
<td>4 spaces</td>
<td></td>
</tr>
</tbody>
</table>

**PROCESS** How to Write Scientific Notation in Standard Form

- **Move the decimal point** a number of spaces equal to the absolute value of the exponent.
- **Move right** if the exponent is positive.  
  **Move left** if the exponent is negative.

**IMPORTANT NOTES**

- The decimal point moves the number of places equal to the absolute value of the exponent.
- For a positive number, multiplying by a positive power of 10 results in a larger number; multiplying by a negative power of 10 results in a smaller number.

**EXAMPLE 2**

Write the following numbers in standard form.

<table>
<thead>
<tr>
<th>Scientific Notation</th>
<th>Standard Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $7.8 \times 10^{-3}$</td>
<td>0.0078</td>
</tr>
<tr>
<td>The exponent is $-3$; so move 3 left.</td>
<td></td>
</tr>
<tr>
<td>b) $6.2 \times 10^6$</td>
<td>6,200,000</td>
</tr>
<tr>
<td>The exponent is $+6$; so move 6 right.</td>
<td></td>
</tr>
<tr>
<td>Multiplying 7.8 by a negative power of 10 results in a smaller number.</td>
<td></td>
</tr>
<tr>
<td>Multiplying 6.2 by a positive power of 10 results in a larger number.</td>
<td></td>
</tr>
</tbody>
</table>
## Your Turn 2

Write the following numbers in standard form.

<table>
<thead>
<tr>
<th>Scientific Notation</th>
<th>Standard Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) (9.52 \times 10^6)</td>
<td>(9.52 \times 10^6)</td>
</tr>
<tr>
<td>b) (7.9 \times 10^{-8})</td>
<td>(7.9 \times 10^{-8})</td>
</tr>
</tbody>
</table>

Will multiplying 9.52 by \(10^6\) result in a larger or smaller number?

Will multiplying 7.9 by \(10^{-8}\) result in a larger or smaller number?

Go to the eText for more practice.

## On Your Own 2

Write the following numbers in standard form.

a) \(5.6 \times 10^{-5}\)  

b) \(1.002 \times 10^{-7}\)  

c) \(9.1 \times 10^5\)

Go to the eText for the online Reading ✓ exercise.

## Multiply and Divide with Scientific Notation

*(Pairs with Skill Section 5.2)*

### Main Idea

Multiplying or dividing with scientific notation numbers follows the same process used with standard form numbers. It just looks different because the multiplication is written as \(\times\) in the scientific notation. Here is a multiplication example comparing the two forms.

\[
\begin{align*}
\text{Rearrange by the Commutative Property.} & \quad (2)(10^3)(3)(10^5) \\
\text{Multiply} & \quad 2 \cdot 3 = 6 \\
\text{and} \ 10^3 \cdot 10^5 = 10^8. \\
\end{align*}
\]

These look different, but they represent the same value.

### Process

How to Multiply and Divide with Scientific Notation

- **Multiply or divide** the coefficients.
- **Use the rules of exponents** to determine the final exponent of the 10.
- **Write the final answer in scientific notation.**
Section 5.8

**IMPORTANT NOTES**

- If applying an exponent rule results in $10^0$, remember that $10^0 = 1$.
- Use the Exponent Rule Summary Chart as needed.

**EXAMPLE 3**

Perform the indicated operation. Write your final answer in scientific notation.

a) \((1.3 \times 10^6)(1.2 \times 10^3)\)

Multiply the coefficients. Use the product rule for exponents.

\[(1.3 \cdot 1.2) \times 10^{6+3}\]

\[1.56 \times 10^9\]

When multiplying expressions with same bases, add the exponents.

b) \(\frac{4.25 \times 10^{-3}}{8.5 \times 10^{12}}\)

Divide the coefficients. Use the quotient rule for exponents.

\[\left(\frac{4.25}{8.5}\right) \times 10^{-3-12}\]

\[0.5 \times 10^{-15}\]

For scientific notation, the coefficient must have exactly one nonzero digit to the left of the decimal point.

Write in scientific notation. Write 0.5 as \(5 \times 10^{-1}\).

\((5 \times 10^{-1}) \times 10^{-15}\)

Use the product rule.

\[5 \times 10^{-16}\]

**YOUR TURN 3**

Perform the indicated operation. Write your final answer in scientific notation.

a) \(\frac{2.53 \times 10^5}{1.1 \times 10^2}\)

Divide the coefficients and use the quotient rule for exponents.

b) \((5.3 \times 10^{-8})(4.2 \times 10^3)\)

Multiply the coefficients and use the product rule for exponents.

Remember, in your final answer, there should be exactly one nonzero digit to the left of the decimal point.

Go to the eText for more practice.
5.8-72

**ON YOUR OWN 3**

Perform the indicated operation. Write your final answer in scientific notation.

a) \((2 \times 10^4)(4 \times 10^{-9})\)

b) \(\frac{8 \times 10^{-7}}{2 \times 10^6}\)

c) \(\frac{1.4 \times 10^6}{7 \times 10^{-2}}\)

---

**Solve Applications Involving Scientific Notation**

**MAIN IDEA**

When applications involve very large or very small numbers, scientific notation is usually used. Your goal here is to become comfortable using scientific notation for applications that may or may not include calculations.

**IMPORTANT NOTES**

- Use the RTSC Approach to guide you through the problems.

**EXAMPLE 4**

**U.S. Population**

The population of the United States is approximately 309,000,000. Write this number in scientific notation.

\[309,000,000 \text{ people} = 3.09 \times 10^8 \text{ people}\]

Value of \(n\): 8 spaces to 3.09

Sign of \(n\): 309,000,000 is larger than 3.09; so the exponent is +8.

**YOUR TURN 4**

**Wolffia Flower**

One of the smallest flowering plants in the world, the *Wolffia* flower, weighs approximately \(5.3 \times 10^{-6}\) ounce. Write this number in standard form.

Will multiplying 5.3 by \(10^{-6}\) result in a larger or smaller number? Enter your answer in the eText.

Go to the eText for more practice.
Section 5.8

**EXAMPLE 5**

**Space Shuttle Travel**

The space shuttle must reach a speed of \(1.75 \times 10^4\) miles per hour to remain in orbit. If the space shuttle maintains this speed, how far will it travel in 4 hours? Express your answer in scientific notation.

- **Read**: 1.75 \(\times\) 10\(^4\) miles per hour for 4 hours
- **Translate**: \((1.75 \times 10^4) \cdot 4\)
- **Solve**: \((4 \cdot 1.75) \times 10^4\)
  \[7 \times 10^4\text{ miles}\]
- **Check**: Does the answer make sense? Yes, the distance traveled in 4 hours should be larger than the distance, 1.75 \(\times\) 10\(^4\) miles, that can be traveled in 1 hour.

**YOUR TURN 5**

**Mosquitoes**

Twenty mosquitoes weigh approximately \(3 \times 10^{-2}\) gram. Approximately how much does one mosquito weigh?

- **Read**
- **Translate**
- **Solve**
- **Check**

Go to the eText for more practice.

**ON YOUR OWN 4**

- **a)** It is estimated that approximately 250,000,000 people in the United States subscribe to a cell phone plan. Write this number in scientific notation.

- **b)** Recently, a group of 8 college students shared a winning lottery ticket that paid a total of \(1.05 \times 10^8\). How much did each student win if the winnings were split evenly? Express your answer in both standard form and scientific notation.

Go to the eText for the online Reading ✓ exercise.

Go to the eText for your instructor’s online assignment.
5.8 Section Summary

Fill in the blanks to make true statements.

### KEY CONCEPTS

#### How to Write a Number in Scientific Notation
- **Write** $a$ by moving the ________ ________ until there is exactly ________ nonzero digit to the left of the decimal.
- **Count the number of places** you moved the ________ ________. This is the absolute value of the exponent, $n$.
- **Find the sign** of the exponent.
  - If the standard form number is ________ than $a$, the exponent is positive.
  - If the standard form number is ________ than $a$, the exponent is negative.
- **Write the final answer in the form** ________.

#### How to Write Scientific Notation in Standard Form
- **Move the decimal point** a number of spaces equal to the absolute value of the ________.
- **Move** ________ if the exponent is positive.
  - Move ________ if the exponent is negative.

#### How to Multiply and Divide with Scientific Notation
- **Multiply or divide** the ________.
- **Use the rules of exponents** to determine the final exponent of the ________.
- **Write the final answer in scientific notation.**

### EXAMPLES

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>Scientific Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>6,780,000</td>
<td>$6.78 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

6,780,000 is ________ than 6.78; so the exponent is ________.

### Vocabularies
- **scientific notation**, **standard form**, **coefficient**

### Additional Exercises

1. The first step in writing 62,000,000 in scientific notation is to ________.
   - a) move the decimal point to the left 6 places
   - b) move the decimal point to the left 7 places
   - c) move the decimal point to the right 7 places

2. The first step in writing 0.0000925 in scientific notation is to ________.
   - a) move the decimal point to the right 7 places
   - b) move the decimal point to the right 6 places
   - c) move the decimal point to the right 5 places

3. The first step in writing $1.03 \times 10^{-4}$ in standard form is to ________.
   - a) move the decimal point 4 places to the left
   - b) move the decimal point 4 places to the right
   - c) move the decimal point 3 places to the left
Section 5.8

4. The first step in writing \( 9.02 \times 10^5 \) in standard form is to \___________.
   a) move the decimal point \( 4 \) places to the right
   b) move the decimal point \( 5 \) places to the right
   c) move the decimal point \( 5 \) places to the left

5. The first step in performing the indicated operation for \( (1.7 \times 10^{-3})(2 \times 10^8) \) is to \___________.
   a) find \( 1.7 \times 2 \)
   b) find \( (1.7)(2) \)
   c) find \( 1.7 \div 2 \)

6. The first step in performing the indicated operation for \( (6.2 \times 10^{-3}) \div (3.1 \times 10^5) \) is to \___________.
   a) find \( (6.2)(3.1) \)
   b) find \( 6.2 \div 3.1 \)
   c) find \( 6.2 + 3.1 \)

Choose the best translation of the given word problem.

7. If 75% of \( 3.09 \times 10^8 \) people own cell phones, how many people own a cell phone?
   a) \( (3.09 \times 10^8)(75) \)
   b) \( (3.09 \times 10^8)(0.75) \)
   c) \( (3.09 \times 10^8) \div (0.75) \)

8. If \( 2.5 \times 10^3 \) standard staples have a mass of 7.5 ounces, what is the mass of one staple in ounces?
   a) \( \frac{2.5 \times 10^3}{7.5} \)
   b) \( (2.5 \times 10^3)(7.5) \)
   c) \( \frac{7.5}{2.5 \times 10^3} \)

PRACTICE PROBLEMS

A Write the following numbers in scientific notation. See Example 1.

9. \( 5,020,000 \)
10. \( 0.00045 \)
11. \( 0.000098 \)
12. \( 10,300 \)
13. \( 6,030,000,000 \)
14. \( 0.00785 \)
15. \( 0.0000905 \)
16. \( 2,300 \)

B Write the following numbers in standard form. See Example 2.

17. \( 3.1 \times 10^5 \)
18. \( 2.75 \times 10^{-3} \)
19. \( 6.18 \times 10^{-6} \)
20. \( 9.1 \times 10^7 \)
21. \( 1.17 \times 10^5 \)
22. \( 4.41 \times 10^{-4} \)
23. \( 5.11 \times 10^{-2} \)
24. \( 7.1 \times 10^8 \)

C Perform the indicated operation. Write your final answer in scientific notation. See Example 3.

(5.2)

25. \( (3.5 \times 10^5)(2.1 \times 10^{-3}) \)
26. \( (2.0 \times 10^{-5})(4 \times 10^{12}) \)
27. \( 9.0 \times 10^{-6} \div 2.0 \times 10^7 \)
28. \( 7.0 \times 10^{-3} \div 4.0 \times 10^{-9} \)
29. \( (6.0 \times 10^{-13})(1.2 \times 10^6) \)
30. \( 4.2 \times 10^6 \div 1.4 \times 10^{-3} \)
31. \( 6.4 \times 10^{-8} \div 1.6 \times 10^{-2} \)
32. \( (3.2 \times 10^{-12})(3.1 \times 10^9) \)
33. \( 2.8 \times 10^{-6} \div 7 \times 10^{-3} \)
34. \( 2.72 \times 10^5 \div 6.8 \times 10^{-3} \)
35. \( (8 \times 10^{-7})(2.3 \times 10^{15}) \)
36. \( (6.7 \times 10^6)(5.0 \times 10^{-8}) \)
Solve the following applications. See Examples 4 and 5. Write the following numbers in scientific notation.

37. The Trans-Alaska pipeline was completed at a cost of $7,000,000,000.
38. The world’s smallest chameleon is 0.0000254 inch in length.
39. One ounce equals approximately 0.0283 kilogram.
40. The sun is approximately 93,000,000 miles from the Earth.
41. The Earth’s circumference is approximately 40,000,000 meters.
42. Each adult has roughly 2,500,000,000,000 red blood cells in his or her body.

Write the following numbers in standard form.

43. Approximately $3.09 \times 10^8$ people live in the United States.
44. Alaska was purchased from Russia for approximately $7.2 \times 10^6$.
45. Did you know that a dust particle may weigh only $7.53 \times 10^{-10}$ kilogram?
46. The smallest insect, the fairy fly, is no longer than $1.39 \times 10^{-2}$ millimeter.
47. The speed of light is approximately $3 \times 10^8$ meters per second.
48. The smallest bird, the bee hummingbird, weighs only $3.937 \times 10^{-3}$ pound.

Solve the following problems requiring calculations with scientific notation.

49. Travel to the Moon A rocket ship travels at $2 \times 10^4$ miles per hour. If the moon is $2.5 \times 10^5$ miles from Earth, how long will it take the rocket to make the trip from Earth to the moon? (Hint: Divide the distance to the moon by the speed.) Write your answer in scientific notation.
50. Public Debt The outstanding public debt is approximately $1.24 \times 10^{13}$. With the U.S. population at approximately $3.09 \times 10^8$, what is each person’s equal share of the debt? (Hint: Divide the outstanding public debt by the U.S. population.) Write your answer in scientific notation. Round your answer to two decimal places.
51. Professional Athlete Pay A professional athlete signed a contract that would pay her $1.6 \times 10^6$ per month. How much money will the athlete have made after 2 years? Write your answer in scientific notation.
52. The Human Heart The average human heart pumps $1.9 \times 10^3$ gallons of blood per day. How many gallons of blood are pumped in 2 months? (Use 30 days per month.) Write your answer in scientific notation.

CHAPTER ✓

Simplify. Write your final answer using only positive exponents. Assume that all variables represent nonzero real numbers.

53. \((3m^{-2}n)^{-1}(5m^3n^{-2})^{-3}\) 54. \(\frac{(x^{-7}y^2)^{-5}}{(2x^2y^{-3})^{-2}}\)
Section 5.9

Perform the indicated operation.

55. \((6x^2 - 7x + 12) + (-8x^2 - 4)\)
56. \((2x^2 - 5x + 6) - (-3x^2 + 2x - 11)\)
57. \((4x - 1) + (-6x - 7) - (-8x + 2)\)
58. \((-5m^n)(-2m^n)\)
59. \((3t - 7)(2t - 6)\)
60. \((a - 2)(a^2 - a + 3)\)
61. \(-45y^5 - 9y^4 \div -9y^3\)
62. \((2x^2 + 3x - 2) \div (2x - 1)\)
63. Write \(6.18 \times 10^{-6}\) in standard form.
64. Write 0.00785 in scientific notation.

5.9-77

5.9 Applying Polynomials

OBJECTIVES
A Evaluate formulas involving polynomials
B Solve applications involving adding and subtracting polynomials
C Solve applications involving multiplying polynomials
D Solve applications involving dividing polynomials

PAIRS WITH VOCABULARY
Section 5.3 divisor
Section 5.4 dividend
Sections 5.5, 5.6
Section 5.7

Evaluate Formulas Involving Polynomials
(Pairs with Skill Section 5.3)

MAIN IDEA
Evaluate polynomials.

Given a polynomial that represents a real-life situation, evaluate the value of the polynomial at a specific value for the variable.

IMPORTANT NOTES
- Use the RTSC Approach to guide you through the problems.
- Always ask yourself what the answer means in terms of the real situation the polynomial represents.
- Often in application problems, a polynomial will be a reliable representation of the real situation for only certain values of the variable.
EXAMPLE 1

**Business**

The marketing department of a manufacturing firm estimates that \( n \) months after the newest product is introduced to the market, the number of thousands of households using the product will be given by

\[-n^2 + 12n, \text{ for } 0 \leq n \leq 12.\]

Estimate how many thousands of households will be using the product 3 months after it is introduced to the market.

**Read / Translate**

Evaluate \(-n^2 + 12n\) for \( n = 3 \).

**Solve**

Rewrite the expression. \(-n^2 + 12n\)

Replace \( n \) with 3. \(-(3)^2 + 12(3)\)

Simplify. Evaluate the exponent, \((3)^2 = 9\). \(-9 + 12(3)\)

Multiply \(12(3)\). \(-9 + 36\)

Add. \(27\)

After 3 months, approximately **27,000 households** will be using the product.

**Check**

Does the answer make sense? Yes, 27,000 is a realistic answer for the number of households. (An unrealistic number would have been a negative number, for example.)

YOUR TURN 1

**Business**

A manufacturer determines that when he has \( m \) employees working on a unit, the total number of units produced per day is approximated by

\[-0.5m^2 + 20m.\]

Estimate the number of units produced per day when 20 employees are working on the unit.

**Read / Translate**

**Solve**

Replace \( m \) with 20; then simplify.

Is your answer realistic?

**Check**

Go to the eText for more practice.
ON YOUR OWN 1

Biology
In a biology class experiment, the average weight gain (in grams) of rats fed a protein mix containing $P$ percent yeast was found to be approximately

$$-0.03P^2 + 2P + 20$$

for $0 \leq P \leq 100$.

Estimate the average weight gain when the percent yeast was 10%.

Go to the eText for the online Reading ✓ exercise.

Solve Applications Involving Adding and Subtracting Polynomials
(Pairs with Skill Section 5.4)

MAIN IDEA
Given more than one polynomial that represents a real situation, find the new polynomial requested by adding or subtracting the given polynomials.

IMPORTANT NOTES
• Use the RTSC Approach to guide you through the problems.
• To find the perimeter of a shape, add the lengths of all of its sides.
• Use the Formula Chart as needed.

EXAMPLE 2

Perimeter
Find the polynomial that represents the perimeter of the square shown.

Read
The perimeter equals the sum of the lengths of all sides.

Translate
$$2(x^2 + x) + (2x^2 + x) + (2x^2 + x) + (2x^2 + x)$$

Solve
Distribute to remove parentheses.
$$2x^2 + x + 2x^2 + x + 2x^2 + x + 2x^2 + x$$
Combine like terms.
$$8x^2 + 4x$$

Check
Does the answer make sense? Yes, because the expression for perimeter equals a larger value than the expression for the length of a side any time a positive number is substituted for $x$.

YOUR TURN 2

Perimeter
Find the polynomial that represents the perimeter of the triangle shown.

Read

Translate
5.9-80

Solve

Check

Go to the eText for more practice.

ON YOUR OWN 2

Perimeter
Find the polynomial that represents the perimeter of the rectangle shown.

\[(x + 3)\]

\[(2x^2 + 3x + 1)\]

Go to the eText for the online Reading ✓ exercise.

Solve Applications Involving Multiplying Polynomials
(Pairs with Skill Sections 5.5, 5.6)

MAIN IDEA

Given more than one polynomial that represents a real situation, find the new polynomial requested by multiplying the given polynomials.

When you have two binomials being multiplied, determine whether you can use a special product formula.

IMPORTANT NOTES

- Use the RTSC Approach to guide you through the problems.
- The area of a square is the length of a side squared; the area of a rectangle is length times width; the area of a triangle is \( \frac{1}{2} \) base times height.
- When you have two binomials to which a special product formula applies, you can use the special product formula or just multiply the binomials using the general process for multiplying polynomials.
- Use the Formula Chart as needed.
EXAMPLE 3

**Area**

Find the polynomial that represents the area of the square shown.

\[(x + 2)^2\]

**Read / Translate**

The area of a square is the length of a side squared.

**Solve**

Determine whether a special product formula can be used.

Use the formula for squaring a binomial.

\[x^2 + 2(2x) + 2^2\]

\[x^2 + 4x + 4\]

**Check**

Does the answer make sense? Yes. Here’s why: Suppose \(x = 1\). Then the square would have sides of length 3, and the area would be \(3^2 = 9\). Letting \(x = 1\) in our answer of \(x^2 + 4x + 4\), we get the same area, 9.

**Alternative Method:** Use FOIL (or the general process for multiplying polynomials).

The area of a square is the length of a side squared.

\[(x + 2)^2\]

\[(x + 2)(x + 2)\]

Distribute using FOIL.

\[x \cdot x + x \cdot 2 + 2 \cdot x + 2 \cdot 2\]

Multiply.

\[x^2 + 2x + 2x + 4\]

Combine like terms.

\[x^2 + 4x + 4\]

**YOUR TURN 3**

**Area**

Find the polynomial that represents the area of the rectangle shown.

\[(x + 9)(2x + 3)\]

**Read**

**Translate**

**Solve**

Is there a special product formula for something like \((2x + 3)(x + 9)\)?

If not, use FOIL (or the general process for multiplying two polynomials).

**Check**

Go to the eText for more practice.
Area
Find the polynomial that represents the area.

a) \((x + 3)\)  
\((4x + 5)\)

b) \((x + 5)\)  
\((x + 5)\)

Go to the eText for the online Reading ✓ exercise.

Solve Applications Involving Dividing Polynomials
(Pairs with Skill Sections 5.7)

MAIN IDEA

Given more than one polynomial that represents a real situation, find the new polynomial requested by dividing the given polynomials.

IMPORTANT NOTES

• Use the RTSC Approach to guide you through the problems.
• Remember, order matters for division. It’s important to correctly identify the divisor and the dividend.
• Always identify whether you’re dividing by a monomial or a polynomial. That will determine which process to use.

EXAMPLE 4

Find the Width

The area of a rectangle is \(x^2 + 4x + 3\) square inches, and its length is \((x + 3)\) inches. Find its width.

Read
The area of a rectangle is length times width.
area = \((x^2 + 4x + 3)\); length = \((x + 3)\)

Translate
To find the width, divide area by length.

\[
rac{(x^2 + 4x + 3)}{(x + 3)}
\]

Width = \(\frac{\text{Area}}{\text{Length}}\)
Section 5.9

Find the Length

The area of a rectangle is $12x^2 + 3x + 1$ square inches, and its width is $(x + 3)$ inches. Find its length.

**Read**

$(4x^2 + 8x)$

? 

**Translate**

First, decide if it is area divided by width or width divided by area.

Then decide if you’re dividing by a monomial or a polynomial and use the appropriate process.

**Solve**

Round 1

\[
\begin{align*}
\frac{x + 1}{x + 3} & \frac{x^2 + 4x + 3}{(x^2 + 3x)} \\
& \frac{x + 1}{x + 3} \\
& 0
\end{align*}
\]

The width is the quotient, $x + 1$.

**Check**

Does the answer make sense? Yes, because the width (answer) times the given length equals the given area. 

\[(x + 1)(x + 3) = x^2 + 3x + x + 3 = x^2 + 4x + 3\]

**YOUR TURN 4**

Find the Length

The area of a rectangle is $(4x^2 + 8x)$ square inches, and its width is $2x$ inches. Find its length.

**Read**

$(4x^2 + 8x)$

$2x$

? 

**Translate**

First, decide if it is area divided by width or width divided by area.

Then decide if you’re dividing by a monomial or a polynomial and use the appropriate process.

**Solve**

Round 1

\[
\begin{align*}
\frac{x + 1}{x + 3} & \frac{x^2 + 4x + 3}{(x^2 + 3x)} \\
& \frac{x + 1}{x + 3} \\
& 0
\end{align*}
\]

The width is the quotient, $x + 1$.

**Check**

Does the answer make sense? Yes, because the width (answer) times the given length equals the given area. 

\[(x + 1)(x + 3) = x^2 + 3x + x + 3 = x^2 + 4x + 3\]
Fill in the blanks to make true statements.

**KEY CONCEPTS**

**How to Evaluate a Polynomial (5.3)**
- _______ the variable with the given value.
- Simplify the result using the correct _______ _______ _______.

**EXAMPLES**

The marketing department of a manufacturing firm estimates that \( n \) months after the newest product is introduced to the market, the number of thousands of households using the product will be given by \(-n^2 + 12n\), for \( 0 \leq n \leq 12\). Estimate how many thousands of households will be using the product 3 months after it is introduced to the market.

Evaluate \(-n^2 + 12n\) for \( n = __\).

\[ -(__)^2 + 12(__) \]

\[ -- + -- \]

Answer: _______ households

**How to Add and Subtract More Than Two Polynomials (5.4)**
- _______ to remove parentheses and to change any subtractions to “add the opposite.”
- Combine _______ _______.

Find the polynomial that represents the perimeter of the square shown.

The perimeter equals the sum of the lengths of all sides.

\((2x^2 + x)\)

\((2x^2 + x) + (2x^2 + x) + (______) + (______)\)

\(2x^2 + x + 2x^2 + x + _____ + _____\)

\(_____x^2 + _____x\)

**How to Multiply Two Polynomials (5.5, 5.6)**
- Determine whether a special product formula can be used. Apply if possible.
  \((a + b)^2 = a^2 - 2ab + b^2;\)
  \((a - b)^2 = a^2 - 2ab + b^2\)

If not:
- Distribute and multiply each _______ in the first polynomial by each _______ in the second polynomial.
- Combine like _______.

Find the polynomial that represents the area of the square shown.

The area of a square is the length of a side squared.

\((x + 2)\)

\((x + 2)(x + 2)\)

\((x + 2)\)

\((x + 2)\)

\(-x + __(2x) + 2--\)

\(-x + _x + _\)
Section 5.9

How to Divide a Polynomial by a Monomial (5.7)
• Divide each _________ of the polynomial by the _________.
• Simplify.

How to Divide a Polynomial by a Polynomial (5.7)
• Write the polynomial in descending _________.
• Add _________ for any missing terms.
• For as many rounds as necessary: _________ _________ _________ _________

Example 1
The area of a rectangle is \((x^2 + 4x + 3)\) square inches, and its length is \((x + 3)\) inches. Find its width.

The area of a rectangle is length times width. To find the width, divide area by length.

\[
\frac{x + 3}{x^2 + 4x + 3}
\]

Round 1
\[
\begin{align*}
1 &+ \\
\hline \\
1 &+ \\
\end{align*}
\]

Round 2
\[
\begin{align*}
1 &+ \\
\hline \\
1 &+ \\
\end{align*}
\]

Answer: _________ + _________

Vocabulary divisor, dividend

5.9 Additional Exercises

Reading ✓

Choose the best response for the following problems.

1. A marketing firm used the polynomial \((-n^2 + 15n)\) to predict the number of thousands of households using a new product \(n\) months after its release. Three months after release, the number of households using the product was actually 40,000. Choose the best response.
   a) The prediction was 10,000 too high.
   b) The prediction was correct.
   c) The prediction was 4,000 too low.

2. A biology lab assistant uses the polynomial \((-0.04P^2 + 2P + 25)\) to predict the average weight gain (in grams) of rats fed a protein mix containing \(P\) percent yeast. The actual weight gain when \(P = 5\) was measured to be 34 grams. Choose the best response.
   a) The prediction was 12 grams too high.
   b) The prediction was correct.
   c) The prediction was 8 grams too low.

3. Choose the correct first step in finding the perimeter of the following rectangle.
   a) \((x^2 + 5x) + (x + 2)\)
   b) \((x + 2) + (x + 2) + (x^2 + 5x) + (x^2 + 5x)\)
   c) \((x^2 + 5x)(x + 2)\)

4. Choose the correct first step in finding the perimeter of the following triangle.
   a) \((x^2 + 2x)(x^2 + 2x)(3x^3 + x)\)
   b) \((x^2 + 2x) + (x^2 + 2x) + (3x^3 + x)\)
   c) \((x^2 + 2x) + (x^2 + 2x) + (3x^3 + x)\)

\[
\begin{align*}
&+ \\
\hline \\
&+ \\
\end{align*}
\]
5. Choose the correct first step in finding the area of the following rectangle.
   a) \((x + 1)(x^2 + 5x + 4)\)
   b) \((x^2 + 5x + 4) + (x + 1)\)
   c) \((x + 1)(x^2 + 5x + 4)(x + 1)(x^2 + 5x + 4)\)

6. Choose the correct first step in finding the area of the following triangle.
   a) \((x + 3)(x + 4)\)
   b) \(\frac{(x + 3) + (x^2 + 4x)}{2}\)
   c) \(\frac{1}{2}(x^2 + 4x)(x + 3)\)

7. Given the area \((x^2 + 7x + 12)\) and the width shown, choose the correct expression for finding the missing length of the rectangle.
   a) \(\frac{(x + 4)}{(x^2 + 7x + 12)}\)
   b) \(\frac{(x^2 + 7x + 12)}{(x + 4)}\)
   c) \((x + 4)(x^2 + 7x + 12)\)

8. Given the area \((x^2 + 8x + 12)\) and the length shown, choose the correct expression for finding the missing width of the rectangle.
   a) \(\frac{(x^2 + 8x + 12)}{(x + 2)}\)
   b) \(\frac{(x^2 + 8x + 12)}{(x + 2)}\)
   c) \((x^2 + 8x + 12)(x + 2)\)

PRACTICE PROBLEMS

A. Evaluate formulas involving polynomials. See Example 1.

9. Business The marketing department of a manufacturing firm estimates that \(n\) months after the newest product is introduced to the market, the number of thousands of households using the product will be given by
   \[-n^2 + 12n, \text{ for } 0 \leq n \leq 12.\]
   Estimate how many households will be using the product 6 months after it is introduced to the market.

10. Business The marketing department of a manufacturing firm estimates that \(n\) months after the newest product is introduced to the market, the number of thousands of households using the product will be given by
    \[-0.5n^2 + 6n, \text{ for } 0 \leq n \leq 12.\]
    Estimate how many households will be using the product 6 months after it is introduced to the market.

11. Business In the gardening department of an outdoor store, the daily profit is approximately
    \[-x^2 + 16x + 140,\]
    where \(x\) is the number of plants sold. Estimate the daily profit when 8 plants are sold.
12. **Business**  In the gardening department of an outdoor store, the daily profit is approximately 

\[-2x^2 + 40x + 120,
\]

where \(x\) is the number of plants sold. Estimate the daily profit when 10 plants are sold.

13. **Physics**  During a science fair competition, a student's mini-rocket was shot straight up from the ground at a speed of 80 feet per second. Ignoring air resistance, the height (in feet) of the mini-rocket at time \(t\) is given by

\[-16t^2 + 80t.
\]

What is the height of the mini-rocket 3 seconds after launch?

14. **Physics**  During a science fair competition, a student's mini-rocket was shot straight up from the ground at a speed of 75 feet per second. Ignoring air resistance, the height (in feet) of the mini-rocket at time \(t\) is given by

\[-16t^2 + 75t.
\]

What is the height of the mini-rocket 2 seconds after launch?

15. **Criminal Justice**  The re-offense rate (in percent) at a state prison was approximated by the polynomial

\[-0.01x^2 + 1.44x + 5, \text{ for } 0 \leq x \leq 60,
\]

where \(x\) is the number of months after release. Estimate the re-offense rate 24 months after release.

16. **Criminal Justice**  The re-offense rate (in percent) at a state prison was approximated by the polynomial

\[-0.01x^2 + 1.5x + 4, \text{ for } 0 \leq x \leq 60,
\]

where \(x\) is the number of months after release. Estimate the re-offense rate 60 months after release.

17. **Surface Area**  The surface area (in \(\text{cm}^2\)) of a cylindrical can with height 6 cm and radius \(r\) can be approximated by

\[6.28r^2 + 37.68r.
\]

What is the surface area when the radius is 4 cm?

18. **Surface Area**  The surface area (in \(\text{cm}^2\)) of a cylindrical can with height 6 cm and radius \(r\) can be approximated by

\[6.28r^2 + 37.68r.
\]

What is the surface area when the radius is 6 cm?

---

**B** Solve applications involving adding and subtracting polynomials. *See Example 2.*

19. **Perimeter**  Find the polynomial that represents the perimeter of the square shown.

\[(3x^2 + 2x)\]

20. **Perimeter**  Find the polynomial that represents the perimeter of the triangle shown.

\[(2x^2 + x) \quad (2x^2 + x) \quad (3x^3 + 4x)\]
21. Surface Area  A cylindrical can has a height of 2 units. The surface area of each circular end of the can is approximately $3.14r^2$, where $r$ is the radius. The surface area of the wall of the can is approximately $12.56r$. Find the polynomial that represents the total surface area.

22. Surface Area  A cylindrical can has a height of 4 units. The surface area of each circular end of the can is approximately $3.14r^2$, where $r$ is the radius. The surface area of the wall of the can is approximately $25.12r$. Find the polynomial that represents the total surface area.

23. Physics  Two toy rockets were shot straight up at the same time from the top of a 500-foot building. The first one was shot at a speed of 20 feet per second; the second, at 30 feet per second. Ignoring air resistance, the height (in feet) of each toy rocket at time $t$ (in seconds) is given as follows:

First rocket: $-16t^2 + 20t + 500$  
Second rocket: $-16t^2 + 30t + 500$

Find a polynomial that represents the difference in the height between the second rocket and the first rocket at time $t$.

24. Physics  Two toy rockets were shot straight up at the same time from the top of a 200-foot building. The first one was shot at a speed of 25 feet per second; the second, at 30 feet per second. Ignoring air resistance, the height (in feet) of each toy rocket at time $t$ (in seconds) is given as follows:

First rocket: $-16t^2 + 25t + 200$  
Second rocket: $-16t^2 + 30t + 200$

Find a polynomial that represents the difference in the height between the second rocket and the first rocket at time $t$.

25. Area  Find the polynomial that represents the area of the triangle shown. (Hint: The area, $A$, of a triangle with base, $b$, and height, $h$, is $A = \frac{1}{2}bh$.)

26. Area  Find the polynomial that represents the area of the triangle shown. (Hint: The area, $A$, of a triangle with base, $b$, and height, $h$, is $A = \frac{1}{2}bh$.)

27. Physics  A mini-rocket is shot straight up from the ground at 48 feet per second. Ignoring air resistance, the height (in feet) of the mini-rocket at time $t$ (in seconds) is given as follows:

$-16(t - 3)$.

Find the polynomial that represents the height at time $t$. 

C Solve applications involving multiplying polynomials. See Example 3. (5.5, 5.6)
28. **Physics**  A mini-rocket is shot straight up from the ground at 64 feet per second. Ignoring air resistance, the height (in feet) of the mini-rocket at time $t$ (in seconds) is given as follows:

$$-16(t - 4).$$

Find the polynomial that represents the height at time $t$.

29. **Area**  Find the polynomial that represents the area of the square shown.

![Square with side length $(2x + 5)$](image1)

30. **Area**  Find the polynomial that represents the area of the square shown.

![Square with side length $(3x + 2)$](image2)

31. **Money**  A student deposits $100 in an account at an interest rate of $r$. If the interest is compounded annually, after 2 years, the amount of money in the account is given by

$$100(1 + r)^2.$$ 

Find the polynomial that represents the amount of money in the account after 2 years.

32. **Money**  A student deposits $500 in an account at an interest rate of $r$. If the interest is compounded annually, after 2 years, the amount of money in the account is given by

$$500(1 + r)^2.$$ 

Find the polynomial that represents the amount of money in the account after 2 years.

---

**D** Solve applications involving dividing polynomials. *See Example 4.*

(5.7)

33. **Find the Width**  The area of a rectangle is $(4x^2 + 8x)$ square meters, and its length is $2x$ meters. Find its width.

![Rectangle with area $(4x^2 + 8x)$ and length $2x$](image3)

34. **Find the Width**  The area of a rectangle is $(2x^2 + 10x)$ square meters, and its length is $2x$ meters. Find its width.

![Rectangle with area $(2x^2 + 10x)$ and length $2x$](image4)

35. **Find the Length**  The area of a rectangle is $(x^2 + 8x + 15)$ square inches, and its width is $(x + 3)$ inches. Find its length.

![Rectangle with area $(x^2 + 8x + 15)$ and width $(x + 3)$](image5)
36. Find the Length
The area of a rectangle is \((x^2 + 6x + 5)\) square inches, and its width is \(2x\) inches.

Find its length.

\[
\begin{align*}
(x^2 + 6x + 5) & \quad 2x \\
? & 
\end{align*}
\]

CHAPTER ✔

Simplify. Write your final answer using only positive exponents. Assume that all variables represent nonzero real numbers.

37. \[
\frac{(x^3y^4)^5}{(3x^3y^{-1})^{-3}}
\]

38. \[
(2m^{-2}n)^{-1}(4m^5n^{-4})^{-3}
\]

Perform the indicated operation.

39. \((3x^2 - 5x) + (6x^2 + 2x - 1)\)

40. \((x^2 - 4x + 9) - (-5x^2 + 3x - 4)\)

41. \((2x - 1) + (-7x - 9) - (-2x + 5)\)

42. \((-4m^5n)(-2m^4n^3)\)

43. \((6t - 5)(3t - 1)\)

44. \((a + 2)(a^2 - a + 4)\)

45. \[
\frac{-30y^6 + 4y^5}{-2y^3}
\]

46. \((3x^2 + 13x + 4) + (3x + 1)\)

47. Write \(7.05 \times 10^{-8}\) in standard form.

48. Write \(0.0000315\) in scientific notation.

49. Business
A manufacturer determines that when he has \(m\) employees working on a unit, the total number of units produced per day is approximated by the polynomial \((-0.4m^2 + 15m)\). Estimate the number of units produced per day when 25 employees are working on the unit.

50. Area
Find the polynomial that represents the area of the square shown.

\[
(2x + 1)
\]
Answers

CHAPTER 5  Section 5.1 Your Turn 1: a) 128; b) −125; 2: a) (−4)8; b) m7; c) −10x6; d) 16x11y5; 3: a) 221; b) −y6; 4: a) 64a4; b) 128x21y28; c) $x^{\frac{3}{7}}$

Section 5.1 On Your Own 1: a) 48; b) $\frac{8}{25}$; c) −1215; 2: a) (−3)10; b) y14; c) 3x4y11; 3: a) 28; b) y18; c) −z21; 4: a) 27p5; b) 125a4b3c6; c) $\frac{8x^6}{125r^7}$

Section 5.1 Additional Exercises For the Section Summary, see Chapter 5 Chapter Summary in the eText.

Section 5.2 Your Turn 1: a) 1; b) 1; c) −1; d) 1; 2: a) $\frac{1}{64}$ b) $\frac{6}{a^5}$ c) −49; 3: a) $\frac{3}{2x}$ b) $\frac{2}{y}$ c) $\frac{2y^6}{x^3}$; 4: a) $\frac{4x^3}{3r^5}$ b) 40x3y2

Section 5.2 On Your Own 1: a) 1; b) 1; c) 2; 2: a) $\frac{1}{x^3}$ b) $\frac{81}{16}$ c) $\frac{5x^3y^2}{y^4}$ d) $\frac{3}{4}$; 3: a) $\frac{1}{216}$ b) $\frac{27}{m^9}$ c) $\frac{4x^3y^5}{5y^7}$; d) $\frac{b^9}{10a^3}$

Section 5.2 Additional Exercises For the Section Summary, see Chapter 5 Chapter Summary in the eText.

Section 5.3 Your Turn 1: −2x, −2, 1; 6x3, 6, 5; −3x4y3, −3, 5, 8, 8, 0; 2: a) $8x^8 − 2x^3 − 3x + 5$; b) 8; c) 4, none of these; 3: 152; 4: $−10y^6 + 6y^3$

Section 5.3 On Your Own 1: a) $−9x^3$, −9, 3; −2x2, −2, 2; 5x, 5, 1; 16, 16, 0 b) $6x^2$, 6, 2; 4x3y2, 4, 4 c) $\frac{x}{2}$, $\frac{1}{2}$; 1; 2x2, 2, 3; −5, −5, 0; 2: a) $−7x^8 + 27$, 8, binomial b) 43x, 1, monomial c) $−2x^4 + 3x^2 + 16$, 4, trinomial; 3: a) 4; b) −436; c) 41; 4: a) −5ab; b) $6m^2 − 3mn$; c) $\frac{5}{3}z^2$

Section 5.3 Additional Exercises For the Section Summary, see Chapter 5 Chapter Summary in the eText.

Section 5.4 Your Turn 1: −12x3 + x2 + 5x − 11; 2: $18y^3 + 9y^2 − 10y$; 3: $−15a^2 − 5u + 8$; 4: $5a^2b − a^2 + 9b$
Section 5.4 On Your Own
1: \(-2x^2 - 2x^2 + 11x; b) -2x^2 - 3x - 6; c) 7x^4 + 3x^3 - 6x^2 - 2x - 4;
2: a) \(10x^2 + 5x + 9; b) -7y^2 - 11y - 6; c) -11r^3 - 6r^2 + 5r - 22; 3: a) -5m^2 + 2m - 3; b) 5a^2 + 2a - 5; 
4: a) 13p^2 - 9y + 6q; b) 9xy^2 - 19x + 11y

Section 5.4 Additional Exercises
For the Section Summary, see Chapter 5 Chapter Summary in the eText.

Section 5.5 On Your Own
1: \(-24x^2y^3t^4; 2: -15b^5 + 25b^4 - 35b^3 - 15b^2; 3: 3y^2 - 13y - 30; 4: 6x^3 - 2x^2 - 46x + 42

Section 5.5 Additional Exercises
For the Section Summary, see Chapter 5 Chapter Summary in the eText.

Section 5.6 On Your Own
1: a) \(x^2 - 2x - 35; b) 16r^2 - 46r + 15; 2: a) r^2 - 16; b) 25a^2 - 49; 3: a) y^2 + 4y + 4;
b) 4a^2 - 20a + 25; 4: a) \(x^2 - 15a^2 + 75a - 125; b) x^2 + 8x + 24x^2 + 32x + 16

Section 5.6 Additional Exercises
For the Section Summary, see Chapter 5 Chapter Summary in the eText.

Section 5.7 On Your Own
1: a) \(10x^4 - 2x^2; b) 6x^4 + 3x^2 - \frac{1}{x} 2: 2x + 1; 3: 2b - 1 - \frac{4}{2h - 1}

e) \(x^2 - x + 1

Section 5.7 Additional Exercises
For the Section Summary, see Chapter 5 Chapter Summary in the eText.

Answers

\[ A-2 \]
Answers

Section 5.8 Your Turn 1: a) $8.0 \times 10^{-3}$; b) $2.45 \times 10^{7}$; 2: a) 9,520,000; b) 0.000000079; 3: a) $2.3 \times 10^{3}$; b) $2.226 \times 10^{-4}$; 4: $0.0000053$ oz; 5: $1.5 \times 10^{-3}$ g

Section 5.8 On Your Own 1: a) $5.0 \times 10^{-4}$; b) $2.1 \times 10^{6}$; c) $6.3 \times 10^{-2}$; 2: a) 0.000056; b) 0.000001002; c) 910,000; 3: a) $8.0 \times 10^{-3}$; b) $4.0 \times 10^{-11}$; c) $2.0 \times 10^{7}$; 4: a) $2.5 \times 10^{7}$ people; b) $1.3125 \times 10^{7}$, or $13,125,000$

Section 5.8 Additional Exercises 1: b); 3: a); 5: b); 7: b); 9: $5.02 \times 10^{6}$; 11: $9.8 \times 10^{-3}$; 13: $6.03 \times 10^{3}$; 15: $9.05 \times 10^{-3}$; 17: 310,000; 19: 0.000000618; 21: 117,000; 23: 0.0511; 25: $7.35 \times 10^{2}$; 27: $4.5 \times 10^{-11}$; 29: $7.2 \times 10^{-2}$; 31: $4.0 \times 10^{-4}$; 33: $4.0 \times 10^{4}$; 35: $1.84 \times 10^{5}$; 37: $7.0 \times 10^{5}$; 39: $2.83 \times 10^{-3}$ kg; 41: $4.0 \times 10^{7}$ m; 43: 309,000,000 people; 45: 0.00000000753 kg; 47: 300,000,000 m/s;
49: $1.25 \times 10$ hours; 51: $3.84 \times 10^{7}$; 53: $\frac{1}{375m^2}$; 55: $-2x^2 - 7x + 8$; 57: $6x - 10$; 59: $6r^2 - 32r + 42$; 61: $5y^2 + y$; 63: 0.000000618

Section 5.9 Your Turn 1: 200 units per day; 2: $3x^3 + 2x^2 + 5x$; 3: $2x^2 + 21x + 27$; 4: $2x + 4$

Section 5.9 On Your Own 1: 37 g; 2: $4x^2 + 8x + 8$; 3: a) $4x^2 + 17x + 15$; b) $x^2 + 10x + 25$; 4: $2x - 3 + \frac{10}{x + 3}$

Section 5.9 Additional Exercises For the Section Summary, see Chapter 5 Chapter Summary in the eText.
1. c); 3: b); 5: a); 7: b); 9: about 36,000 households; 11: $204$; 13: $96$ ft; 15: $33.8$%; 17: $251.2$ cm$^2$; 19: $12x^2 + 8x$; 21: $(6.28r^2 + 12.56\pi)$ sq units; 23: 10 ft; 25: $x^3 + 4x^2 + 4x$; 27: $(-16r^2 + 48\pi)$ ft; 29: $4x^2 + 20x + 25$; 31: $100\pi^2 + 200\pi + 100$; 33: $(2x + 4)$ m; 35: $(x + 5)$ in.; 37: $\frac{27x^2}{\pi}$; 39: $9x^2 - 3x - 1$; 41: $-3x - 15$; 43: $18r^2 - 21r + 5$; 45: $15y^3 - 2y^2$;
47: 0.0000000705; 49: 125 units per day